Physics 8510

Problem Set 8

1) GaAs has a conduction band that can be described near the band edge by an expression in the form $E(k) = \hbar^2 k^2 / 2m^*$, where m * = 0.067 m_0. An electric field of 10 kV/cm (along the x-axis is applied at time t = 0 to an electron at the bottom of the conduction band. What is the energy of the electron after a time interval of 1, 2, 10 picoseconds. (Band structure is periodic in the reciprocal lattice space. If the electron exceeds the zone edge k value, it will undergo a reflection by a RLV-Reciprocal Lattice Vector.)

2) Consider a square lattice in 2-d with a background potential $U(x, y) = U_0 \cos 2\pi\ x/a\ \ \cos 2\pi\ y/a$, where U_0 =2.0 eV. Set up the 2x2 secular equation to calculate the bandgap at the point $(\pi / a, \pi / a)$ of the Brillouin zone. (Hint: Potential energy contains the 4 reciprocal lattice vectors)

3) In GaAs, once the channel dimensions are less than 1000 Å, much of the transport occurs without scattering. In such ballistic transport, the electron climbs up along the E vs K diagram in accordance with the effective Newton's equation of motion.

Assume a parabolic band relation $E = \hbar^2 k^2 / 2m^*$ with m* = 0.067 m_0. Calculate the electron transit time and final energy for a transit length of 0.1μm and an electric field of 10^3 V/cm and 10^5 V/cm.

4) Consider a 1-d solid of length L = Na made up of N diatomic molecules, the interatomic spacing within a molecule is b (b<a/2). The centers of adjacent molecules are a distance a apart. Potential energy can be represented as a sum of delta functions centered on each atom with A a positive quantity and n = 0, 1, 2, ………N-1.(shown below)

$$V = -A \sum_{n=0}^{N-1} [\delta(x-na+b/2) + \delta(x-na-b/2)]$$

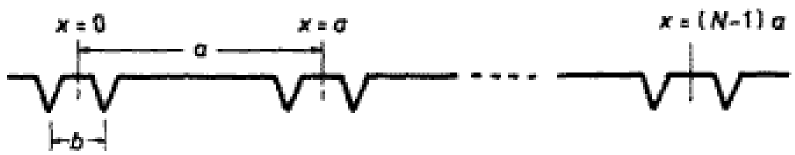
(a) Consider free electrons (V = 0) in the solid and periodic boundary conditions. Derive the allowed values of the electron wave vectors k, and normalize the wave function.

(b) Expressing the potential as a Fourier series $V = \sum V_G e^{iGx}$, find the allowed values of G and the coefficients V_G.

(c) Assuming A to be small, show that for certain values of k there are energy gaps. Derive a formula for the gaps and show in particular that the gap energy at the top of the first zone is proportional to $\cos(\pi b / a)$. [Hint: use the result of (b)]

(d) Derive an expression for the number of states that are in the first zone. If each atom has one electron, will the substance be a conductor or an insulator? (Hint: Each state corresponds to 2 spin values. N diatomic molecules give 2N atoms each atom contributes one electron. If the first zone states are not completely full it will be a conductor, if completely full an insulator.)

(e) If b= a/2, how will the answers (a-d) change?



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