Physics 8100 - Electromagnetic Theory I

Assignment # 2 (due to Monday, September 25, 2017)

1) **Problem 1.1**, Jackson textbook (page 50 / 51): Use the Gauss's theorem to prove the following:

a) A closed hollow conductor shields its interior from fields due to charges outside, but does not shield its exterior from field due to charges placed inside.

b) The electric field at the surface of a conductor is normal to the surface and has the magnitude σ / ε_0, where σ is the charge density per unit area on the surface.

2) **Problem 1.2**, Jackson textbook (page 51): The Dirac delta function in three dimensions can be taken as the improper limit as $\alpha \rightarrow 0$ of the Gaussian function

$$D(\alpha; x, y, z) = (2 \cdot \pi)^{-3/2} \cdot \alpha^{-3} \cdot \exp\left[-\frac{1}{2 \cdot \alpha^2} \cdot \left(x^2 + y^2 + z^2 \right) \right] \quad \alpha^{-1} = \alpha_o / 2, \alpha_o$$

Consider a general orthogonal coordinate system specified by the surfaces u = constant, v = constant, w = constant, with the length elements du/U, dv/V and dw/W in the three perpendicular directions. Show that

$$\delta(\vec{x} - \vec{x}') = \delta(u - u') \cdot \delta(v - v') \cdot \delta(w - w') \; U \cdot V \cdot W$$

by considering the limit of the Gaussian above. Note that as $\alpha \rightarrow 0$ only the infinitesimal length element need to be used for the distance between the points in the exponent.

3) **Problem 1.5**, Jackson textbook (page 51): The time-average potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4 \cdot \pi \cdot \varepsilon_o} \cdot \frac{e^{-\alpha \cdot r}}{r} \cdot \left(1 + \frac{\alpha \cdot r}{2} \right) \quad \alpha^{-1} = \alpha_o / 2, \alpha_o$$

where q is the magnitude if the electric charge, and $\alpha^{-1} = \alpha_o / 2$, α_o being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

4) An infinitely long cylinder of radius R has a line charge density λ_o. Find the electric field and its scalar potential λ_o everywhere for

b) If the charge density is uniformly distributed on the surface (i.e. the cylinder is a conductor) and

b) If the charge density is uniformly distributed over the whole volume (i.e. the cylinder is an insulator). Assume that the potential is zero at r = a, with a < R.

5) The electric potential of a dipole $\vec{P}$ at origin is $\Phi = \frac{1}{4 \cdot \pi \cdot \varepsilon_o} \cdot \frac{\vec{P} \cdot \vec{X}}{r^3}$

a) Find the electric field using $E = -\nabla \Phi$

b) If $\vec{P} = P \cdot \vec{k}$ (parallel to z-axis), find the spherical field components E_r, E_θ, and E_Φ.