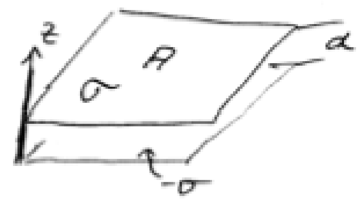
Physics 8100 - Electromagnetic Theory I

Fall 2017 **N. Diet**

Solutions for HW # 4

1) The parallel-plate capacitor: (30 points)

Parallel-plate capacitor of area A separated by distance d with charge density $\sigma = q/A$ / $-q/A$



Gauss' law $\oint \vec{E} \cdot d\vec{r}^2 = \oint_S \frac{\sigma}{\varepsilon} d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma \cdot A}{\varepsilon_0}$

$\oint \vec{E} \cdot \hat{n}\, d^2r = \int_A \vec{E} \cdot \hat{z}\, dA = E_z \cdot A \stackrel{!}{=} \frac{\sigma A}{\varepsilon_0}$

$\hookrightarrow \boxed{E_z = \frac{\sigma}{\varepsilon_0}}$ potential V between plates:

$V = -\int_0^d E_z\, dl = \frac{\sigma}{\varepsilon_0} \cdot d \cdot Q$

the capacitance $C = \frac{Q}{V} = \frac{Q}{Q\, d/\varepsilon_0 A} = \frac{\varepsilon_0 \cdot A}{d}$

2) The spherical capacitor: (30 points)

Spherical capacitor with shells of radii r_1 and r_2

Gauss' law $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl.}}{\varepsilon_0}$; radial symmetric problem

which reduces to E_r $\rightsquigarrow \oint E\, dr = 4\pi r_{12}^2 \cdot E_r = Q/\varepsilon_0$

$\rightsquigarrow E_r = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_{12}^2}$

potential $V = \int_{r_1}^{r_2} \vec{E} \cdot d\vec{r} = -\frac{1}{4\pi\varepsilon_0} q \int_{r_1}^{r_2} \frac{1}{r^2} dr = \frac{1}{4\pi\varepsilon_0} \cdot q \cdot \frac{(r_1 - r_2)}{r_1 \cdot r_2}$

Capacitance: $C = \frac{q}{V} = \frac{4\pi\varepsilon_0 \cdot r_1 \cdot r_2}{r_1 - r_2}$

3) The cylindrical capacitor: (40 points)

problem #3: Cylindrical Capacitor

↳ Gauss' law in cylindrical coords.

$$\oint \vec{E}\cdot d\vec{s} = E_r \cdot 2\pi \cdot r \cdot h = q/\varepsilon_0$$ for surface of radius r

↳ $$\vec{E} = \frac{q}{2\pi r h \cdot \varepsilon_0} \cdot \hat{\rho}$$

Potential V: $$V = -\int \vec{E}\cdot d\vec{r} = \frac{-q}{2\pi h \cdot \varepsilon} \int_{r_1}^{r_2} \frac{1}{r}\, dr$$

$$= \frac{-q}{2\pi h \varepsilon_0} \ln r \Big|_{r_1}^{r_2}$$

$$= \frac{+q}{2\pi h \varepsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

Capacitance $$C = \frac{q}{V} = \frac{2\pi h \cdot \varepsilon_0}{\ln\left(\frac{r_1}{r_2}\right)}$$