Phys8500

Solution Homework #3

1.

for reaction 1:
$$\Delta H_f'$$
: $H_2S(g) = -20.5 [KJ \cdot mol^{-1}]$; $Zn(s) = 130.4 [KJ \cdot mol^{-1}]$; $ZnS(s) = -205.9 [KJ \cdot mol^{-1}]$; $H_2(g) = 0 [KJ \cdot mol^{-1}]$;

 \Rightarrow Enthalpy change $\Delta H < 0$

for reaction 2:
$$\Delta H'_f$$
: $H_2S(g) = -20.5 [KJ \cdot mol^{-1}]$; $Cd(s) = 111.8 [KJ \cdot mol^{-1}]$; $CdS(s) = -162 [KJ \cdot mol^{-1}]$; $H_2(g) = 0 [KJ \cdot mol^{-1}]$;

 \Rightarrow Enthalpy change $\Delta H < 0$

So, both reactions are exothermic! An alternative way is the following:

Form
$$\Delta H^0 = \Delta G^0 + T \cdot \Delta S^0$$
 and $\Delta S^0 < 0$ follow that $\Delta H^0 < 0$ if $\Delta G^0 < 0$ Reaction 1:
$$\Delta G^0 = -76400 + 82.1 \cdot T - 5.9 \cdot T \cdot \ln(T) = \\ (@T = 680 ^{\circ}C = 953 \text{K}) = -36728 \text{ [cal/mol]} = -152588 \text{ [J/mol]}$$
 Reaction 2:
$$\Delta G^0 = -50000 + 82.2 \cdot T - 6.64 \cdot T \cdot \ln(T) = \\ (@T = 600 ^{\circ}C = 873 \text{K}) = -17494 \text{ [cal/mol]} = -73256 \text{ [J/mol]}$$

- a) both reactions are exothermic!! If you go extreme high temperature, the reaction may become endothermic Most likely, however, the ΔG reaction equation is not longer valid!
- b) In equilibrium: $\Delta G = \Delta G^0 + \ln \left[K_p \right] = 0;$ with $K_p = \left(\frac{p_{ZnS} \times p_{H_2}}{p_{Zn} \times p_{H_2S}} \right)$ and $K_p = \left(\frac{p_{CdS} \times p_{H_2}}{p_{Cd} \times p_{H_2S}} \right)$, for reaction 1 and 2, respectively.

$$\Rightarrow \frac{p_{ZnS} \times p_{H_2}}{p_{Zn} \times p_{H_2S}} = \frac{p_{H_2}}{p_{Zn} \times p_{H_2S}} = exp \left[\frac{-\Delta G^0}{R \times T} \right] = exp \left[\frac{152588 \text{ [J/mol]}}{8.31 \text{ [J/mol K]} \times 953 \text{K}} \right] = exp \left[19.26 \right] = 2.33 \times 10^8$$
 and
$$\frac{p_{CdS} \times p_{H_2}}{p_{Cd} \times p_{H_2S}} = \frac{p_{H_2}}{p_{Cd} \times p_{H_2S}} = exp \left[\frac{-\Delta G^0}{R \times T} \right] = exp \left[\frac{73256 \text{ [J/mol]}}{8.31 \text{ [J/mol K]} \times 873 \text{K}} \right] = exp \left[10.10 \right] = 2.43 \times 10^4.$$

$$\Rightarrow \log(p_{Zn}) = -\frac{7070}{T} + 9.41 = -\frac{7070}{953} + 9.41 = 1.991 \Rightarrow p_{Zn} = 98.02$$

$$\Rightarrow \frac{p_{H_2S}}{p_{H_2}} = \left[\frac{1}{2 \cdot 33.10^8.98.02}\right] = 4.37.10^{-11}$$

Similarly for reaction 2,

$$\Rightarrow \log(p_{Cd}) = -\frac{5940}{T} + 9.02 = -\frac{5940}{873} + 9.02 = 2.21 \Rightarrow p_{Cd} = 164.39$$

$$\Rightarrow \frac{p_{H_2S}}{p_{H_3}} = \left[\frac{1}{2 \cdot 43.10^4.164.39} \right] = 2.57.10^{-7}$$

2. Si deposition via

Si + SiCl₄
$$\Leftrightarrow$$
 2SiCl₂: $\Delta G^0 = 83000 - 89.4 \cdot T + 3.64 \cdot T \cdot \ln(T)$
source @ 900°C: $\Delta G^0 = 83000 - 89.4 \cdot 1173K + 3.64 \cdot 1173K \cdot \ln(1173K)$
= 8309 [Cal / mol] = 34746 [J / mol]
substrate @ 750°C: $\Delta G^0 = 83000 - 89.4 \cdot 1023K + 3.64 \cdot 1023K \cdot \ln(1023K)$
= 17351 [Cal / mol] = 72558 [J / mol]

The partial pressure gradient Δp for SiCl₂ / SiCl₄ at source and substrate is given by

@ 900°C:
$$\frac{\left[p_{\text{SiCl}_2}\right]^2}{p_{\text{SiCl}_4}} = exp\left[\frac{-\Delta G^0}{R \cdot T}\right] = exp\left[\frac{-34746 \text{ J/mol}}{8.31 \text{ [J/mol K]} \cdot 1173\text{K}}\right] = 2.83 \cdot 10^{-2}$$
with $p_{\text{SiCl}_2} + p_{\text{SiCl}_4} = 1$ and $\left[p_{\text{SiCl}_2}\right]^2 = exp\left[\frac{-\Delta G^0}{R \cdot T}\right] \cdot p_{\text{SiCl}_4} = exp\left[\frac{-\Delta G^0}{R \cdot T}\right] \cdot (1 - p_{\text{SiCl}_2})$
we get
$$\frac{\left[p_{\text{SiCl}_2}\right]^2 + 2.83 \cdot 10^{-2} \cdot p_{\text{Sicl}_2} - 2.83 \cdot 10^{-2} = 0,}{p_{\text{Sicl}_1} = -0.0141 \pm \sqrt{1.988 \cdot 10^{-4} + 2.83 \cdot 10^{-2}} = 0.155}$$

@ 750°C:
$$\frac{\left[p_{SiCl_2}\right]^2}{p_{SiCl_4}} = exp \left[\frac{-\Delta G^0}{R \cdot T}\right] = = exp \left[\frac{-72558 \text{ J/mol}}{8.31 \text{ [J/mol K]} \cdot 1023\text{K}}\right] = 1.96 \cdot 10^{-4}$$
with $p_{SiCl_2} + p_{SiCl_4} = 1$ and $\left[p_{SiCl_2}\right]^2 = 1.96 \cdot 10^{-4} \cdot (1 - p_{SiCl_2})$
we get
$$\left[p_{SiCl_2}\right]^2 + 1.96 \cdot 10^{-4} \cdot p_{SiCl_2} - 1.96 \cdot 10^{-4} = 0,$$

$$p_{SiCl_2} = -9.8 \cdot 10^{-5} \pm \sqrt{9.604 \cdot 10^{-9} + 1.96 \cdot 10^{-4}} = 0.0139$$

The partial pressure difference $\Delta p = 0.155 - 0.0139 = 0.1411$

The flux is given by the difference in the partial pressure between source and substrate:

$$Flow = \frac{\pi \cdot r^4}{8 \cdot \eta \cdot l} \cdot \Delta p = \frac{\pi \cdot 0.075^4 \text{ [m}^4]}{8.0 \cdot 8.10^4 \text{[Pa} \cdot s] \cdot 0.25 \text{ [m]}} \cdot 0.1411.1 \cdot 013.10^5 \text{ [Pa]} = 8870 \text{ } m^3 \text{s}^{-1} = 887.10^7 \text{ [cm}^3 \cdot \text{s}^{-1]}$$

$$Flux = Flow/area = \frac{r^2 \cdot \Delta p}{8 \cdot \eta \cdot l} = \dots = 49 \cdot 6.10^6 \text{ [cm} \cdot \text{s}^{-1]}$$

b.)
$$D \cdot \frac{d^2C}{dx^2} - v \cdot \frac{dC}{dx} - K \cdot C = 0,$$
 with BC: $C(x=0) = 1$ and $C(x=1 \text{ m}) = 0$, $D = 1000 \text{ cm}^2/\text{s}$, $v = 100 \text{ cm/s}$, and $K=1 \text{ s}^{-1}$;
$$C(x) = \left(1 - 7.56 \cdot 10^{-6}\right) \cdot \exp[-0.009 \cdot x] - 7.56 \cdot 10^{-6} \cdot \exp[0.109 \cdot x]$$