Chapter 9. Impulse and Momentum

Explosions and collisions obey some surprisingly simple laws that make problem solving easier when comparing the situation *before* and *after* an interaction.

Chapter Goal: To introduce the ideas of impulse and momentum and to learn a new problem-solving strategy based on conservation laws.

Chapter 9. Impulse and Momentum

Topics:

- Momentum and Impulse
- Solving Impulse and Momentum Problems
- Conservation of Momentum
- Inelastic Collisions
- Explosions
- Momentum in Two Dimensions

Momentum

After the collision

What is the velocity of ball A after the collision? ball B?

What is conserved during the collision?

MOMENTUM

$$\vec{p} = m\vec{v}$$

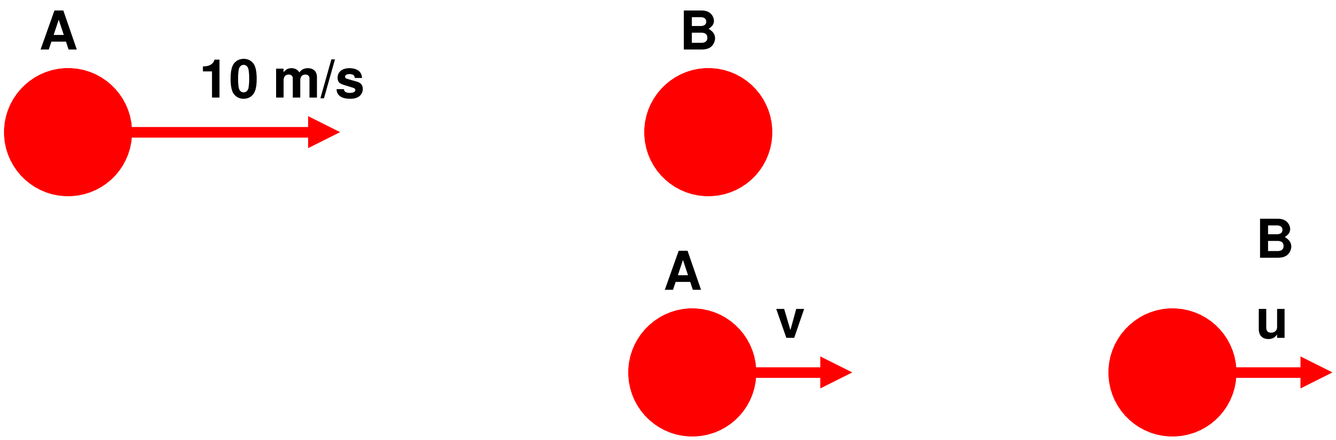
The total momentum is the sum of momentum of ball A and momentum of ball B.

Momentum

$$\vec{p} = m\vec{v}$$

The total momentum of the system is conserved during the collision:

$$m_A v_{A,i} = m_A v + m_B u$$

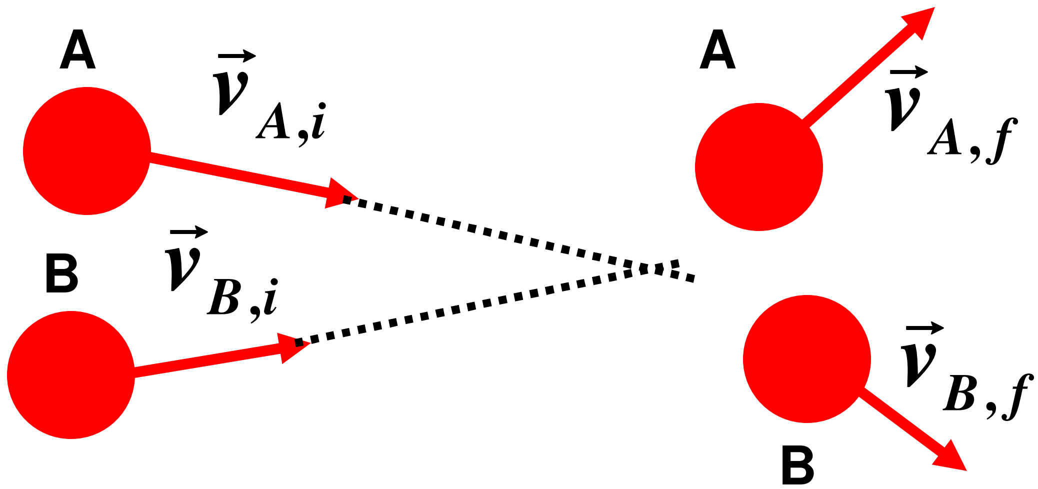


- **Momentum is a vector. It has the same direction as corresponding velocity.**
- **General expression for the momentum conservation: the total momentum before the collision is equal to the total momentum after the collision**

Momentum

$$\vec{p} = m\vec{v}$$

• General expression for the momentum conservation: the total momentum before the collision is equal to the total momentum after the collision



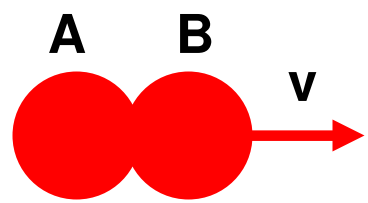
$$m_A\vec{v}_{A,i} + m_B\vec{v}_{B,i} = m_A\vec{v}_{A,f} + m_B\vec{v}_{B,f}$$

Usually this equation is written in terms of components.

Example:

$m_A = 1kg$ $\qquad$ $m_B = 4kg$

After the collision the balls are moving together (have the same velocity). What is their velocity?



Momentum before the collision: $p_i = m_A v_{A,i} = 10\frac{kg \cdot m}{s}$

Momentum after the collision: $p_f = (m_A + m_B)v = 5v$

Conservation of momentum: $p_i = p_f$

$10 = 5v \qquad v = 2m/s$

Why do we have conservation of total momentum?

Newton's second law:

$$\vec{F}_{net} = m\vec{a}$$

Acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Then

momentum

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

After integration

$$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}_{net}dt$$

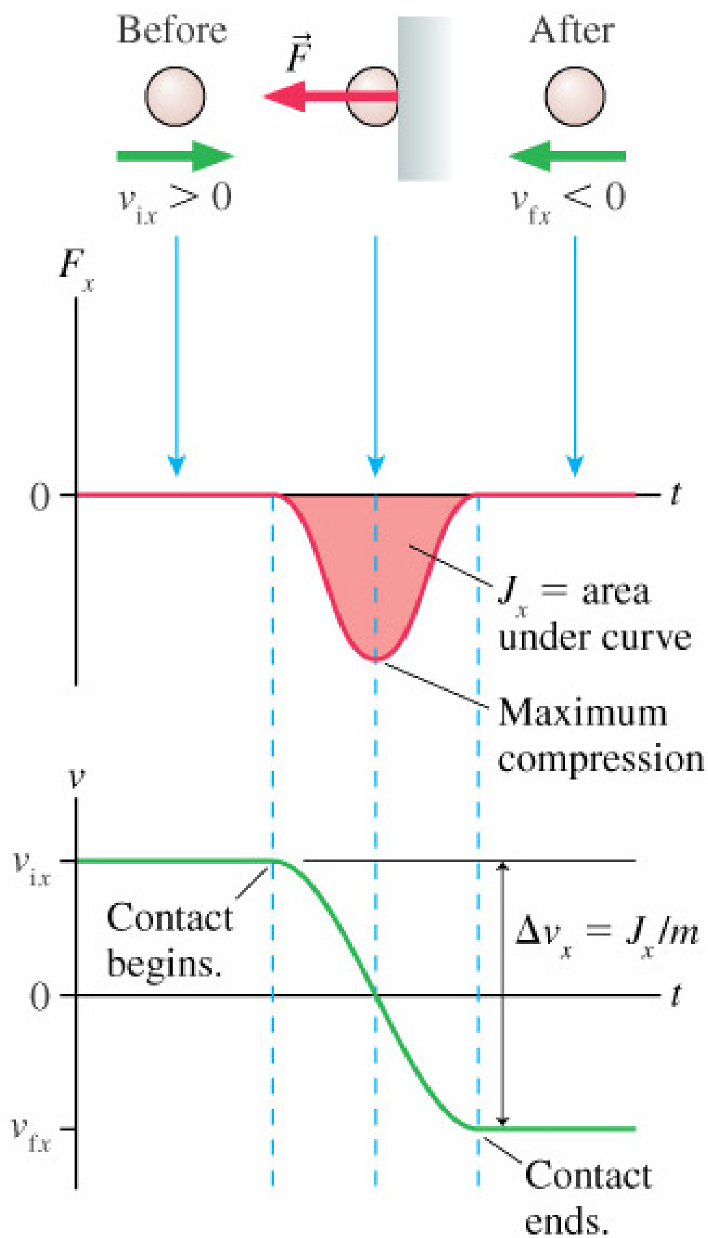
The area under $\vec{F}_{net}(t)$ curve.

It is called IMPULSE, J.

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{net}dt$$

The impulse of the force is equal to the change of the momentum of the object.

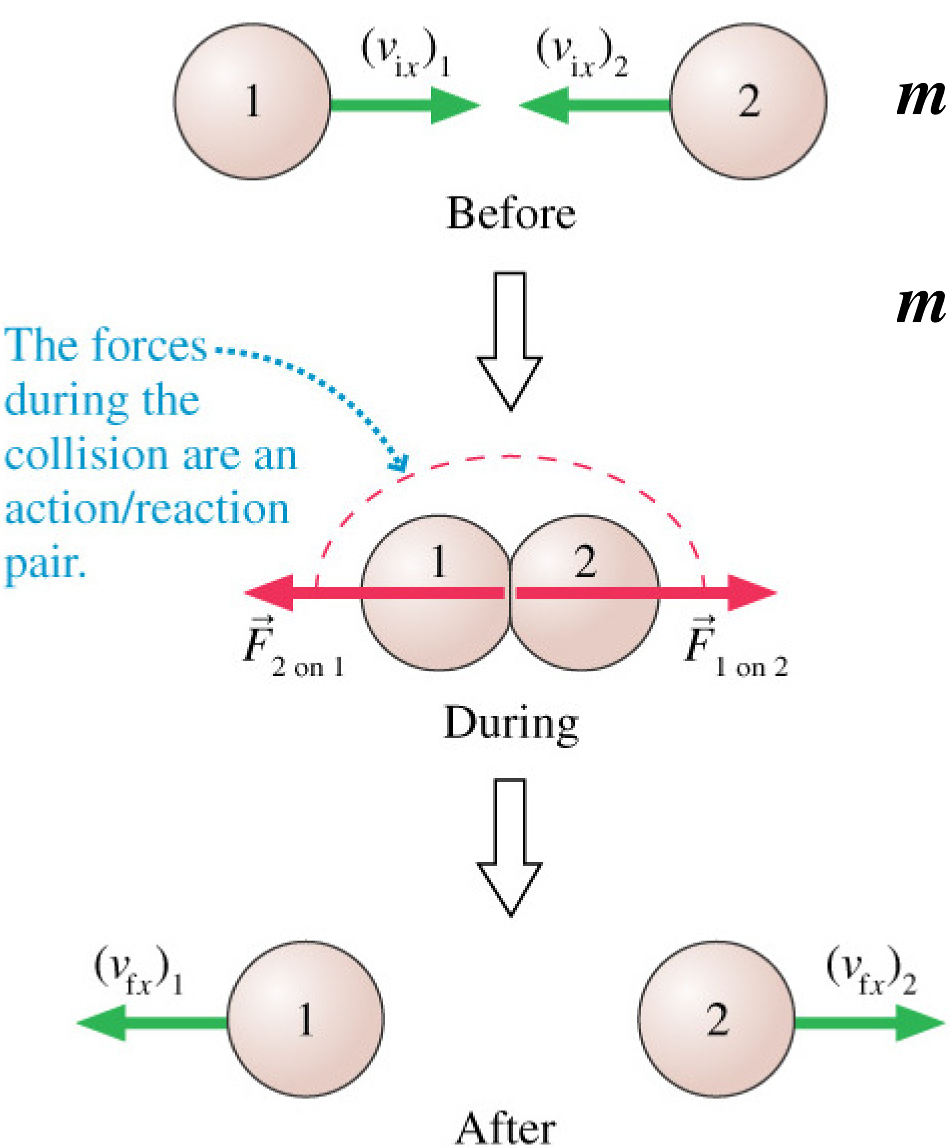
$$\Delta\vec{p} = \vec{J}$$



$$p_i = mv_{ix}$$

$$p_f = mv_{fx} < 0$$

$$J_x = p_f - p_i < 0$$



$$m_1 v_{fx,1} - m_1 v_{ix,1} = \int_{t_1}^{t_2} F_{x,2on1}\,dt$$

$$m_2 v_{fx,2} - m_2 v_{ix,2} = \int_{t_1}^{t_2} F_{x,1on2}\,dt$$

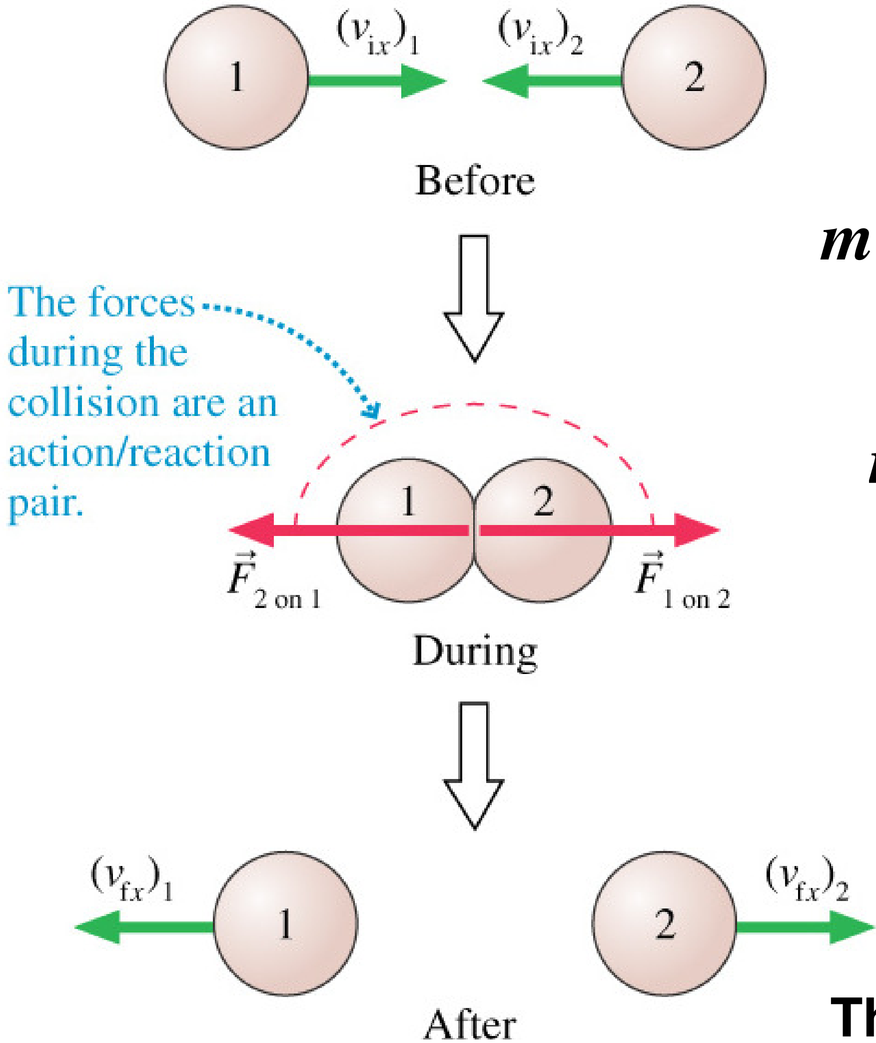
Newton's third law:

$$F_{x,1on2} = -F_{x,2on1}$$

Then

$$\int_{t_1}^{t_2} F_{x,1on2}\,dt = -\int_{t_1}^{t_2} F_{x,2on1}\,dt$$

$$m_2 v_{fx,2} - m_2 v_{ix,2} = -\left(m_1 v_{fx,1} - m_1 v_{ix,1}\right)$$



$$m_2 v_{fx,2} - m_2 v_{ix,2} = -\left(m_1 v_{fx,1} - m_1 v_{ix,1}\right)$$

$$m_1 v_{ix,1} + m_2 v_{ix,2} = m_1 v_{fx,1} + m_2 v_{fx,2}$$

$$p_{ix,1} + p_{ix,2} = p_{fx,1} + p_{fx,2}$$

$$p_{ix,total} = p_{fx,total}$$

The law of conservation of momentum

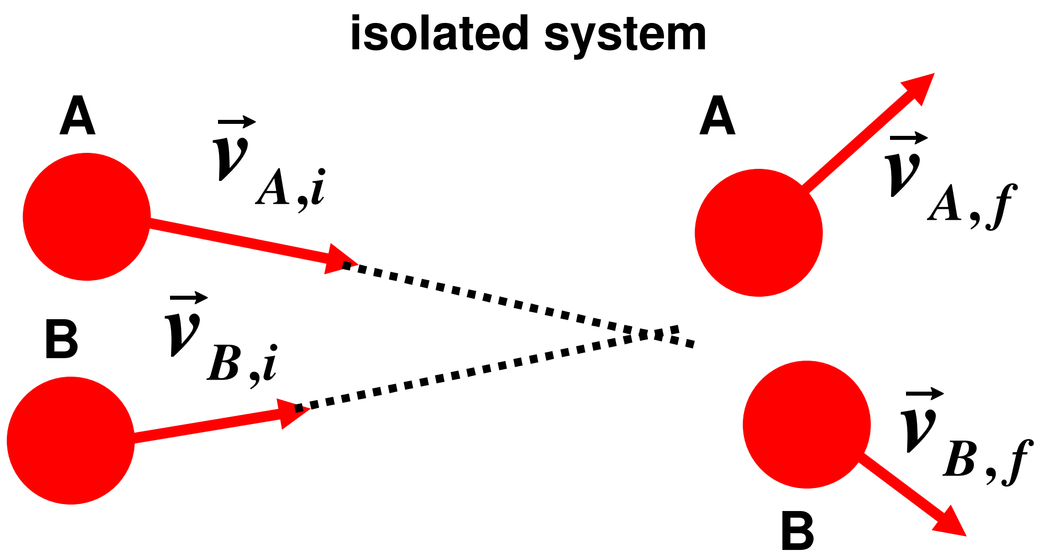
Momentum

$$\vec{p} = m\vec{v}$$

The law of conservation of momentum:

The total momentum of an isolated system (no external forces) does not change.

Interactions within system do not change the system's total momentum



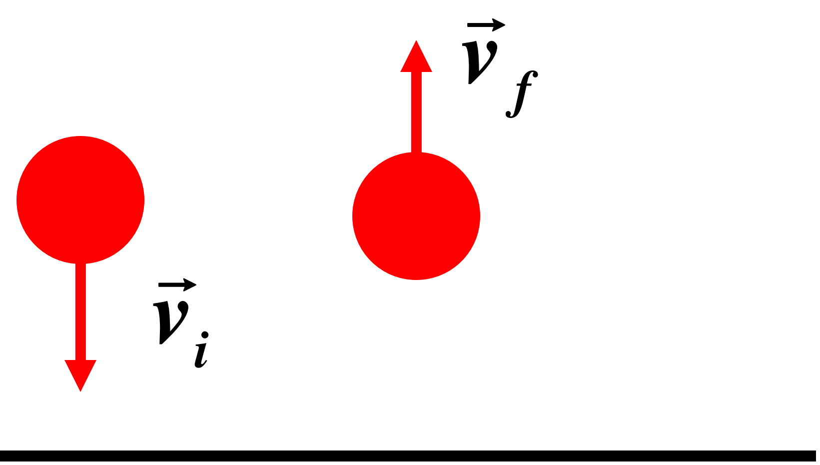
$$m_A\vec{v}_{A,i} + m_B\vec{v}_{B,i} = m_A\vec{v}_{A,f} + m_B\vec{v}_{B,f}$$

Momentum

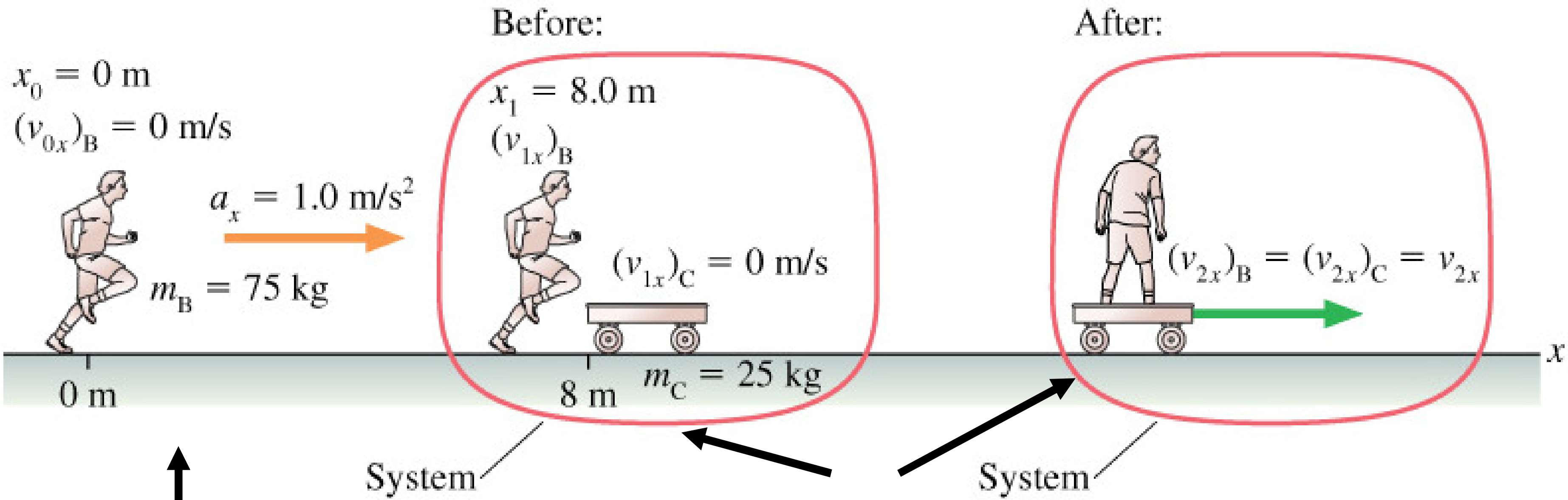
$$\vec{p} = m\vec{v}$$

The ball is dropped onto a hard floor:

- **The ball is not an isolated system (interaction with the floor)**
- **no conservation of momentum for the ball**
- **Initial momentum is** $\vec{p}_i = m\vec{v}_i$
- **Final momentum (after collision) is** $\vec{p}_f = m\vec{v}_f$
- **The ball+ the floor is an isolated system**
- **The total momentum (ball+floor) is conserved**



Example: Find v_{2x}



Isolated system

Motion with constant acceleration:

$$(v_{1x,B})^2 = 2a_x x_1 = 16$$

$$v_{1x,B} = 4m/s$$

Momentum before the "collision":

$$p_{i,total} = m_B v_{1x,B} + m_C v_{1x,C} = m_B v_{1x,B} = 75 \cdot 4 = 300 \frac{kg \cdot m}{s}$$

Momentum after the "collision":

$$p_{f,total} = m_B v_{2x,B} + m_C v_{2x,C} = (m_B + m_C) v_{2x} = 100 v_{2x}$$

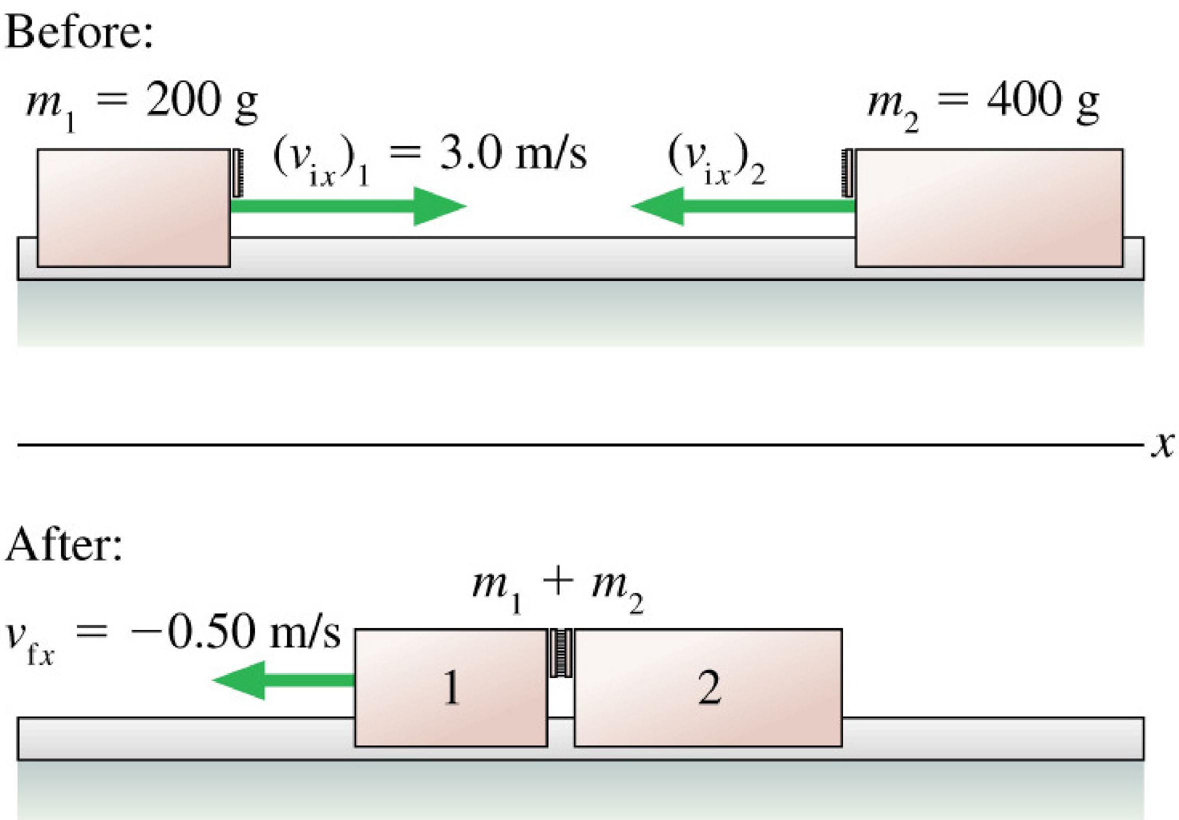
Conservation of momentum:

$$p_{f,total} = p_{i,total}$$

$$100 v_{2x} = 300 \qquad v_{2x} = 3m/s$$

Perfectly inelastic collision:

A collision in which the two objects stick together and move with a common final velocity.



$$p_{i,total} = m_1 v_{ix,1} + m_2 v_{ix,2}$$

$$p_{f,total} = (m_1 + m_2) v_{fx}$$

Find: $(v_{ix})_2$

$$m_1 v_{ix,1} + m_2 v_{ix,2} = (m_1 + m_2) v_{fx}$$

$$v_{ix,2} = \left(\frac{m_1}{m_2} + 1\right) v_{fx} - \frac{m_1}{m_2} v_{ix,1} = -\frac{3}{2} \cdot 0.5 - \frac{3}{2} = -2.25 m/s$$

Chapter 9. Summary Slides

General Principles

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Newton's Second Law

In terms of momentum, Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$

General Principles

Solving Momentum Conservation Problems

MODEL Choose an isolated system or a system that is isolated during at least part of the problem.

VISUALIZE Draw a pictorial representation of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

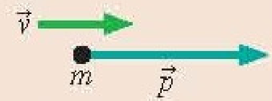
$$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

ASSESS Is the result reasonable?

Important Concepts

Momentum $\vec{p} = m\vec{v}$

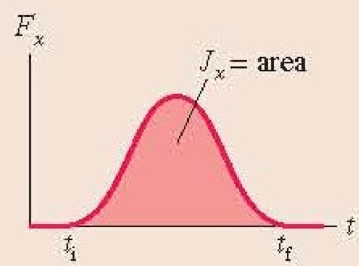


Impulse $J_x = \int_{t_i}^{t_f} F_x(t)\,dt$ = area under force curve

Impulse and momentum are related by the **impulse-momentum theorem**

$$\Delta p_x = J_x$$

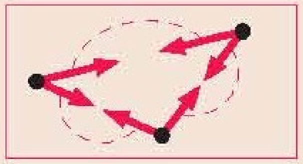
This is an alternative statement of Newton's second law.



Important Concepts

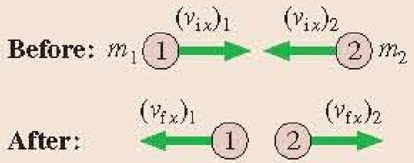
System A group of interacting particles.

Isolated system A system on which there are no external forces or the net external force is zero.



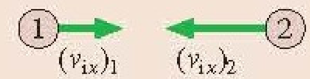
Before-and-after pictorial representation

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.



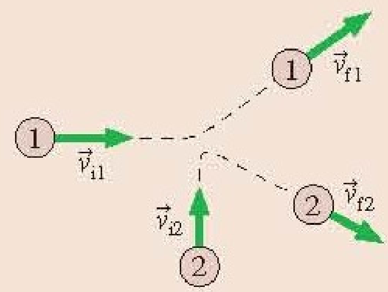
Applications

Collisions Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.



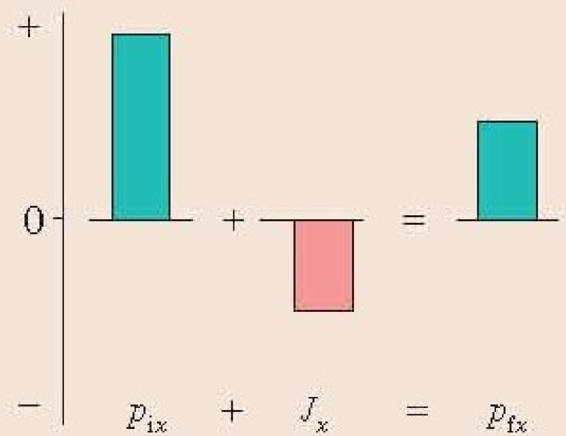
Explosions Two or more particles move away from each other.

Two dimensions No new ideas, but both the x- and y-components of P must be conserved, giving two simultaneous equations.



Applications

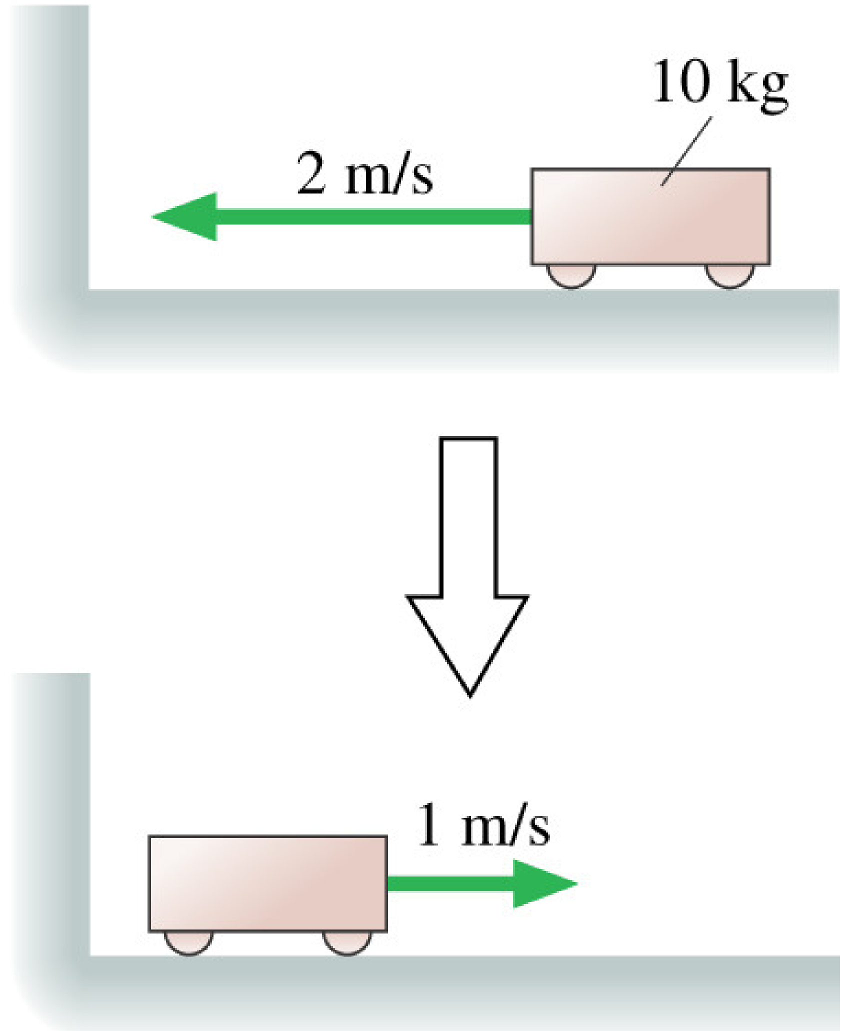
Momentum bar charts display the impulse-momentum theorem $p_{fx} = p_{ix} + J_x$ in graphical form.



Chapter 9. Questions

The cart's change of momentum is

A. 30 kg m/s.
B. 10 kg m/s.
C. –10 kg m/s.
D. –20 kg m/s.
E. –30 kg m/s.



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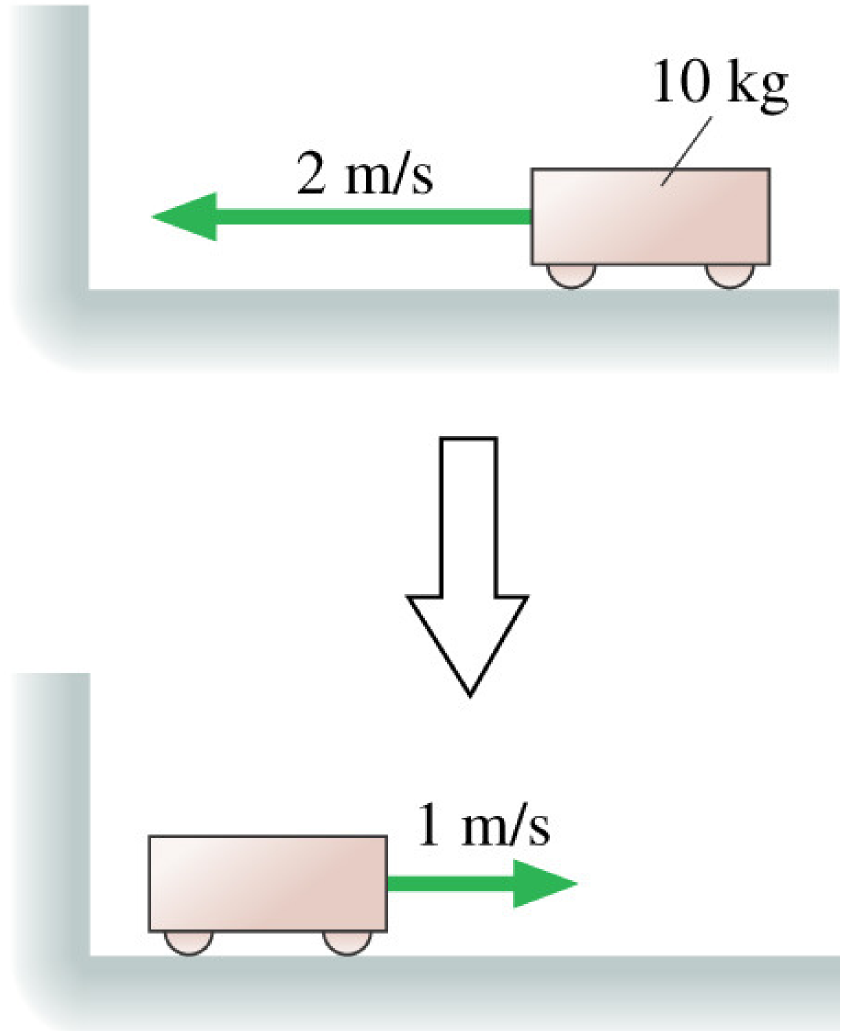

A 10 g rubber ball and a 10 g clay ball are thrown at a wall with equal speeds. The rubber ball bounces, the clay ball sticks. Which ball exerts a larger impulse on the wall?

A. They exert equal impulses because they have equal momenta.
B. The clay ball exerts a larger impulse because it sticks.
C. Neither exerts an impulse on the wall because the wall doesn't move.
D. The rubber ball exerts a larger impulse because it bounces.

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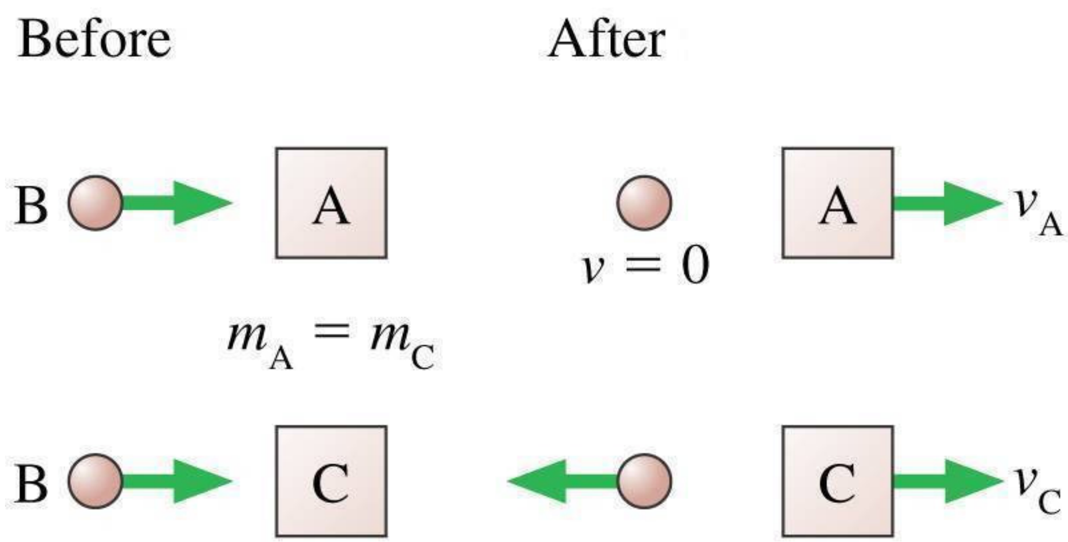
✔ **D. The rubber ball exerts a larger impulse because it bounces.**

Objects A and C are made of different materials, with different "springiness," but they have the same mass and are initially at rest. When ball B collides with object A, the ball ends up at rest. When ball B is thrown with the same speed and collides with object C, the ball rebounds to the left. Compare the velocities of A and C after the collisions. Is v_A greater than, equal to, or less than v_C?



A. $v_A > v_C$

B. $v_A < v_C$

C. $v_A = v_C$

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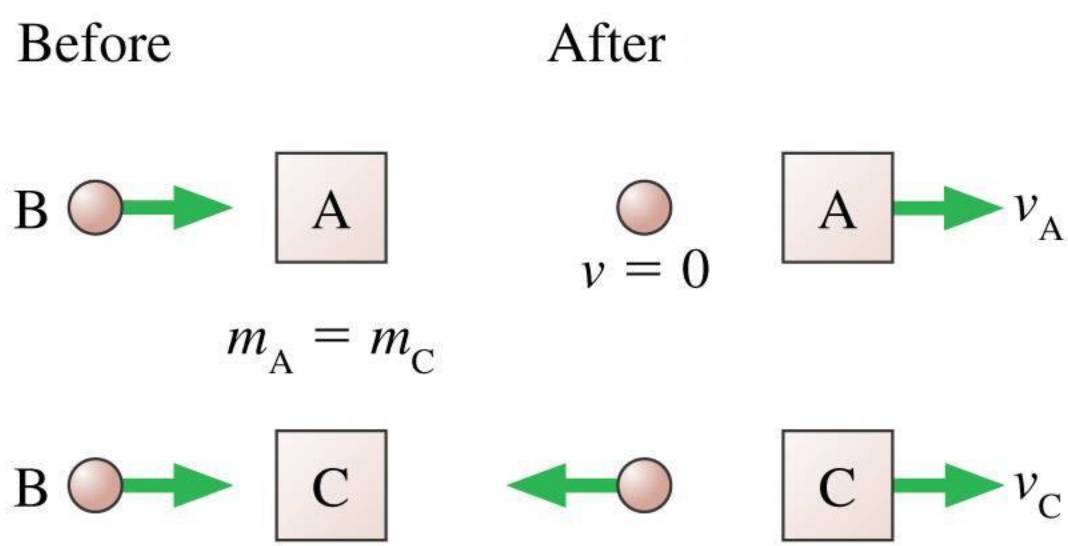

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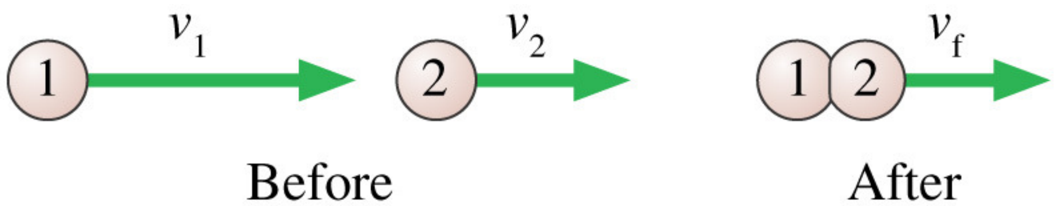


The two particles are both moving to the right. Particle 1 catches up with particle 2 and collides with it. The particles stick together and continue on with velocity v_f. Which of these statements is true?

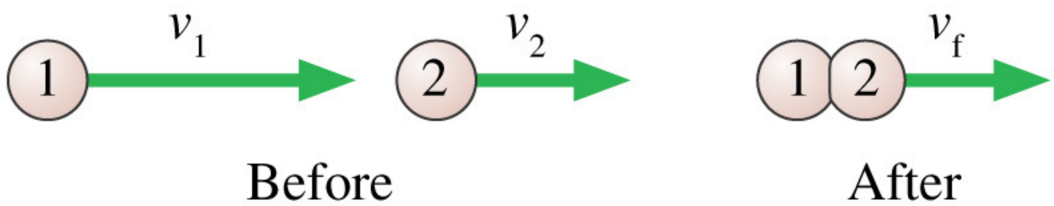
A. $v_f = v_2$.

B. v_f is less than v_2.

C. v_f is greater than v_2, but less than v_1.

D. $v_f = v_1$.

E. v_f is greater than v_1.



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D. $v_f = v_1$.

E. v_f is greater than v_1.

An explosion in a rigid pipe shoots out three pieces. A 6 g piece comes out the right end. A 4 g piece comes out the left end with twice the speed of the 6 g piece. From which end does the third piece emerge?

A. Right end

B. Left end

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