Physics 8100 - Electromagnetic Theory I

Fall 2017

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Solutions to Assignment # 09

Problem 1: *(15 points)*

(a) Use the Biot-Savart Law to calculate the magnetic field produced by a straight, infinitely long wire that carries a current ***I***.

(b) Use the Biot-Savart Law to calculate the magnetic field produced by a circular current loop of radius ***A***. You only need to find the field along the axis of the loop.

a.)

$$B(\rho) = \frac{\mu_0 \cdot J}{2\pi\rho} \cdot \hat{\phi}$$

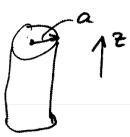
b.)

$$B(z) = \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 \cdot J}{(z^2+a^2)^{3/2}} \cdot \hat{z}$$

Problem 2: *(15 points)*

Use Ampere's Law in integral form to find. The magnetic field in the following situations: (a) A long cylinder of radius A carries an uniformly distributed current - distributed throughout its interior. Find the magnetic field inside and outside the cylinder. (b) A long solenoid of radius A is constructed by wrapping wire around a cylinder, with N turns per unit length. If the wire carries a current I, find the magnetic field inside the solenoid. (c) A thin conducting sheet, of thickness d, carries a current density J = J0 ex . Find the magnetic field just above the sheet.

a.)

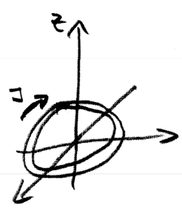


long cylinder current J

$$B(\rho) = \frac{\mu_0 \cdot J}{2\pi}\,\frac{\rho}{a^2}\,\hat{\phi} \quad \text{for } \rho < a$$

$$B(\rho) = \frac{\mu_0\, J}{2\pi\rho} \cdot \hat{\phi} \quad \text{for } \rho > a$$

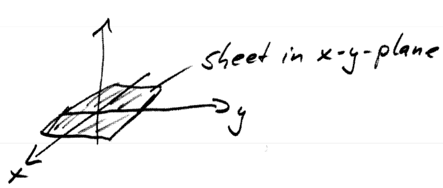
b.)



inside of solenoid

$$\vec{B} = \mu_0 \cdot J \cdot N \cdot \hat{z}$$

c.)



thickness of sheet = d

current density $\vec{j} = j_0 \cdot \hat{x}$

$$\Rightarrow \vec{B} = -\frac{\mu_0\, j_0 \cdot d}{2}\, \text{sign}(z)\, \hat{y}$$

field is in −y-direction above an y-direction below the sheet.

Problem 3: *(35 points)*

An alternating current $I = I_o \cdot \cos(\omega t)$ (amplitude 0.5A, frequency 60Hz) flows down a straight wire, which runs along the axis of a toriodal coil with rectangular cross section (inner radius 1 cm, outer radius 2 cm, height 1 cm, 1000 turns). The coil is connected to a 500 Ohm resistor.

a) In the quasistatic approximation, what emf is induced in the toroid? Find the current, $I_r(t)$, in the resistor.

b) Calculate the back emf in the coil, due to the current $I_r(t)$. What is the ratio of the amplitudes of the back emf and the "direct" emf in (a)?

a) In the quasistatic approximation the magnetic induction field $\vec{B} = \dfrac{\mu_o}{2 \cdot \pi \cdot s} \cdot \hat{\varphi}$

The Flux Φ_1

$$\Phi_1 = \int \vec{B} \cdot d\vec{A} = 2 \cdot \frac{\mu_o \cdot I}{2 \cdot \pi} \cdot \int_a^b \frac{1}{s} \cdot h \cdot ds = \frac{\mu_o \cdot I \cdot h}{2 \cdot \pi} \cdot \ln\left(\frac{b}{a}\right)$$

This is the flux through one turn; the total flux is N times Φ_1:

$$\Phi = \frac{\mu_o \cdot N \cdot h}{2 \cdot \pi} \cdot \ln\left(\frac{b}{a}\right) \cdot I_o \cdot \cos(\omega \cdot t)$$

The emf induced in the toroid is

$$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{\mu_o \cdot N \cdot h}{2 \cdot \pi} \cdot \ln\left(\frac{b}{a}\right) \cdot I_o \cdot \omega \cdot \sin(\omega \cdot t)$$

$$= \frac{4 \cdot \pi \cdot 10^{-7} \cdot 10^3 \cdot 10^2}{2 \cdot \pi} \cdot \ln(2) \cdot (0.5) \cdot (2 \cdot \pi \cdot 60) \cdot \sin(\omega \cdot t) \quad = 2.61 \cdot 10^{-4} \cdot \sin(\omega \cdot t) \ \text{[Volts]}$$

The current in the resistor is

$$I_r = \frac{\mathcal{E}}{R} = \frac{2.61 \cdot 10^{-4}}{500} \cdot \sin(\omega \cdot t) = 5.22 \cdot 10^{-7} \cdot \sin(\omega \cdot t) \ \text{[Amperes]}$$

b) Back emf in the coil

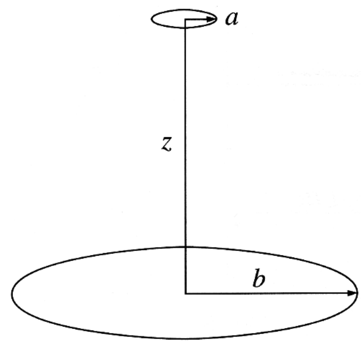
$$\mathcal{E}_b = -L \frac{dI_r}{dt} \quad \text{with } L = \frac{\mu_o \cdot N^2 \cdot h}{2 \cdot \pi} \cdot \ln\left(\frac{b}{a}\right) = \frac{4 \cdot \pi \cdot 10^{-7} \cdot 10^6 \cdot 10^{-2}}{2 \cdot \pi} \cdot \ln(2) = 1.39 \cdot 10^{-3} \ \text{[henries]}$$

therefore: $\mathcal{E}_b = -1.39 \cdot 10^{-3} \cdot 5.22 \cdot 10^{-7} \cdot \omega \cdot \cos(\omega \cdot t) = -2.74 \cdot 10^{-7} \cdot \cos(\omega \cdot t)$ [Volts]

The ratio of amplitudes: $\dfrac{2.74 \cdot 10^{-7}}{2.61 \cdot 10^{-4}} = 1.05 \cdot 10^{-3} = \dfrac{\mu_o \cdot N^2 \cdot h \cdot \omega}{2 \cdot \pi \cdot R} \cdot \ln\left(\dfrac{b}{a}\right)$

Problem 4: *(35 points)*

A small loop of wire (radius a) lies a distance z above the center of a large loop (radius b), as shown in the Fig. on the side. The planes of the two loops are parallel, and perpendicular to the common axis.



a) Suppose current I flow in the big loop. Find the flux through the little loop. The little loop is so small that you can consider the field of the big loop to be essential constant
b) Suppose current I flow in the little loop. Find the flux through the big loop. The little loop is so small that you may treat it as a magnetic dipole.

a) The magnetic flux $\vec{B}_{(2)}$ is given via

$$\vec{B} = \frac{\mu_o \cdot I}{4\cdot\pi}\cdot\int\frac{dl\times\hat{r}}{r^2} = \frac{\mu_o\cdot I}{4\cdot\pi}\cdot\int\frac{dl\cdot\cos\theta}{(z^2+b^2)}\cdot\hat{z} = \frac{\mu_o\cdot I}{4\cdot\pi}\cdot\frac{\cos\theta\cdot 2\cdot\pi\cdot R}{\left(z^2+b^2\right)^{3/2}}\cdot\hat{z} \quad \frac{\mu_o\cdot I\cdot b^2}{2\cdot\left(z^2+b^2\right)^{3/2}}\cdot\hat{z}$$

The Flux Φ of $\vec{B}_{(2)}$ through little loop

$$\Phi = \int\vec{B}\cdot d\vec{A}_{\text{little}} = \vec{B}\cdot\pi\cdot a^2 = \frac{\mu_o\cdot I\cdot\pi\cdot a^2\cdot b^2}{2\cdot\left(z^2+b^2\right)^{3/2}}$$

b) Lets treat the little loop as magnetic dipole. Hence the dipole moment is $\vec{m} = I\cdot\pi\cdot a^2\cdot\hat{z}$
The associated filed is

$$\vec{B}_{\text{dipole}} = \vec{B}\cdot\pi\cdot a^2 = \frac{\mu_o}{4\cdot\pi\cdot r^3}\cdot[3\cdot(\vec{m}\bullet\hat{r})\cdot\hat{r}-\vec{m}] = \frac{\mu_o\cdot m}{4\cdot\pi\cdot r^3}\cdot\left[2\cdot\cos\theta\cdot\hat{r}-\sin\theta\cdot\hat{\theta}\right]$$

The Flux Φ is given by integrating over the spherical cap bounded by the big loop and centered at the little loop

$$\Phi = \int\vec{B}\cdot d\vec{A} = \frac{\mu_o\cdot I\cdot\pi\cdot a^2}{4\cdot\pi\cdot r^3}\cdot\int 2\cdot\cos\theta\, r^2\cdot\sin\theta\cdot d\theta\cdot d\varphi$$

$$= \frac{\mu_o\cdot I\cdot\pi\cdot a^2}{2\cdot r}\cdot 2\cdot\pi\cdot\int_0^{\bar{\theta}}\cos\theta\cdot\sin\theta\cdot d\theta; \quad (\bar{\theta} = b/r)$$

$$= \frac{\mu_o\cdot I\cdot\pi\cdot a^2\cdot b^2}{2\cdot\left(z^2+b^2\right)^{3/2}}$$, the same as in (a)!!