Estimating Granger causality from Fourier and wavelet transforms of time series data - Supplementary Information

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In this supplement, we illustrate further the usefulness of the proposed nonparametric Granger causality techniques by applying them to additional datasets, both simulated and real-world. Figures S1(a-c) are from simulated data, and Figures S2 and S3 (a-b) are the results of analyzing brain signals(local field potentials) recorded from the primary motor area (M1) and posterior parietal area (7b) of a monkey performing a visual motor task (see Reference [1] for further details on the task and data recordings). The simulated data are generated by using the same network model equations as in the Letter. The length of each realization and the number of realizations are varied as parameters. The Granger causality spectra (by parametric (P) and nonparametric (NP) techniques) shown in Figure S1 (a) are computed by breaking two 40000 time points long time series data each into 200 segments, which are treated as different realizations, each of 200 time points long. The spectra in Figures S1 (b) are obtained from 200 such realizations each with 200 time points. These figures show that the results from different methods are in excellent agreement. Furthermore, these results will converge to the results in Figure 1 (c) of the letter, which can be considered the theoretical results, as we increase the length of the time series. Figure S1 (c), which also correctly recovers the true features of Granger causality spectra in terms of directionality and frequency peak location, is obtained by using 15 realizations each with 200 time points. These numerical examples indicate that the nonparametric techniques can be used reliably even when only a small number of realizations are available for analysis.

Figures S2 and S3 (a-b) show the results obtained by applying the nonparametric techniques to the local field potentials (LFPs) recorded from M1 and 7b as mentioned above. The results from applying the parametric Granger causality techniques to these data have been previously reported (see [1], and can be summarized as follows: (I) synchronized beta-frequency (15-30 Hz) oscillations linked together diverse sensorimotor areas (somatosensory (S1), primary motor (M1), posterior parietal areas (7a and 7b)) to form a large-scale brain network, (II) strong Granger causal influences were from S1 to M1 and to 7a and 7b, and (III) 7b exerted further Granger causal influences on M1. Here, we apply the nonparametric techniques to the data, consisting of 1000 trials each of length 125ms, from the recording site pair M1-7b. Figure S2 shows that (i) the casual influence from 7b to M1 (solid line) is statistically significant ($p < 10^{-6}$) and has a spectral peak at around 22 Hz, and (ii) the causal influence from M1 to 7b (dashed line) is below the significant threshold (dotted line). The statistical threshold level ($p < 10^{-}6$) was determined by a random permutation method [1], which involved reshuffling the order of trials randomly a number of times separately in each channel and computing the spectral quantities from the reshuffled trials. Application of the wavelet based technique on the data from the same recording site pair (7b and M1) gives time-frequency Granger causality plots shown in Figures S3 (a-b). The pattern of causal influence is similar to that in Figure S2. Specifically, the spectral peak of the dominant causal influence for 7b to M1 remains around 22 Hz throughout the duration -90 ms to 35 ms, which was the time period when the monkey maintained pressure on a depressed hand lever while anticipating the imminent onset of visuomotor processing. These nonparametric Granger causality results support the hypothesis that the beta oscillation supports isometric pressure maintenance by facilitating the feedback sensory information processing (see [1] for further details).

[1] A. Brovelli, M. Ding, A. Ledberg, Y. Chen, R. Nakamura, and S. L. Bressler, Proc. Natl. Acad. Sci. USA **101**, 9849 (2004).

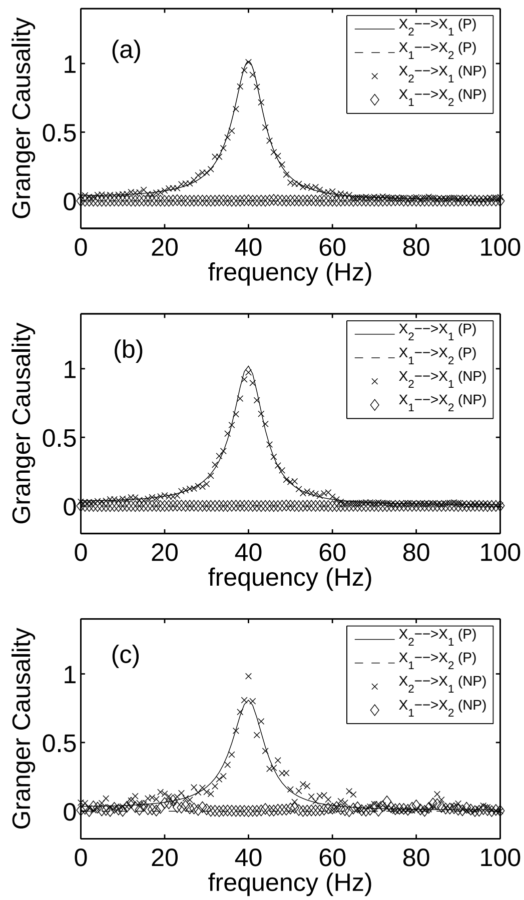

FIG. S1: Parametric (P) and Fourier-transform based Nonparametric (NP) Granger causality spectra computed from simulated data: (a) 200 segments each of 200 time points (these segments were constructed from two long simulated time series each of length 40000 time points), (b) 200 realizations each of 200 time points, and (c) 15 realizations and 200 time points. See the body text for further descriptions on these results.

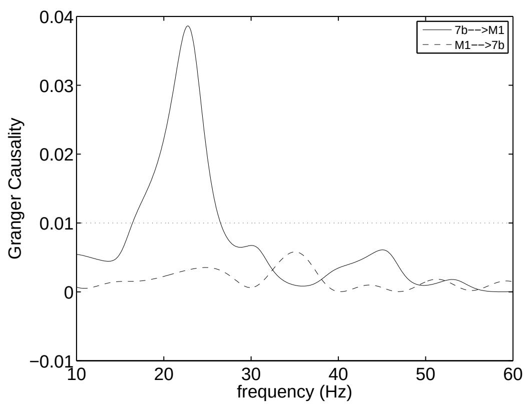

FIG. S2: Application of the Fourier transform-based Granger causality technique on brain signals (LFPs from M1 and 7b). The significant causal driving from 7b to M1 occurs around 22 Hz in the beta frequency range (15-30 Hz) and is consistent with previously reported parametric results [1].

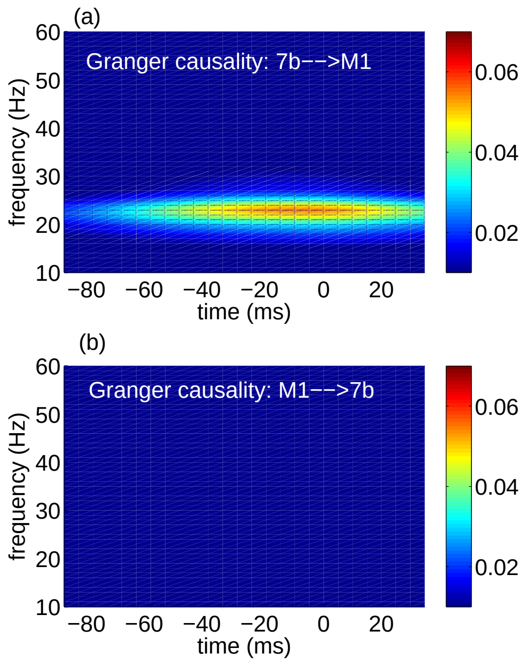

FIG. S3: Application of the wavelet transform-based Granger causality technique on LFPs from M1 and 7b. (a) the significant casual influence from 7b to M1 is around 22 Hz throughout the whole time period, and (b) the causal influence from M1 to 7b remains below the statistical significance level.