Physics 8100 - Electromagnetic Theory I

Fall 2017 **N. Dietz**

Assignment # 6 (due to Monday, October 30, 2017)

1) *(30 points)* An infinite line charge with linear charge density λ is located at a distance h from a grounded conducting plane z=0. Find: (a) the electric field, (b) the potential, (c) the surface charge induced on the conducting plane, and (d) the induced charge per unit length on the conducting plane. *(40 points)*

2) **Jackson Problem 2.5** *(20 points)* (see textbook chapter 2, p. 86): Show that the work done to remove the charge q from a distance r > a to infinity against the force

$$|\vec{F}| = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q^2}{a^2}\cdot\left(\frac{a}{y}\right)^3\cdot\left(1-\frac{a^2}{y^2}\right)^{-2} \qquad \text{(Eq. 2.6)}$$

of a grounded conducting sphere is

$$W = \frac{q^2\cdot a}{8\pi\varepsilon_o\cdot(r^2 - a^2)}$$

Relate this result to the electrostatic potential (Equ. 2.3) and the energy discussion of Section 1.11.

3) **Jackson Problem 2.7** *(30 points)* (see textbook chapter 2, p.87): Consider a potential problem in the half space defined by $z \geq 0$, with dirichlet boundary conditions on the plane $z = 0$ (and at infinity).

a) Write down the appropriate Green function $G(\vec{x},\vec{x}')$.

b) If the potential on the plane $z = 0$ is specified by $\Phi = V$ inside the circle of radius a centered at origin, and $\Phi = 0$ outside of that circle, find an integral expression for the potential at the point P specified in terms of cylindrical coordinated (ρ, ϕ, z).

c) Show that, along the axis of the circle $(\rho = 0)$, the potential is given by

$$\phi = V\cdot\left(1 - \frac{z}{\sqrt{a^2 + r^2)}}\right)$$

d) Show that at large distances $(\rho^2+z^2 >> a^2)$ the potential can be expanded in a power series in $(\rho^2+z^2)^{-1}$, and that the leading terms are

$$\phi = \frac{V\cdot a^2}{2}\cdot\frac{z}{\left(\rho^2 + z^2\right)^{3/2}}\cdot\left(1 - \frac{3\cdot a^2}{4\cdot\left(\rho^2 + z^2\right)} + \frac{5\cdot\left(3\cdot a^2\cdot\rho^2 + a^4\right)}{8\cdot\left(\rho^2 + z^2\right)^2} + \ldots\right)$$

Verify that the result of part c and d are consistent with each other in their common range of validity.

4) **Jackson Problem 2.9** - *(20 points)* (see textbook chapter 2, p.88): An insulated, spherical, conducting shell of radius a is in an uniform electrical field E_o. If the sphere is cut into two hemispheres by a plane perpendicular to the field, find the force required to prevent the hemispheres from separating

a) if the shell is uncharged;

b) if the total charge on the shell is Q.