Estimates of infrared intersubband emission and its angular dependence in GaAs/AlGaAs multiquantum well structures

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This letter describes results of a modeling analysis of IR radiative efficiency for cascade-type quantum well emitter structures, and the angular dependence of spontaneous emission. The radiative decay rates are calculated for different IR wavelength ranges. Estimates of radiative efficiency indicate that the performance of these cascade mode devices in the long- and midwavelength infrared range can be comparable to or superior to that reported at far-infrared range. Based upon calculations of the angular dependence of IR emission, an etched surface grating structure is proposed which should lead to high effective coupling efficiencies.

Recent studies by Helm *et al.*[1] have demonstrated the use of intersubband radiative transitions in GaAs/AlGaAs superlattice structures for the generation of radiation at wavelengths in the far-infrared (FIR) range. They also concluded that radiative transitions from subbands lying at energies higher than the optical phonon energy would be relatively weak due to the dominance of nonradiative processes, which suggests that the use of such structures as infrared sources in the long-wavelength infrared (LWIR) or midwavelength infrared (MWIR) ranges is not likely to be successful. In view of the current technological interest in these shorter wavelength sources, we have undertaken a reevaluation of this situation. A careful consideration in fact suggests that relatively high-efficiency radiation sources in the LWIR and MWIR ranges should be feasible. A key factor in determining overall source characteristics is also the strong angular dependence of emitted radiation. In this letter we discuss approaches for optimizing emission at specific wavelengths through the use of etched surface grating structures.

We first examine the quantum efficiency of the Cascade process, which is the number of photons produced for one electron injected into an excited subband of a well. There are three channels for the electron: (a) radiative relaxation (desired process that produces a photon), (b) nonradiative (NR) process via phonons (ineffective process), and (c) leak by tunneling through the barrier (See Fig. 1).

The radiative transition probability is calculated for spontaneous emission.[2] The Hamiltonian is given by

$$H_{\text{emis}} = -\frac{e}{m}\left(\frac{\hbar(n_{q\lambda}+1)}{2V\epsilon\omega}\right)^{1/2} e^{-iq\cdot r}\varepsilon^{(\lambda)}\cdot p, \qquad (1)$$

where e is the electronic charge, $n_{q\lambda}$ is the number of photons of the type emitted that are present initially, V is the volume of the effective superlattice region, ϵ is the optical permittivity, ω, q, and $\varepsilon^{(\lambda)}$ are, respectively, the angular frequency, the wave vector, and the unit polarization vector of the emitted photon, and m and p are the effective mass and momentum of the electron. In the superlattice the wave function of an electron can be written as $\Psi(r) = e^{ik_x x + ik_y y}u(z)/\sqrt{S_{xy}}$, where $u(z)$ is the wave function in the z direction (the growth direction), S_{xy} is the area of the quantum well, k is the electron wave vector. After some mathematical manipulation we obtain the T matrix element as

$$T \sim \delta^{(2)}(k_{f\perp}+q_\perp-k_{i\perp})(-i\hbar\varepsilon_z^{(\lambda)}) \int u_f^*(z)\frac{\partial}{\partial z}u_i(z)dz$$
$$\propto \varepsilon_z^{(\lambda)}(q), \qquad (2)$$

i.e., the transition amplitude for the emission of a photon with momentum q is proportional to the coefficient of the z component of the polarization vector of the photon. From the symmetry around the z axis we can pick two independent polarization directions as in the inset of Fig. 2. From the figure it is apparent that $\varepsilon^{(2)}$ does not have any z component and $\varepsilon_z^{(1)} = \cos\theta$. Therefore light produced is always in the TM mode. There is no restriction to emission on the azimuthal angle, but as will be explained later, the emission of a photon with $0<|\theta|\leqslant\theta_m$, with θ_m an angle that depends on device parameters, is forbidden. From the Golden Rule the total radiative decay rate is then given by

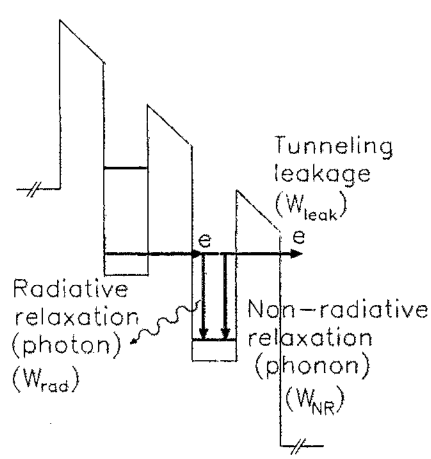

FIG. 1. Illustration of the three available channels for the electrons injected into an excited subband of a well.

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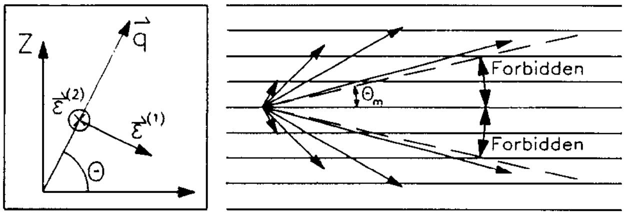

FIG. 2. Intensity distribution of light produced in a superlattice and illustration of the forbidden emission region. Intensities are scaled uniformly and indicated by arrows. Inset: Polarization directions in the superlattice.

$$W_{\rm rad} = \frac{e^2\omega^2}{12\pi\epsilon m v^3}(2 - 3\sin\theta_m + \sin^3\theta_m)f, \qquad (3)$$

where v is the velocity of light in the superlattice and $f = (2m\omega/\hbar)|\langle\Psi_f|z|\Psi_i\rangle|^2$ is known as the oscillator strength.

Using this formula we numerically evaluate the total radiative decay time for three device structures with different parameters aiming for MWIR ($\sim 4\ \mu$m) emission, LWIR ($\sim 10\ \mu$m) emission, and also the FIR ($\sim 100\ \mu$m) emission that has already been observed by Helm *et al.*[1] The structures we consider here consist of 50, 80, 350 Å wells and 100, 120, 100 Å barriers for the three wavelengths, respectively, with undoped wells. In this evaluation the small degree of barrier penetration of wave functions has been taken into account to improve the accuracy. The results are summarized in the first column of Table I. We note here that the radiative decay rate $W_{\rm rad}$ increases almost quadratically with the frequency of light ω. The results do not support Helm's conjecture that the cascade type of light source may only be possible in the FIR region, since he considers only the nonradiative relaxation effects. But at MWIR or LWIR wavelengths the radiative relaxation will also be faster than that at FIR wavelengths, compensating the LO-phonon processes. Hence one should consider the relative efficiency in determining the probability of emission.

Now let us consider the nonradiative decay processes, mainly phonon processes. For this, we refer to the recent experimental work. Tatham *et al.*[3] measured the relaxation time of $\leqslant 1$ ps in 146 Å GaAs quantum wells at 30 K, which is close to the theoretical estimate.[4] The structure we consider here for the 10 μm emission resembles Tatham's case and we choose ~ 1 ps for the LO-phonon relaxation time. We could not find any experimental data for nonradiative decay rate ($W_{\rm NR}$) for the 4.0 μm emission sample (~ 310 meV transition). Since the transition energy is 2.5 times bigger than for the 10 μm emission sample, we assume that the decay rate will be roughly two times slower. (Higher order processes are highly unlikely compared to the lower order ones.) When the energy gap becomes less than the LO-phonon energy (36 meV), the nonradiative relaxation is dominated by the acoustic phonon process. Recently Levenson *et al.*[5] found that the relaxation rate decreases with decreasing transition energy, but in all cases they investigated up to a well width of 240 Å, it was faster than 40 ps. Also they did not see any abrupt slowing down when the intersubband separation is varied through the LO-phonon energy. For the 350 Å Helm's sample we still choose ~ 100 ps[6] for the nonradiative relaxation time, even though there is an indication that it might be lower.[7] These values are summarized in Table I. We use the Wentzel–Kramers–Brillouin (WKB) method to find the tunneling time associated with the leak process ($W_{\rm leak}$), which is usually greater than 100 ps. Since these values are much greater than the nonradiative decay time, we can write the quantum efficiency in terms of relaxation rates associated with the three processes as

TABLE I. Comparison of relative efficiency for various device parameters.

Wavelength	Rad. decay	Nonrad. decay	Rel. Efficiency
111 μm (Helm's)	21 μs	$\sim$100 ps (Acoustic)	1
10 μm	0.20 μs	$\sim$1 ps (Optical)	1
4 μm	0.06 μs	$\sim$2 ps (Optical)	7

$$\eta = \frac{W_{\rm rad}}{W_{\rm rad} + W_{\rm NR} + W_{\rm leak}} \simeq \frac{W_{\rm rad}}{W_{\rm NR}}, \qquad (4)$$

and the relative quantum efficiency for each sample with respect to FIR case is again given in Table I. As is clear from the table, the quantum efficiency for LWIR and MWIR Cascade processes is comparable to or even greater than the FIR case, contradicting the concern by others.[1,8]

Next we consider the task of controlling the propagation of light efficiently. It is known that spontaneous emission is affected by the source's local environment: light is only emitted in a way that goes with the given boundary conditions.[9] One of the standard approaches to this problem is to expand the electromagnetic field in appropriate mode functions that satisfy the given boundary conditions (photon wave functions). This is equivalent to the quantum mechanical problem of finding the wave functions of a particle when a potential is given: for the EM field the tangential components of E and H and the normal components of D and B should be continuous across the boundary when there is no surface charge σ or surface current density j. In certain cases these conditions reduce to the continuity of one field component and its first derivative, as is for the case of quantum mechanical wave function of a particle.[10] Since the superlattice consists of repeating layers of a barrier and a well, the problem becomes very similar to the Kronig–Penney–type potential problem with position-dependent electron mass.[11,12] Yeh *et al.*[10] found that there are forbidden bands for the propagation of light in a stratified media. The condition for these prohibited bands is that the Bloch wave vector Q defined by

$$Q(q_\perp,\omega) = (1/\Lambda)\cos^{-1}\left[\tfrac{1}{2}(A+D)\right], \qquad (5)$$

with

$$A = e^{iq_{wz}a}\left[\cos q_{bz}b + \frac{i}{2}\left(\frac{n_b^2 q_{wz}}{n_w^2 q_{bz}} + \frac{n_w^2 q_{bz}}{n_b^2 q_{bz}}\right)\sin q_{bz}b,\right.$$

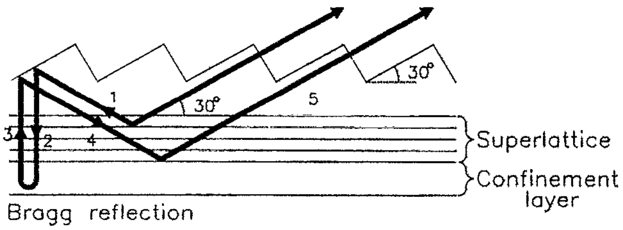

FIG. 3. Prototype surface grating structure that optimizes the extraction of the beam produced inside a superlattice, taking the angular dependence into account.

$$D=e^{-iq_{wz}a}\left[\cos q_{bz}b-\frac{i}{2}\left(\frac{n_b^2 q_{wz}}{n_w^2 q_{bz}}+\frac{n_w^2 q_{bz}}{n_b^2 q_{bz}}\right)\right]\sin q_{bz}b \tag{6}$$

becomes imaginary. In the above

$$q_{iz}=\left[\left(\frac{n_i\omega}{c}\right)^2-q_1^2\right]^{1/2},\quad i=b \text{ or } w, \tag{7}$$

a, b are the thicknesses of the well and barrier, respectively, and $\Lambda = a + b$. This condition is satisfied when $|(A+D)/2|>1$, and spontaneous emission cannot occur in this regime. Numerical calculation of Eq. (5) with the 10 μm emission device parameters ($x\sim0.3$, $a = 80$ Å, $b = 120$ Å) yields emission with $0<|\theta|\leqslant 13°$ is not allowed. We can now summarize the results derived so far in a real scale, in 15° steps in Fig. 2. Incorporating these considerations, a prototype surface grating structure that will optimize the extraction of the produced beam in the superlattice structure is suggested in Fig. 3. In order to optimize the emitted intensity at a selected wavelength, we may also choose a grating period from Michelson's echelon-type condition so that the path difference between successive steps is a multiple of 2π.

In conclusion, our calculations indicate that the relative increase in radiative relaxation processes in the shorter IR wavelength range should lead to a relative efficiency for emission which is comparable to or exceeds the performance thus far obtained at FIR wavelengths. It is also found that optimum MWIR or LWIR emission should occur in an angular range of about 15–45° from the superlattice layers, and that when used in conjunction with a properly designed etched surface grating this effect can lead to additional performance advantages.

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