Radiation by moving charges

Liénard - Wiechert potential:

For current and charge density produced by a charge q in motion we had:

$$\rho(\vec{x},t) = q \cdot \delta(\vec{x} - \vec{x}(t))$$
$$\vec{j}(\vec{x},t) = q \cdot \dot{\vec{x}}\, \delta(\vec{x} - \vec{x}(t))$$

if $\vec{x}(t)$ is the position of the particle at time t and $\dot{\vec{x}} = \frac{d\vec{x}(t)}{dt}$ = velocity $\vec{v}$

In four-vector notation:

$$\boxed{j^\mu(\vec{x},t) = q \cdot c \cdot \beta^\mu\, \delta(\vec{x} - \vec{x}(t))}\,; \quad \beta^\mu = (1, \beta), \quad \beta = \vec{v}/c$$

From $\vec{j}$ and ρ $\rightsquigarrow$ find vector potential $\vec{A}$ and scalar potential ϕ!

- use retarded Green's function

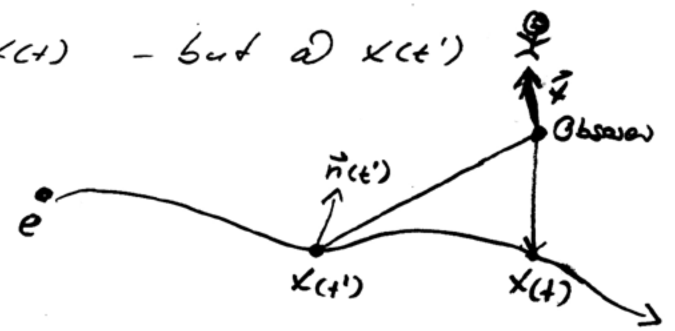
$$G(\vec{x},t,\vec{x}',t') = \frac{\delta\left(t - t' - \frac{|\vec{x}-\vec{x}'|}{c}\right)}{|\vec{x}-\vec{x}'|}$$

$$\rightsquigarrow A^\mu(\vec{x},t) = \int d^3\vec{x}'\,dt'\, G(\vec{x},t,\vec{x}',t') \cdot j^\mu(x',t')$$

$$= \dots = q \int dt'\, \beta^\mu(t')\, \frac{\delta\left(t - t' - \frac{|\vec{x}-\vec{x}'|}{c}\right)}{|\vec{x} - \vec{x}(t')|}$$

$\Longrightarrow$

This form of A^μ reflects the retarded nature of the problem, since a particle is not seen by the observer at its present location $x(t)$ - but at $x(t')$



To evaluate the integral, rewrite the integral in the form

$$\int dt'\, f(t')\, \delta[g(t')] \stackrel{!}{=} f(t_0) \Big/ \left|\frac{dg}{dt'}\right|_{t_0}$$

with $g(t') = t' + \frac{|x - x(t')|}{c} - t = t' + \frac{\sqrt{(x - x(t'))\cdot(x - x(t'))}}{c} - t$

$$\leadsto \frac{dg}{dt'} = 1 - \vec{\beta}(t')\cdot\hat{n}(t') ,$$

$\hat{n}$: unit vector pointing from particle path toward field point $\vec{x}$.

β must be evaluated at time t', which is earlier than t - the time at which the field is evaluated!

(*) $\qquad t = t' + \frac{1}{c}\cdot|x - x(t')|$

$$\hookrightarrow \boxed{A^\mu(\vec{x},t) = q\cdot\left[\frac{\beta^\mu}{|x - x(t)|\cdot(1-\vec{\beta}\cdot\hat{n})}\right]_{ret} = \left[\frac{q\cdot\beta^\mu}{R\cdot K}\right]_{ret}}$$

Jackson 14.8

= Liénard-Wiechert potentials

$R \equiv |x - x(t')|$, $K = 1 - \vec{\beta}\cdot\hat{n}$

$[\ldots]_{ret}$: means that the quantity in bracket has to be evaluated at the retarded time t' determined by (*).

To find the EM field, we have to take derivatives of R with respect to various components of $\vec{x}$.

Start with $\vec{E}$:

$$E(\vec{x},t) = -\nabla\phi(\vec{x},t) - \frac{1}{c}\frac{\partial A(\vec{x},t)}{\partial t}$$

$$= -q\int dt'\,\hat{n}\cdot\frac{d}{dR}\left(\frac{\delta(t'+R/c-t)}{R}\right) + \frac{q}{c}\int dt'\cdot\frac{\vec{\beta}\cdot\delta'(t'+R/c-t)}{R}$$

$$= q\int dt'\left[\hat{n}\,\frac{\delta(t'+R/c-t)}{R} + \frac{1}{c\cdot R}(\vec{\beta}-\hat{n})\,\delta'(t'+R/c-t)\right]$$

$$= q\left[\frac{\hat{n}}{k\cdot R^2}\right]_{ret} - \frac{q}{c}\left[\frac{1}{k}\frac{\partial}{\partial t'}\left(\frac{\vec{\beta}-\hat{n}}{k\cdot R}\right)\right]_{ret.}$$

The expression involves time derivatives of retarded quantities, which we have to work out:

$\hookrightarrow$ $\dfrac{\partial\vec{R}}{\partial t'} = -\dot{\vec{x}} = -\vec{\beta}\cdot c$, $\quad\vec{R}$ is defined via $\hat{n} = \dfrac{\vec{R}}{R}$

and

$$\frac{\partial R}{\partial t'} = \frac{\partial\sqrt{[\vec{x}-\vec{x}(t')]\cdot[\vec{x}-\vec{x}(t')]}}{\partial t} = \frac{1}{2R}\left[-2\cdot(\vec{x}-\vec{x}(t'))\cdot\dot{\vec{x}}(t')\right] = -\hat{n}\cdot\vec{\beta}\cdot c$$

Hence, $$\frac{1}{c}\frac{d\hat{n}}{dt'} = \frac{1}{c\cdot R}\cdot\frac{\partial\vec{R}}{\partial t'} - \frac{1}{c\cdot R^2}\cdot\vec{R}\cdot\frac{\partial R}{\partial t'} = -\frac{1}{R}\left[\vec{\beta}-(\hat{n}\cdot\vec{\beta})\,\hat{n}\right]$$

$$= \frac{1}{R}\,\hat{n}\times(\hat{n}\times\vec{\beta})$$

$\Rightarrow$ $$E(\vec{x},t) = \ldots = q\left[\frac{\hat{n}-\vec{\beta}}{k^2\cdot R^2} + \frac{\hat{n}}{c\cdot k}\cdot\frac{\partial}{\partial t'}\left(\frac{1}{k\cdot R}\right) - \frac{1}{c\cdot k}\frac{\partial}{\partial t}\left(\frac{\vec{\beta}}{k\cdot R}\right)\right]_{ret}$$

$\Rightarrow$

The expression for the magnetic induction can be found by similar manipulation:

$$\underline{\vec{B}(\vec{x},t)} = \dots = q\left[\frac{\vec{\beta}\times\hat{n}}{\kappa\cdot R}\right]_{ret} + \frac{q}{c}\left[\frac{1}{\kappa}\frac{\partial}{\partial t'}\left(\frac{\vec{\beta}}{\kappa R}\right)\times\hat{n}\right]_{ret}$$

$$+ q\left[\frac{1}{\kappa^2 R^2}\left[\hat{n}\left((\hat{n}\times\vec{\beta})\cdot\vec{\beta}\right) - (\hat{n}\times\vec{\beta})\cdot(\hat{n}\cdot\vec{\beta})\right]\right]_{ret}$$

Comparing this with the expression for $\vec{E}(\vec{x},t)$, we see that

$$\boxed{\vec{B}(\vec{x},t) = [\hat{n}]_{ret}\times\vec{E}(\vec{x},t)}$$

Thus we can find $\vec{B}(\vec{x},t)$ easily provided we can find $E(\vec{x},t)$!

To have an explicit expression for $\vec{E}$ with __no__ time derivatives, we need to evaluate

$$\frac{\partial}{\partial t'}\left(\frac{1}{\kappa\cdot R}\right) = -\frac{1}{\kappa^2\cdot R^2}\left[\kappa\cdot(-\hat{n}\cdot\vec{\beta}c) + R\cdot\left(-\hat{n}\cdot\frac{\partial\vec{\beta}}{\partial t'}\right) - R\cdot\vec{\beta}\cdot[(\hat{n}\cdot\vec{\beta})\cdot\hat{n} - \vec{\beta}]\cdot\frac{c}{R}\right]$$

$$= \dots = -\frac{c}{\kappa^2 R^2}\left[\beta^2 - \hat{n}\cdot\vec{\beta} - \frac{R}{c}\cdot(\hat{n}\cdot\dot{\vec{\beta}})\right]$$

$$\hookrightarrow E(\vec{x},t) = q\left[\frac{\hat{n}-\vec{\beta}}{\kappa^2 R^2} + \frac{n}{c\cdot\kappa}\cdot\left(\frac{-c}{\kappa^2 R^2}\right)\cdot\left(\beta^2 - \hat{n}\cdot\vec{\beta} - \frac{R}{c}\,\hat{n}\cdot\dot{\vec{\beta}}\right) - \frac{\dot{\vec{\beta}}}{c\kappa^2 R}\right]_{ret}$$

$$-q\cdot\left[\frac{\vec{\beta}}{c\cdot\kappa}\left(\frac{-c}{\kappa^2 R^2}\right)\left(\beta^2 - \hat{n}\cdot\vec{\beta} - \frac{R}{c}\,\hat{n}\cdot\dot{\vec{\beta}}\right)\right]_{ret}$$

$$= q\left[\frac{(\hat{n}-\vec{\beta})(1-\beta^2)}{\kappa^3 R^2}\right]_{ret} + q\left[\frac{1}{c\kappa^3 R}\left\{(\hat{n}-\vec{\beta})(\hat{n}\cdot\dot{\vec{\beta}}) - (1-\hat{n}\cdot\vec{\beta})\cdot\dot{\vec{\beta}}\right\}\right]_{ret}$$

$\Rightarrow$

~> The second bracket contains the quantity

$$(\hat{n}-\vec{\beta})(\hat{n}\cdot\dot{\vec{\beta}}) - \hat{n}\cdot(\hat{n}-\vec{\beta})\,\dot{\vec{\beta}} = \hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]$$

$$\boxed{\vec{E}(\vec{x},t) = q\left[\frac{(\hat{n}-\vec{\beta})(1-\beta^2)}{\kappa^3 R^2}\right]_{ret} + \frac{q}{c}\left[\frac{\hat{n}\times\{(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\}}{\kappa^3 R}\right]_{ret}} \quad (*)$$

If there is <u>no</u> acceleration ~> $\dot{\vec{\beta}} = 0$ and only the first term in $\vec{E}$ is finite! This terms falls with $1/R^2$ ~> can not give rise to net flux of radiation.

If there is acceleration: The second term of $\vec{E}$ (and the corresponding $\vec{B}$-term) are finite. These terms fall of a $1/R$ and hence give rise to radiation, meaning that the charged particle will emit radiation <u>only</u> if it is <u>accelerated</u>.

From these results when $\dot{\vec{\beta}} = 0$ we should recover the results for the fields of a uniformly moving charge - see <u>Chapt. 11, Jackson 11.152</u>

With a particle moving along x-direction with const. velocity $\vec{v}$, the fields felt by an observer a distance b away are

$$E_{\parallel} = -\gamma\cdot q\, v\cdot t\,\left[b^2 + (\gamma\cdot v\cdot t)^2\right]^{3/2}$$

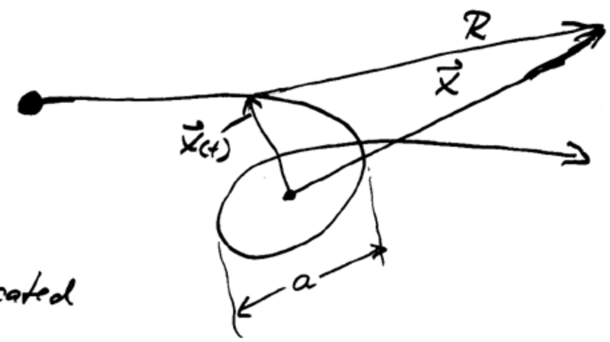
$$E_{\perp} = \gamma\cdot q\cdot b\cdot\frac{1}{\left[b^2+(\gamma\cdot v\cdot t)^2\right]^{3/2}}$$

Where observer and particle are closest at time $t = 0$.

Larmor Formula for radiation from accelerated charge

Even with a explicit form for $\vec{E}(\vec{x},t)$ and $\vec{B}(\vec{x},t)$ the calculation of the emitted radiation is non-trivial, largely due to the effect of retardation. The problem greatly simplifies if the particle is not moving <u>too</u> fast.

Let's consider the trajectory of a particle shown on the side: →



- We will assume that the origin is located in the center of the region of interest.
- region is of linear dimension a (example: e^- bound to atom ⇝ a = Bohr radius)

There are two way in which the problem simplifies in the nonrelativistic limit $\beta \ll 1$.

First, we may approximate $\quad K \approx 1 \,,\ \hat{n}-\vec{\beta} \approx \hat{n}$

$$1-\beta^2 \approx 1$$

Second, and more importantly, we can approximate

$$f\left(t-\frac{|\vec{x}-\vec{x}(t')|}{c}\right) \approx f\left(t-\frac{r}{c}\right) + x(t')\cdot\nabla f\left(t-\frac{r}{c}\right) + \dots \quad \Big\lceil r=|\vec{x}|$$

for $\beta \ll 1$, the second term in the series can be neglected

$$\rightsquigarrow f\left(t-\frac{|\vec{x}-\vec{x}(t')|}{c}\right) \approx f(t-r/c)$$

⇒

$\rightsquigarrow$ This approximation is sometimes call dipol approximation.

$\Rightarrow$ with approximations $\rightsquigarrow$ elect. field becomes

$$E_{rel}(\vec{x},t) = e\left[\frac{\hat{n}}{R^2}\right]_{ret} + \frac{e}{c}\cdot\left[\frac{\hat{n}\times(\hat{n}\times\dot{\vec{\beta}})}{R}\right]_{ret} \qquad \Big|\ e = q$$

The Poynting vector is given by

$$\vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{B}) = \frac{c}{4\pi}(\vec{E}\times([\hat{n}]_{ret}\times\vec{E}) = \frac{c}{4\pi}\left\{[\hat{n}]_{ret}\,\vec{E}\cdot\vec{E} - \vec{E}([\hat{n}]_{ret}\cdot\vec{E})\right\}$$

If we take the limit of large R and just keep the radiation terms $\rightsquigarrow$

$$\vec{E}(\vec{x},t) = \frac{e}{c}\left[\frac{\hat{n}(\hat{n}\cdot\dot{\vec{\beta}}) - \dot{\vec{\beta}}}{R}\right], \quad \vec{B}(\vec{x},t) = [\hat{n}]_{ret}\times\vec{E} = \left[\frac{-e}{c\cdot R}\cdot\hat{n}\times\dot{\vec{\beta}}\right]_{ret}$$

Note: $\hat{n}\cdot\vec{E} = \hat{n}\cdot\vec{B} = 0$ $\rightsquigarrow$ radiation is transverse

Poynting vector: $\vec{S} = \dfrac{e^2}{4\pi R^2\cdot c}\left[\hat{n}\cdot\left|\hat{n}\times(\hat{n}\times\dot{\vec{\beta}})\right|^2\right]_{ret}$

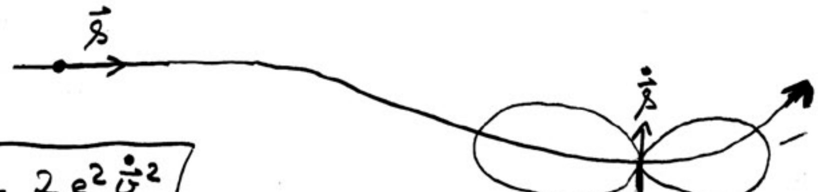
Angular distribution of radiated Power:

$$\frac{dP}{d\Omega} = R^2(\vec{S}\cdot\hat{n}) = \frac{e^2}{4\pi c}\cdot\dot{\vec{\beta}}^2\sin^2\theta = \frac{e^2}{4\pi c}(\dot{\vec{v}}\sin^2\theta)$$

where θ is the angle between $\hat{n}$ and $\dot{\vec{\beta}}$ at the retared time.

total Power:

$$\boxed{P = \int d\Omega\,\frac{dP}{d\Omega} = \frac{2e^2\dot{\vec{v}}^2}{3\cdot c^3}}$$



radiation pattern is characteristic of dipol radiation

Relativistic Larmor Formula

For the relativistic generalization of the Larmor formula, we determine how power transforms under Lorentz transformation.
Since the rate at which energy crosses a closed surface surrounding a particle depends on retardation, we calculate the power as function of t' rather than t. Consider a surface S that encloses the particle at all times during which it is radiating. The power crossing unit area at $\vec{x}$ at time t is $\vec{S}(\vec{x},t)\cdot\hat{n}$ ($\hat{n}$ pointing outwards), and the total energy crossing this unit area is

$$W = \int_{-\infty}^{\infty} dt\, S(\vec{x},t)\cdot\hat{n}$$

transformed to the retarded time t':

$$W = \int_{-\infty}^{\infty} dt' \frac{dt}{dt'} \vec{S}(\vec{x},t(t'))\cdot\hat{n} = \int_{-\infty}^{\infty} dt' \underbrace{\kappa\left[\vec{S}(\vec{x},t(t'))\cdot\hat{n}\right]}_{= dW/dt'}$$

$\frac{dW}{dt'}$ is the instantaneous radiated power, which if multiplied by R^2

becomes $\quad \frac{dP(t)}{d\Omega} = R^2 \frac{dW}{dt} = R^2\cdot \kappa\, \vec{S}\cdot\hat{n}$

If R is large enough that only the radiation field has to be retained

$$\rightsquigarrow \frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c}\cdot \left.\frac{\{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]\}^2}{(1-\hat{n}\cdot\vec{\beta})^5}\right|_{t'} = \frac{e^2}{4\pi c} \left.\frac{\{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]\}^2}{\kappa^5}\right|_{t'}$$

Radiated intensity at time t depends on behavior of particle at time t'! The differential time elements are different as well $\Rightarrow$

$\rightsquigarrow$ with $dt = dt' \cdot (1 - \hat{n}\cdot\vec{\beta})_{ret}$

The total power radiated is given by integration over directions:

$$P(t') = \frac{e^2}{4\pi c} \int \frac{d\Omega}{k^5} \left\{ \hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}] \right\}^2$$

$$= \frac{e^2}{4\pi c} \int \frac{d\Omega}{k^5} \cdot \left\{ (1 - 2\hat{n}\cdot\vec{\beta} + \vec{\beta}^2)(\hat{n}\cdot\dot{\vec{\beta}})^2 - 2(\hat{n}\cdot\dot{\vec{\beta}} - \vec{\beta}\cdot\dot{\vec{\beta}})(\dot{\vec{\beta}}\cdot\hat{n})\cdot(1 - \hat{n}\cdot\vec{\beta}) + \dot{\vec{\beta}}^2 \cdot (1 - 2\vec{\beta}\cdot\hat{n} + (\vec{\beta}\cdot\hat{n})^2 \right\}$$

denote: $\theta :=$ angle between $\vec{\beta}$ and $\hat{n}$

$u = \cos\theta$

$\theta_0 :=$ angle between $\vec{\beta}$ and $\dot{\vec{\beta}}$

$$\hookrightarrow \quad P(t') = \frac{\dot{\beta}^2 e^2}{2 \cdot c} \int_{-1}^{1} \frac{du}{(1-\beta u)^5} \cdot \left\{ (1-u^2) + \sin^2\theta_0 \cdot \left[\left(\frac{3}{2} + \frac{1}{2}\beta^2\right) u^2 - 2\beta u - \frac{1}{2}(1-\beta^2) \right] \right\}$$

next introduce $x \equiv 1 - \beta u$

$$\hookrightarrow \quad P(t') = \frac{\dot{\beta}^2 e^2}{2c} \cdot \int_{1-\beta}^{1+\beta} \frac{dx}{x^5} \left\{ (\beta^2 - 1 + 2x - x^2) + \sin^2\theta_0 \left[\frac{(3+\beta^2)(1-2x+x^2)}{2} + x(\beta^2+2) + \frac{x^2}{2}(3+\beta^2) \right] \right\}$$

$$= \ldots = \frac{\dot{\beta}^2 e^2 \gamma^6}{c\,\beta^3} \left\{ \frac{2}{3}\beta^3 - \frac{2}{5}\beta^5 \sin^2\theta_0 \right\} = \frac{2\dot{\beta}^2 e^2 \gamma^6}{3c}(1 - \beta^2 \sin^2\theta_0)$$

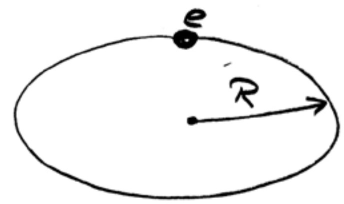
Putting result back in terms of vectors and their products

$$\hookrightarrow \quad \boxed{P = \frac{2e^2\gamma^6}{3c} \cdot \left[\dot{\vec{\beta}}^2 - (\vec{\beta} \times \dot{\vec{\beta}})^2 \right]}$$

relativistic generalization of Larmor formula

Example: Synchrotron

An electron moves in a circle of radius R at constant speed.



The acceleration is entirely centripetal and has the magnitude $\dot{\beta} = \frac{c \cdot \beta^2}{R}$

$$\hookrightarrow \dot{\beta}^2 - (\vec{\beta} \times \dot{\vec{\beta}})^2 = \frac{c^2 \beta^4}{R^2}(1 - \beta^2) = \frac{c^2 \beta^4}{R^2 \gamma^2}$$

and so $\boxed{P = \frac{2e^2 c \cdot \gamma^4 \beta^4}{3R^2}}$ which can be reformulated in terms of the particle's kinetic energy as

$$P = \frac{2e^2 c}{3R^2}\left(\frac{E}{mc^2}\right)^4 \beta^4$$

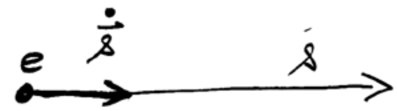
The energy emitted per cycle of the motion is $\Delta E = P \cdot \tau$, where τ is the period of motion: $\tau = \frac{2\pi R}{\beta \cdot c}$

$$\rightsquigarrow \Delta E = \frac{4\pi e^2 \beta^3}{3R}\left(\frac{E}{mc^2}\right)^4$$

An electron of energy $E = 500$ MeV in a synchrotron of radius $R = 10^2$ cm will radiate in each cycle and energy $\Delta E \approx 10^4$ eV, which is a non-trivial loss which has to replenished by an acceleration voltage of at least 10 kV during each cycle in order to break even.
This radiation is the reason why circular very-high-energy e^--accelerators don't exist. However, the synchrotron is a great X-ray source!

Example: Linear Acceleration

An e^- is accelerated in the direction of its velocity



$$\vec{\beta} \parallel \dot{\vec{\beta}}$$

In this instance one trivially find that

$$P = \frac{2 \cdot e^2 \cdot \gamma^6}{3 \cdot c} \cdot \dot{\beta}^2$$

In a linear accelerator, there is much less acceleration required to produce radiation. To show this, let's rewrite the acceleration in terms of the time rate of change of the particle's momentum:

$$\dot{\vec{v}} = \frac{d\vec{v}}{dt} = \frac{1}{m \cdot \gamma^3} \frac{d\vec{p}}{dt}$$

$$\hookrightarrow \quad P = \frac{2e^2}{3m^2c^3} \left(\frac{d\vec{p}}{dt}\right)^2$$

Now relate the rate of change of momentum to the rate of change of energy of the particle:

$$\frac{d\vec{p}}{dt} = \vec{F} = \frac{d\vec{E}}{dx} \quad \leadsto \quad P = \frac{2e^2}{3m^2c^3} \cdot \left(\frac{d\vec{E}}{dx}\right)^2$$

$$\text{or} \quad \frac{P}{dE/dt} = \frac{2e^2}{3m^2c^3} \frac{dE/dx}{dx/dt} = \frac{2}{3\beta} \cdot \frac{(e^2/mc^2)}{mc^2} \cdot \frac{dE}{dx}$$

which says that the power radiated away is quite negligible in comparison with the rate at which energy is pumped into the particle.

Bremsstrahlung: Radiation emitted during Collisions

As discussed before, radiation by acceleration is the general way – for both nonrelativistic and relativistic charged particles – of generation.

__But!__ – You can also decelerate (negative acceleration) to obtain radiation!

↳ __Collision__ and __Inelastic Scattering__

(Jackson Chapter 15)

Nonrelativistic Bremsstrahlung:

Obey energy and momentum conservation for scattering on a fixed (or massive) center of force:

$$E = E' + \hbar\omega$$
$$Q^2 = (\vec{p} - \vec{p}\,' - \vec{k})^2 \approx (\vec{p} - \vec{p}\,')^2$$

where $E = p^2/2M$, $E' = p'^2/2M$ are the kinetic energies

$\hbar\omega$ - energy photon

$\vec{k} = \frac{\hbar\omega\vec{n}}{c}$ - momentum of photon

Q: momentum transfered to scattering center

The radiation cross-section $d\chi/d\omega$ (Jackson 15.29) depends on properties of particles involved in the collision as $\frac{Z^2 \cdot z^4}{M^2}$ / M: mass, charge: $z \cdot e$, Z: # of electrons

→ Total energy loss in radiation by a particle with unit thickness of matter containing N fixed charges $Z\cdot e$ per unit volume

$$\frac{dE_{rad}}{dx} = N \cdot \int_0^{\omega_{max}} \frac{d\chi(\omega)}{d\omega} \cdot d\omega$$

$$= \frac{16}{3} N \cdot Z \cdot \left(\frac{Z \cdot e^2}{\hbar c}\right) \cdot \frac{z^4 e^4}{Mc^2} \int_0^1 \ln\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right) dx$$

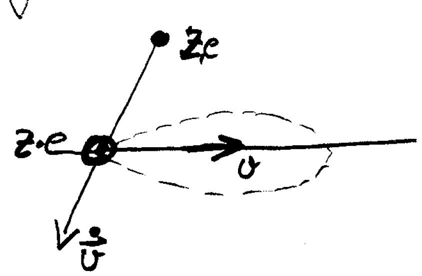
Ratio of radiative energy loss to collision energy loss

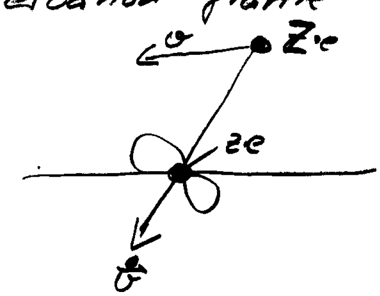
$$\frac{dE_{rad}}{dE_{col}} \simeq \frac{4}{3\pi} z^2 \frac{Z}{137} \frac{m}{M} \left(\frac{v}{c}\right)^2 \frac{1}{\ln B_q} \quad \Big/ \; B_q = \frac{2\gamma^2\beta^2 mc^2}{\hbar \langle\omega\rangle}$$

for $v \ll c$ radiative loss is negligible compared to collision loss!

Relativistic Bremsstrahlung:

Use Lorentz transformation to transform radiation generated in Lab-frame in observation-frame





radiation in Lab-frame K
nucleus in rest!

radiation emitted in frame K'
(incident particle in rest!)

$\rightsquigarrow$ angular distribution of radiation

$$\lim_{\omega \to 0} \frac{d^2 I}{d\omega d\Omega} \simeq \frac{z^2 e^2 \gamma^4}{\pi^2 c} |\Delta \vec{\beta}|^2 \frac{(1 - \gamma^4 \theta^4)}{(1 + \gamma^2 \theta^2)^4} \quad (15.11)$$

with energy radiated per unit frequency interval and per solid angle for $\hbar\omega \ll E$ is

$$\frac{d^2 \gamma_E}{d\omega \, d\Omega_\gamma} \simeq \frac{3}{2\pi} \gamma^2 \frac{(1 + \gamma^4 \theta^4)}{(1 + \gamma^2 \theta^2)^4} \cdot \frac{d\gamma_E}{d\omega}$$

with θ : angle of emission