Encyclopedia of Computational Neuroscience
DOI 10.1007/978-1-4614-7320-6_420-1

Spectral Interdependency Methods

Mukesh Dhamala*
Department of Physics and Astronomy, Neuroscience Institute, Georgia State University, Atlanta, Georgia, USA

Synonyms

Coherence and Granger causality spectral analysis; Multivariate spectral analysis; Oscillatory network activity analysis

Definition

Spectral interdependency methods are a means of statistically quantifying the interrelationship between a pair of dynamic processes as a function of frequency or time period of oscillation. The measures of spectral interdependency are derived from the time series recordings of dynamic systems either by using autoregressive modeling (parametric method) or by using direct Fourier or wavelet transforms (nonparametric method). For a pair of multivariate stationary processes (1 and 2), there are three measures that characterize the spectral interdependency between these processes. They are total interdependence ($M_{1,2}$), Granger causality (one-way effect or directional influence from the first process to the second process, $M_{1\rightarrow 2}$, or from the second to the first, $M_{2\rightarrow 1}$), and instantaneous causality (measure of reciprocity, $M_{1.2}$). In general, the total interdependence is the sum of directional influences and instantaneous causality frequency by frequency ($M_{1,2} = M_{1\rightarrow 2} + M_{2\rightarrow 1} + M_{1.2}$) and is related to coherence (C_{12}) as $M_{1,2} = -\ln(1 - C_{12})$. These measures can also be estimated for nonstationary processes by using moving time-windowed autoregressive modeling or Fourier transformations or wavelet transformations. Coherence and spectral Granger causality are well-accepted measures in neuroscience to characterize frequency-specific interdependence between multiple time series from multisite neurophysiological recordings.

Detailed Description

Many processes in nature, including brain processes, have oscillatory motion, and the time series measurements of their activity are rich in oscillatory content, lending them naturally to spectral analysis. Spectral interdependency methods are used to study the relationship in the frequency domain between multiple processes from their simultaneously recorded time series signals. Consider a pair of zero-mean, stationary processes (1 and 2) in which simultaneously measured time series at a sampling rate of f_s are represented as 1: $X_1(1), X_1(2), \ldots, X_1(t), \ldots$ and 2: $X_2(1), X_2(2), \ldots, X_2(t)$, ... The spectral interdependency measures as defined above are derived from the spectral matrix (S) and/or from the transfer function (H) and noise covariance matrix ($\sum$), which can be estimated by the parametric (Ding et al. 2006) or nonparametric approaches applied to these time series

*Email: mdhamala@gmail.com
*Email: mdhamala@phy-astr.gsu.edu

DOI 10.1007/978-1-4614-7320-6_420-1

(Dhamala et al. 2008a, b). Brief mathematical derivations and descriptions of these interdependency measures (C_{12}, $M_{1,2}$, $M_{1\rightarrow 2}$, $M_{2\rightarrow 1}$ and $M_{1.2}$) are included below.

Directed transfer function (DTF) (Kaminski et al. 2001) and partial directed coherence (PDC) (Baccala and Sameshima 2001) are the accepted alternative measures of directional influence, equivalent to the measures for $M_{1\rightarrow 2}$ and $M_{2\rightarrow 1}$ as defined above. DTF is obtained from H and PDC from the Fourier transform of model coefficients in the parametric approach.

Parametric Approach. Jointly, X_1 and X_2 series can be represented as the following bivariate autoregressive (AR) models:

$$\begin{aligned} X_1(t) &= \sum\nolimits_{j=1}^{\infty} a_{11,\,j} X_1(t-j) + \sum\nolimits_{j=1}^{\infty} a_{12,\,j} X_2(t-j) + \varepsilon(t) \\ X_2(t) &= \sum\nolimits_{j=1}^{\infty} a_{21,\,j} X_1(t-j) + \sum\nolimits_{j=1}^{\infty} a_{22,\,j} X_2(t-j) + \eta(t), \end{aligned} \tag{1}$$

where ε and η are residual (one-step-ahead prediction) errors and are uncorrelated over time. After Fourier transforming the bivariate AR representation (1) and applying proper ensemble average, we obtain the following spectral density matrix S as a function of frequency (f):

$$S(f) = H(f) \sum H^*(f), \tag{2}$$

where * denotes the matrix adjoint. Here, the noise covariance matrix $\sum$ is computed from the residual errors $\varepsilon(t)$, $\eta(t)$ and the transfer function matrix **H(f)** is constructed from the matrix inverse of the Fourier transforms of the coefficients a's:

$$\begin{aligned} \sum &= \begin{pmatrix} \mathrm{var}(\varepsilon) & \mathrm{cov}(\varepsilon,\eta) \\ \mathrm{cov}(\varepsilon,\eta) & \mathrm{var}(\eta) \end{pmatrix} \\ H_{lm}(f) &= \left(\delta_{lm} - \sum_{k=1}^{\infty} a_{lm,\,k} e^{-i2\pi fk} \right)^{-1}, \end{aligned} \tag{3}$$

where δ_{lm} is the Kronecker delta function with the matrix element index lm.

Nonparametric Approach. S, H, and $\sum$ can also be estimated by using the nonparametric spectral methods (Dhamala et al. 2008a, b) without explicitly fitting the time series $X_1(t)$ and $X_2(t)$ in autoregressive models. In this approach, S is constructed by Fourier transforming X_1 and X_2 and properly averaging over ensembles usually with multitapers (Mitra and Pesaran 1999):

$$S_{lm} = \langle x_1(f) x_m(f)^* \rangle, \tag{4}$$

where $_{\mathrm{lm}}$ is the index for time series and matrix element, < > represents averaging over ensemble, and X's are the direct Fourier transforms of X's. H and $\sum$ can be derived from the minimum-phase factors of S:

$$\begin{aligned} H &= \psi A_0^{-1} \\ \sum &= A_0 A_0^T, \end{aligned} \tag{5}$$

where T stands for matrix transposition, $\psi(e^{i\theta}) = \sum_{k=0}^{\infty} A_k e^{ik\theta}$ defined on the unit circle and $A_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \psi\left(e^{i\theta}\right) e^{-ik\theta} d\theta.$

Encyclopedia of Computational Neuroscience
DOI 10.1007/978-1-4614-7320-6_420-1

Spectral Interdependence. The coherence function C(f) is derived from the cross spectra normalized by the product of the individual auto spectra and measures the amount of interdependence (synchrony) as a function of frequency:

$$C(f) = \frac{|S_{12}(f)|^2}{S_{11}(f)S_{22}(f)}. \tag{6}$$

C(f) is sensitive to both amplitude and phase relationships between processes at f and its value ranges from 0 (no interdependence) to 1 (maximum interdependence). C(f) is related to Geweke's measure of total interdependence ($M_{1,2}$) (Ding et al. 2006):

$$M_{1,2}(f) = -\ln(1 - C(f)), \tag{7}$$

whose value ranges from 0 to infinity. The total spectral interdependence ($M_{1,2}$) is equal to the sum of Granger causality or directional influences (one-way effects, $M_{1\rightarrow 2}$ and $M_{2\rightarrow 1}$) and instantaneous causality ($M_{1.2}$) (Geweke 1982):

$$\mathrm{M}_{1,2} = \mathrm{M}_{1\rightarrow 2} + \mathrm{M}_{2\rightarrow 1} + \mathrm{M}_{1.2} \tag{8}$$

Here, directional influences between 1 and 2 are given by (Geweke 1982)

$$M_{1\rightarrow 2}(f) = \ln\frac{S_{22}(f)}{\tilde{H}_{11}(f)\sum_{11}\tilde{H}^*_{11}(f)} \qquad M_{2\rightarrow 1}(f) = \ln\frac{S_{11}(f)}{\tilde{H}_{22}(f)\sum_{22}\tilde{H}^*_{22}(f),} \tag{9}$$

where $\tilde{H}_{11} = H_{11} + \frac{\sum_{12}}{\sum_{11}}H_{12}, \tilde{H}_{22} = H_{22} + \frac{\sum_{12}}{\sum_{22}}H_{21}$, and
instantaneous causality $M_{1.2}$ is given by

$$M_{1.2}(f) = -\ln\frac{|S(f)|}{\left(\tilde{H}_{11}(f)\sum_{11}\tilde{H}^*_{11}(f)\right)\left(\tilde{H}_{22}(f)\sum_{22}\tilde{H}^*_{22}(f)\right)}. \tag{10}$$

Granger causality at low frequencies between a pair of cointegrated series depends only on a few statistically interpretable coefficients from the error correction model if Hosoya's spectral decomposition (Hosoya 1991) is used (Granger and Lin 1995).

The time-domain counterparts of these spectral measures in Eq. 8 are obtained by integrating the spectral measures over the entire frequency range as $\frac{2}{f_s}\int_0^{\frac{f_s}{2}} M(f)\,df$. The integral of total interdependence measures the total amount of mutual information, and the other integrals are simply the respective time-domain measures of Granger causality.

Spectral Interdependence for Nonstationary Processes. The spectral interdependency measures can also be defined for nonstationary processes by treating their time series in sufficiently short windows as locally stationary processes. They can be estimated as a function of time and frequency by using moving time-windowed autoregressive modeling (Liang et al. 2000), moving time-windowed Fourier transformations (Shiogai et al. 2012), or wavelet transformations (Dhamala et al. 2008a, b).

Encyclopedia of Computational Neuroscience
DOI 10.1007/978-1-4614-7320-6_420-1

Numerical Examples. Here, we generate time series (500 trials) from jointly stationary and nonstationary processes and illustrate the estimation of coherence and Granger causality spectra using the parametric (P) and nonparametric (NP) approaches. We consider two interacting autoregressive processes, 2: $\{X_2(t)\}$ driving 1: $\{X_1(t)\}$, similar to the network model considered in Dhamala et al. 2008b, where $X_1 = 0.55\,X_1(t-1) - 0.8X_1(t-2) + C(t)X_2(t-1) + \varepsilon(t)$ and $X_2(t) = 0.55X_2(t-1) - 0.8X_2(t-2) + \eta(t)$. Here, $(\varepsilon(t), \eta(t))$'s are independent white noise processes with zero means and unit variances, the sampling rate is considered to be 200 Hz, and the coupling strength C(t) remains 0.25 for time t in the stationary condition and slowly changes from 0.25 to 0 around t = 2 s in the case of nonstationary processes. P and NP approaches agree well in coherence and Granger causality spectra (Fig. 1a, b). Morlet wavelet transform-based method yields complete time-frequency maps of coherence (Fig. 1c) and Granger causality (Fig. 1d), consistent with the trend of the 2 to 1 coupling as shown on the right side of Fig. 1d.

Extensions of Spectral Interdependency Methods. As an extension of the ordinary coherence described above, block coherence (Nedungadi et al. 2011) can estimate coherence spectra between pairs of nonoverlapping time series. Conditional Granger causality (Geweke 1984; Hosoya 2001) measures directional influences between two processes eliminating the effect of a third process,

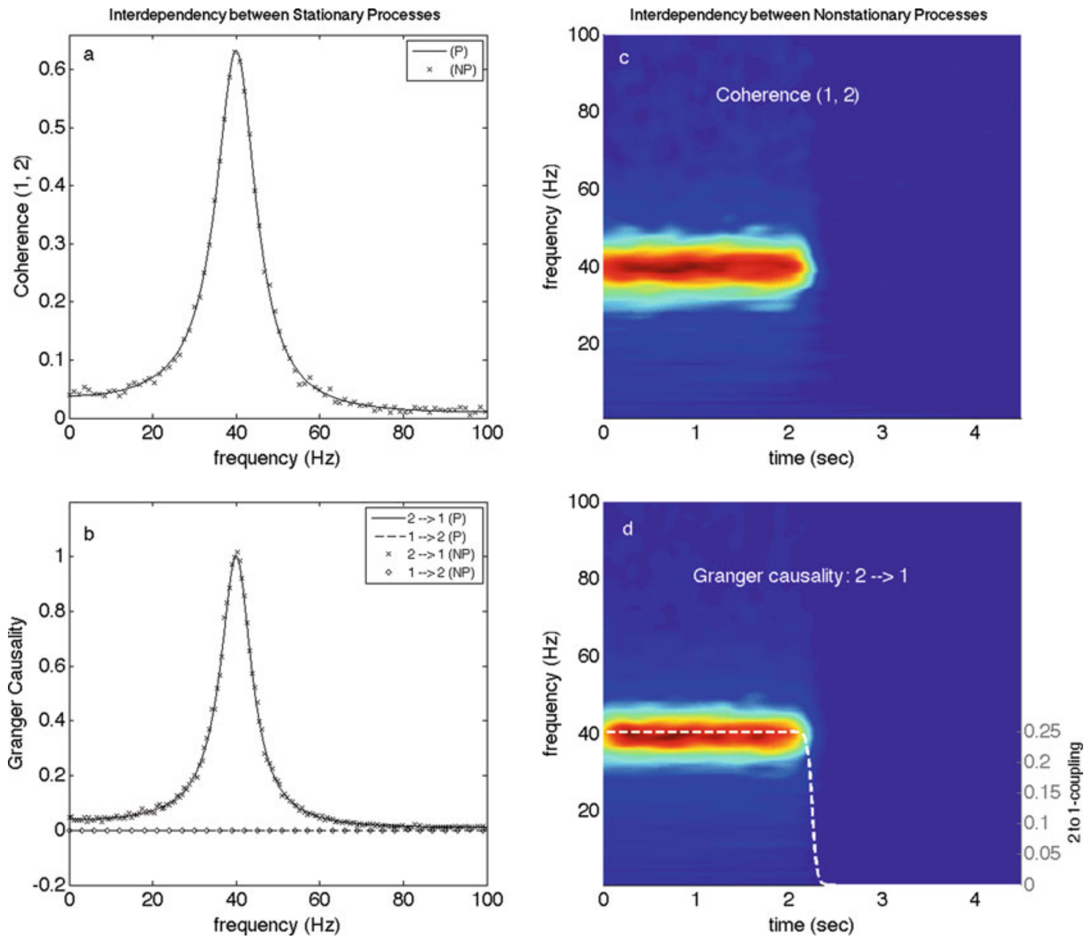

Fig. 1 Coherence (**a**, **c**) and Granger causality (**c**, **d**) spectra between stationary (**a**, **b**) and nonstationary (**c**, **d**) processes (1 and 2). Parametric (*P*) and Fourier transform-based nonparametric (*NP*) methods are used here to evaluate these quantities (**a**, **b**) between the stationary processes. The wavelet transform-based time-frequency maps of coherence (**c**) and Granger causality (**d**) recover the time-varying nature of 2 to 1 coupling (shown on the right y-axis in **d**)

DOI 10.1007/978-1-4614-7320-6_420-1

thereby distinguishing between direct and mediated causality (Ding et al. 2006; Dhamala 2008b). The multivariate version of the spectral Granger causality between two processes makes use of this idea of eliminating the causal effect from all other interrelated processes as an extension of the conditional Granger causality defined for three processes.

Applications in Neuroscience. Spectral interdependency measures have been instrumental in attempts to understand the relationships between oscillatory brain processes at various spatial scales from multisite brain activity recordings. Coherence is widely used in neuroscience (Siegel et al. 2012; Roberts et al. 2013). The insights provided by this measure have even led to the "neuronal communication through coherence" hypothesis (Fries 2005; Roberts et al. 2013). Spectral Granger causality and equivalent directional measures have been used in a variety of brain signal recordings, such as local field potentials, EEG, MEG, and fMRI, in animals and humans (see Bressler and Seth 2011; Friston et al. 2012 for reviews). Because of the unknown theoretical distributions of coherence and spectral Granger causality, establishing statistical significance in these measures derived from experimental time series requires data resampling (surrogate) methods such as jackknifing (Bokil et al. 2010), bootstrapping, and random permutation (Seth 2010).

References

Baccala LA, Sameshima K (2001) Partial directed coherence: a new concept in neural structure determination. Biol Cybern 84:463–474

Bokil H, Andrews P, Kulkarni JE, Mehta S, Mitra PP (2010) Chronux: a platform for analyzing neural signals. J Neurosci Methods 192:146–151

Bressler SL, Seth AK (2011) Wiener-Granger causality: a well established methodology. Neuroimage 58:323–329

Dhamala M, Rangarajan G, Ding M (2008a) Estimating Granger causality from Fourier and wavelet transforms of time series data. Phys Rev Lett 100(018701):1–4

Dhamala M, Rangarajan G, Ding M (2008b) Analyzing information flow in brain networks with nonparametric Granger causality. Neuroimage 41:354–362

Ding M, Chen Y, Bressler SL (2006) Granger causality: basic theory and application to neuroscience. In: Schelter S, Winterhalder N, Timmer J (eds) Handbook of time series analysis. Wiley, Berlin, pp 437–459

Fries P (2005) A mechanism for cognitive dynamics: neuronal communication through neuronal coherence. Trends Cogn Sci 9:474–480

Friston K, Moran R, Seth AK (2012) Analysing connectivity with Granger causality and dynamic causal modeling. Curr Opin Neurobiol 23:172–178

Geweke J (1982) Measurement of linear-dependence and feedback between multiple time-series. J Am Stat Assoc 77:304–313

Geweke J (1984) Measures of conditional linear dependence and feedback between time series. J Am Stat Assoc 79:907–915

Granger CWJ, Lin J-L (1995) Causality in the long run. Econom Theory 11:530–536

Hosoya Y (1991) The decomposition and measurement of the interdependence between second-order stationary processes. Prob Theory Relat Fields 88:429–444

Hosoya Y (2001) Elimination of third-series effect and defining partial measures of causality. J Time Ser Anal 22:537–554

DOI 10.1007/978-1-4614-7320-6_420-1

Kaminski M, Ding M, Truccolo WA, Bressler SL (2001) Evaluating causal relations in neural systems: Granger causality, directed transfer function and statistical assessment of significance. Biol Cybern 85:145–157

Liang H, Ding M, Nakamura R, Bressler SL (2000) Causal influences in primate cerebral cortex during visual pattern discrimination. Neuroreport 11:2875–2880

Mitra PP, Pesaran B (1999) Analysis of dynamic brain imaging data. Biophys J 76:691–708

Nedungadi A, Ding M, Rangarajan G (2011) Block coherence: a method for measuring the interdependence between two blocks of neurobiological time series. Biol Cybern 104:197–207

Roberts M, Lowet E, Brunet N, Ter Wal M, Tiesinga P, Fries P, De Weerd P (2013) Robust gamma coherence between macaque V1 and V2 by dynamic frequency matching. Neuron 78:523–536

Seth AK (2010) A MATLAB toolbox for Granger causal connectivity analysis. J Neurosci Methods 186:262–273

Shiogai Y, Dhamala M, Oshima K, Hasler M (2012) Cortico-cardio-respiratory network interactions during anesthesia. Plos One 7:e44634

Siegel M, Donner TH, Engel AK (2012) Spectral fingerprints of large-scale neuronal interactions. Nat Rev Neurosci 13:121–134