Chapter 7. Newton's Third Law

The harder one sumo wrestler pushes, the harder the other pushes back. Newton's third law describes how two objects interact with each other.

Chapter Goal: To use Newton's third law to understand interacting objects.

Chapter 7. Newton's Third Law

Topics:

- Interacting Objects
- Analyzing Interacting Objects
- Newton's Third Law
- Ropes and Pulleys
- Examples of Interacting-Object Problems

Interacting Objects

If object A exerts a force on object B, then object B exerts a force on object A. The pair of forces, as shown, is called an **action/reaction pair**.

FIGURE 7.2 An action/reaction pair of forces.

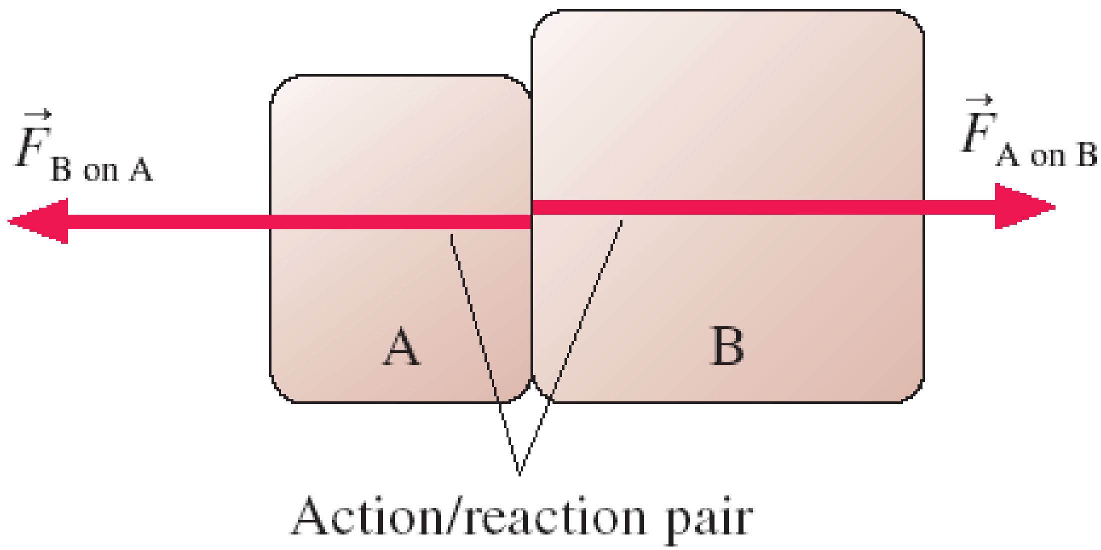
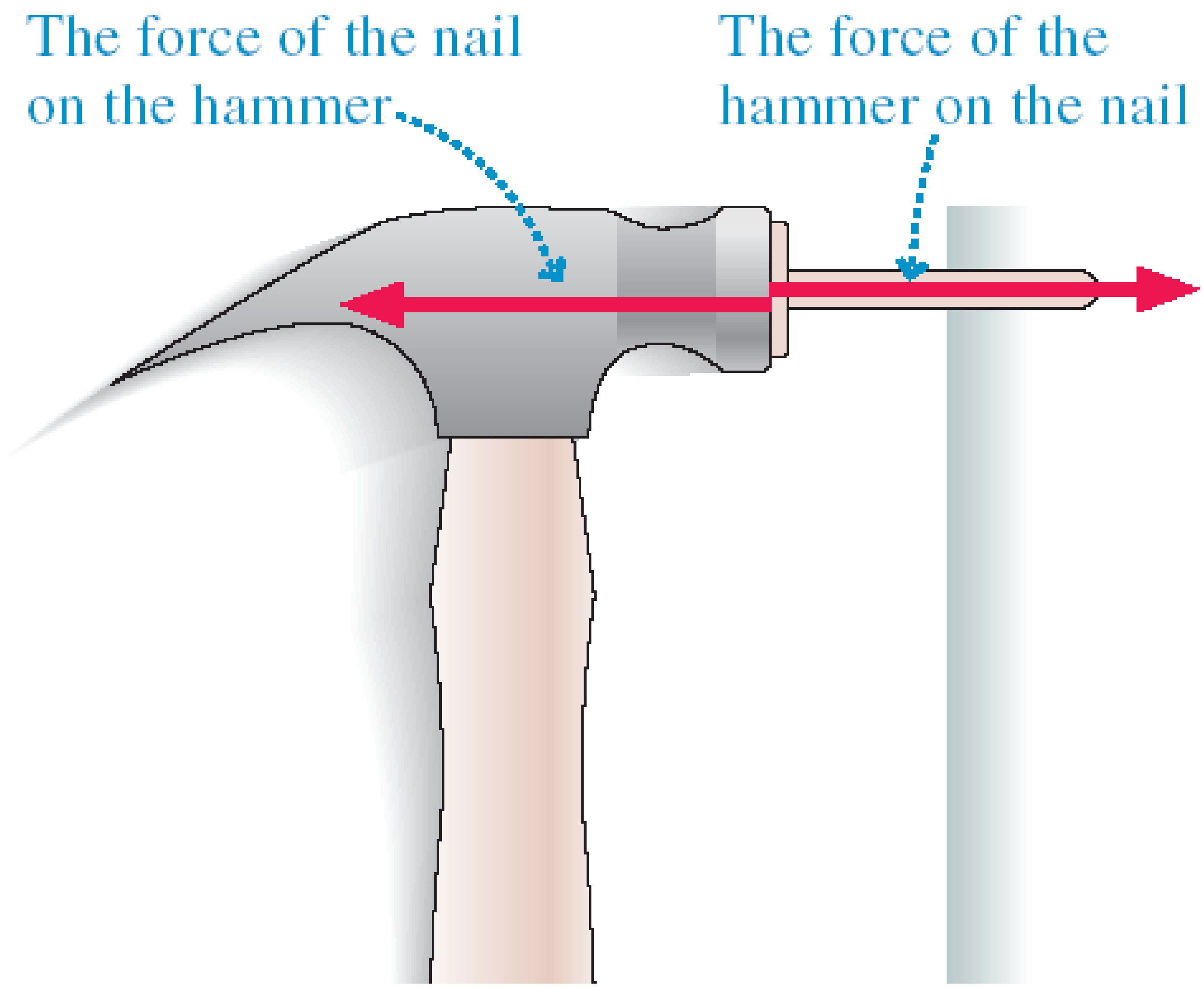
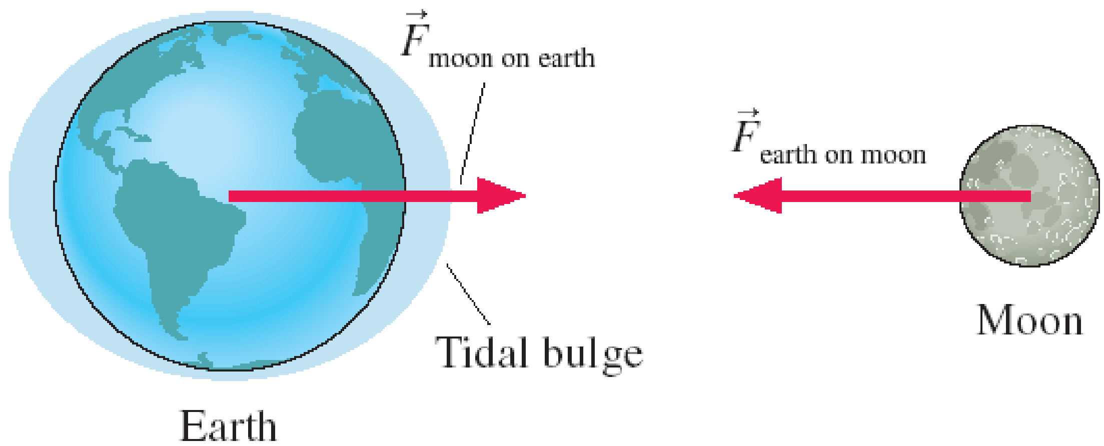


FIGURE 7.1 The hammer and nail are interacting with each other.



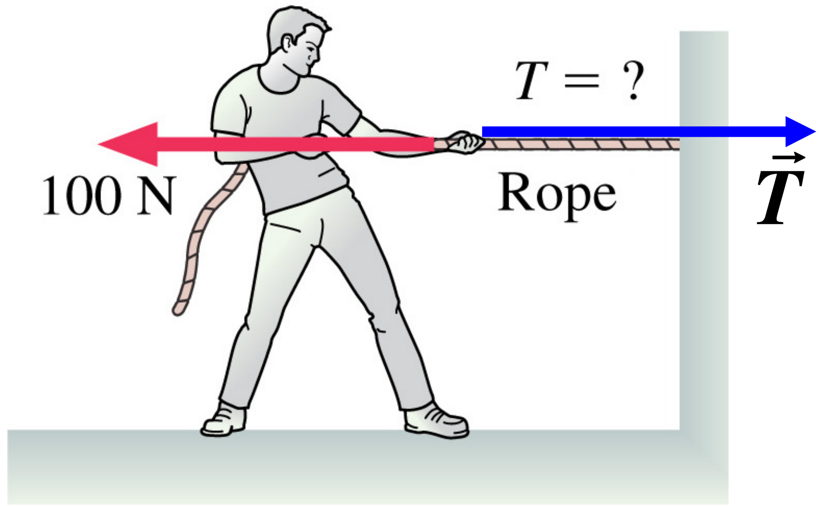
The action-reaction pair never occur on the same object !

FIGURE 7.3 The ocean tides are an indication of the long-range gravitational interaction of the earth and the moon.



Newton's third law Every force occurs as one member of an action/reaction pair of forces.

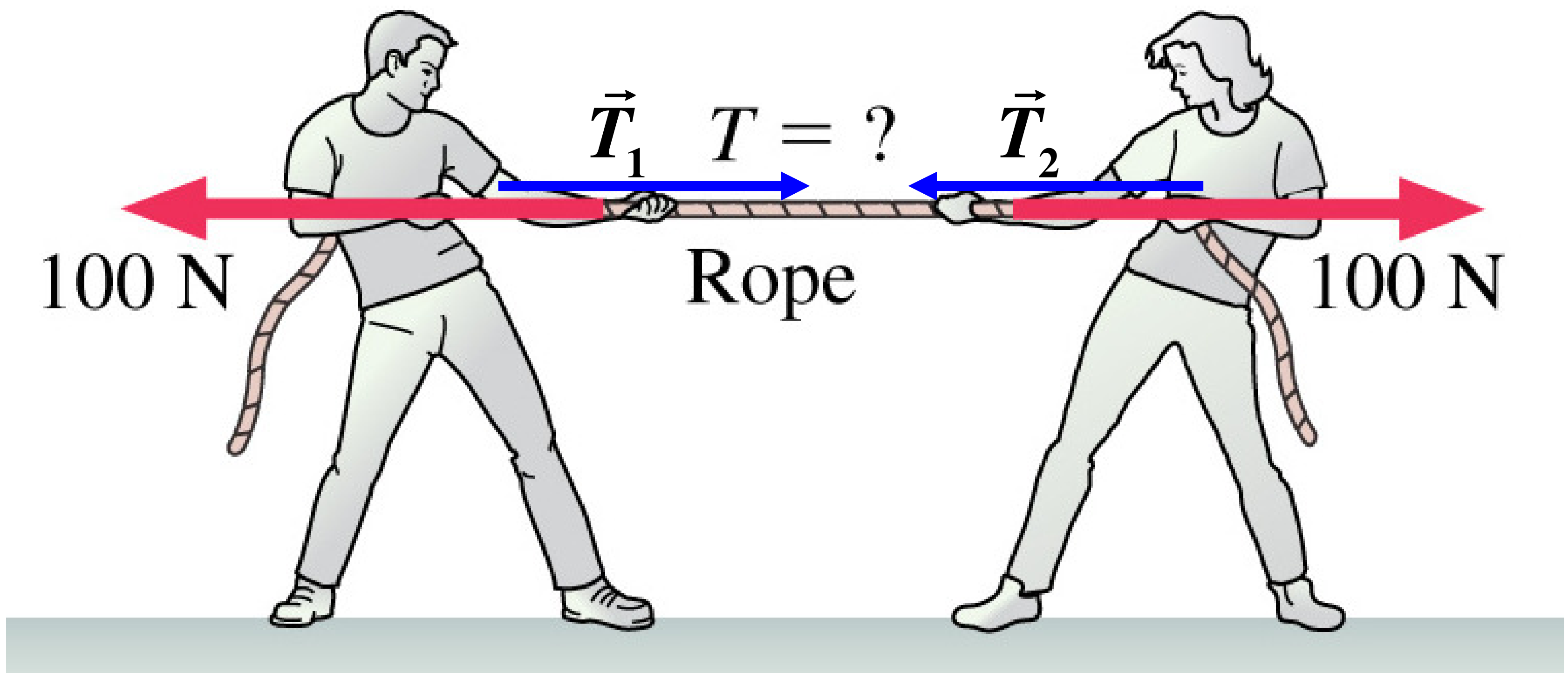
- The two members of an action/reaction pair act on two *different* objects.
- The two members of an action/reaction pair are equal in magnitude but opposite in direction: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$.



Newton's Third Law:

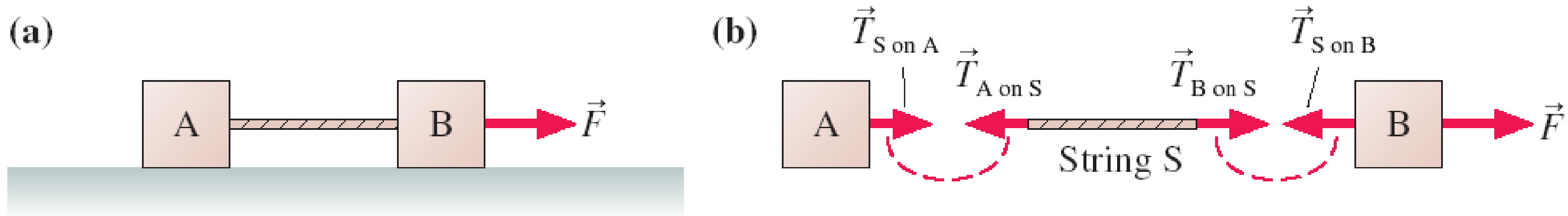
$$T = 100N$$

$$T_1 = T_2 = 100N$$



The Massless String Approximation

FIGURE 7.22 The string's tension pulls forward on block A, backward on block B.



Often in physics and engineering problems the mass of the string or rope is much less than the masses of the objects that it connects. In such cases, we can adopt the following **massless string approximation**:

$$T_{\text{B on S}} = T_{\text{A on S}} \qquad \text{(massless string approximation)}$$

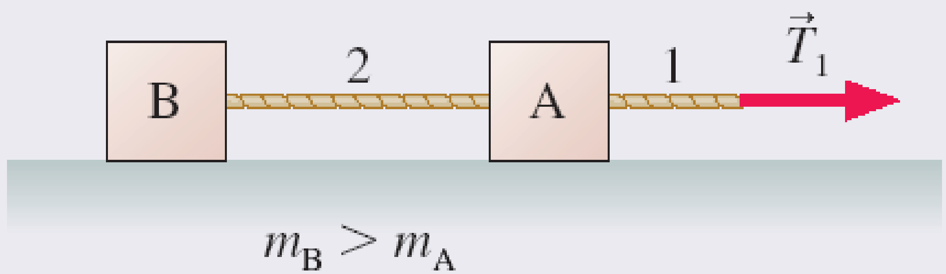
EXAMPLE 7.6 Comparing two tensions

QUESTION:

EXAMPLE 7.6 Comparing two tensions

Blocks A and B in **FIGURE 7.24** are connected by massless string 2 and pulled across a frictionless table by massless string 1. B has a larger mass than A. Is the tension in string 2 larger than, smaller than, or equal to the tension in string 1?

FIGURE 7.24 Blocks A and B are pulled across a frictionless table by massless strings.



EXAMPLE 7.6 Comparing two tensions

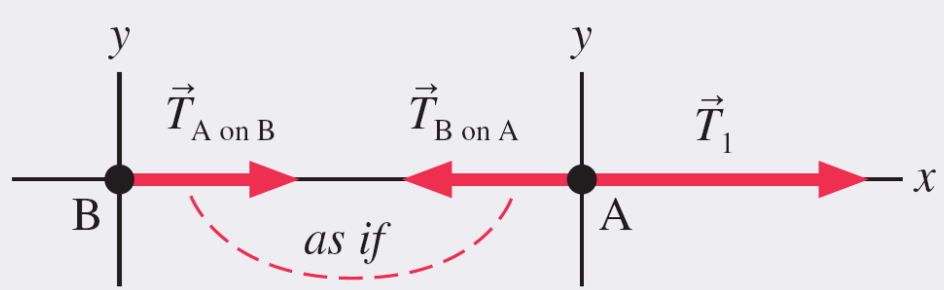
MODEL The massless string approximation allows us to treat A and B *as if* they interact directly with each other. The blocks are accelerating because there's a force to the right and no friction.

EXAMPLE 7.6 Comparing two tensions

SOLVE B has a larger mass, so it may be tempting to conclude that string 2, which pulls B, has a greater tension than string 1, which pulls A. The flaw in this reasoning is that Newton's second law tells us only about the *net* force. The net force on B *is* larger than the net force on A, but the net force on A is *not* just the tension $\vec{T}_1$ in the forward direction. The tension in string 2 also pulls *backward* on A!

FIGURE 7.25 shows the horizontal forces in this frictionless situation. Forces $\vec{T}_{\text{A on B}}$ and $\vec{T}_{\text{B on A}}$ act *as if* they are an action/reaction pair.

FIGURE 7.25 The horizontal forces on blocks A and B.



EXAMPLE 7.6 Comparing two tensions

From Newton's third law,

$$T_{\text{A on B}} = T_{\text{B on A}} = T_2$$

where T_2 is the tension in string 2. From Newton's second law, the net force on A is

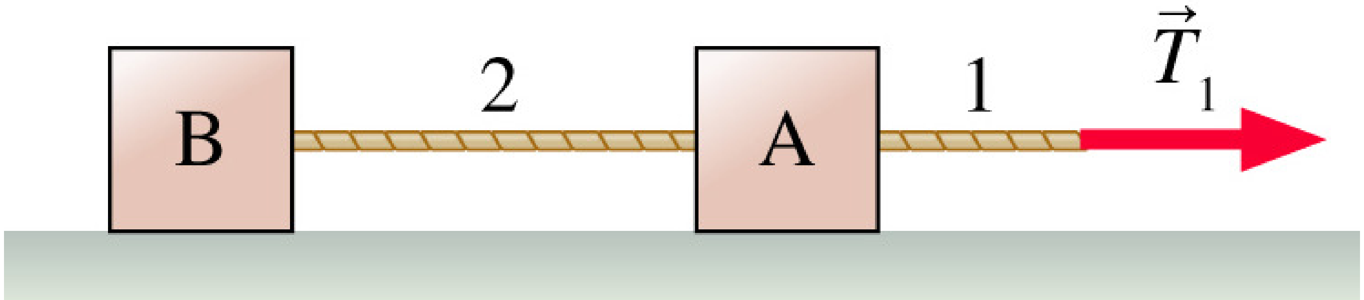
$$(F_{\text{A net}})_x = T_1 - T_{\text{B on A}} = T_1 - T_2 = m_{\text{A}} a_{\text{A}x}$$

The net force on A is the *difference* in tensions. The blocks are accelerating to the right, making $a_{\text{A}x} > 0$, so

$$T_1 > T_2$$

The tension in string 2 is *smaller* than the tension in string 1.

Find tension of rope 2 $T_2 = ?$



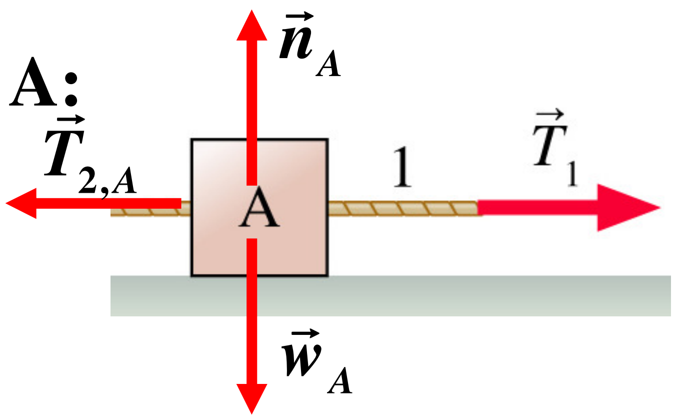
$T_1 = 100N$

$m_A = 1kg$

$m_B = 4kg$

No friction

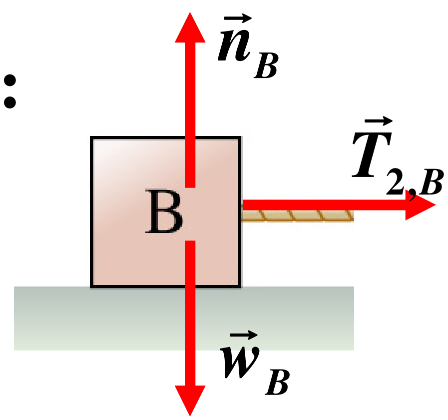
System A:



$$\vec{F}_{A,net} = \vec{T}_{2,A} + \vec{n}_A + \vec{w}_A + \vec{T}_1 = m_A \vec{a}$$

$$T_1 - T_{2,A} = m_A a$$

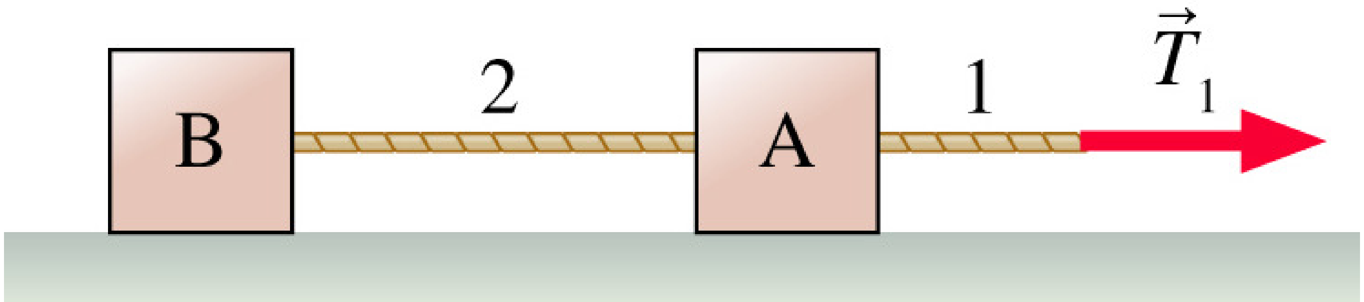
System B:



$$\vec{F}_{B,net} = \vec{T}_{2,B} + \vec{n}_B + \vec{w}_B = m_B \vec{a}$$

$$T_{2,B} = m_B a$$

Find tension of rope 2 $T_2 = ?$



$T_1 = 100N$

$m_A = 1kg$

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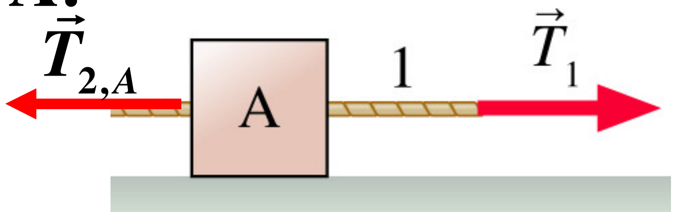
No friction

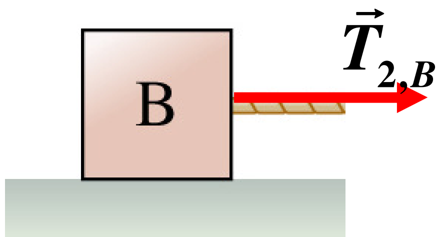
System A:



$\vec{a}$

$$T_1 - T_{2,A} = m_A a$$

System B:



$\vec{a}$

$$T_{2,B} = m_B a$$

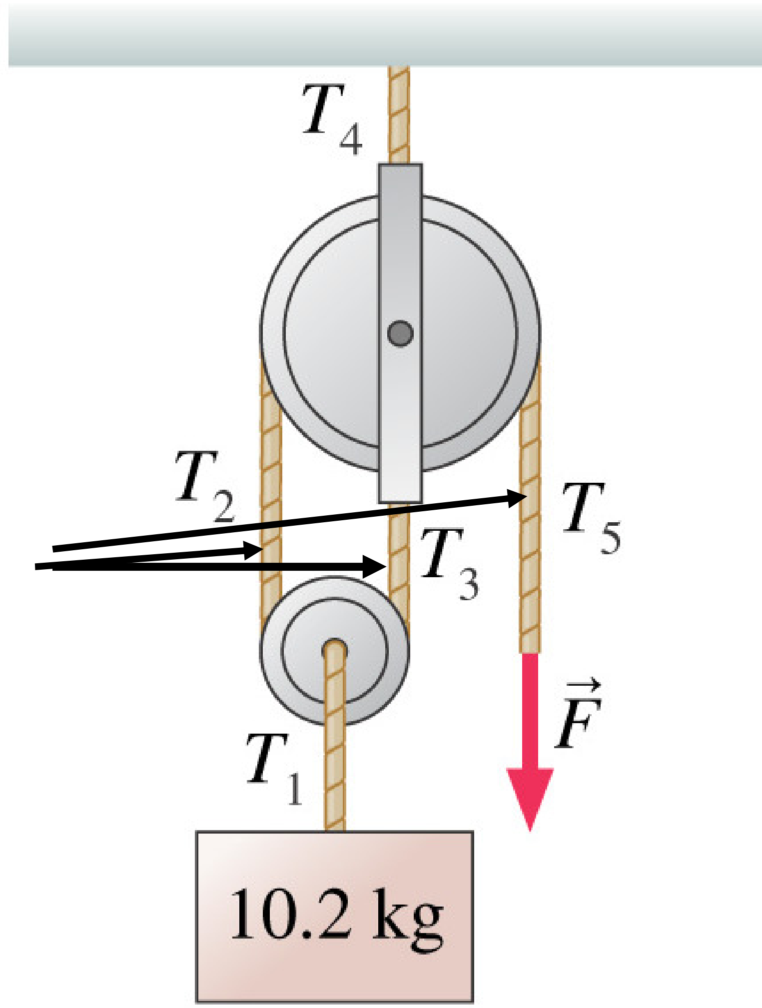
Newton's Third Law: $T_{2,B} = T_{2,A} = T_2$

$$T_1 - T_2 = m_A a$$
$$T_2 = m_B a$$

$$a = \frac{T_1}{m_A + m_B}$$

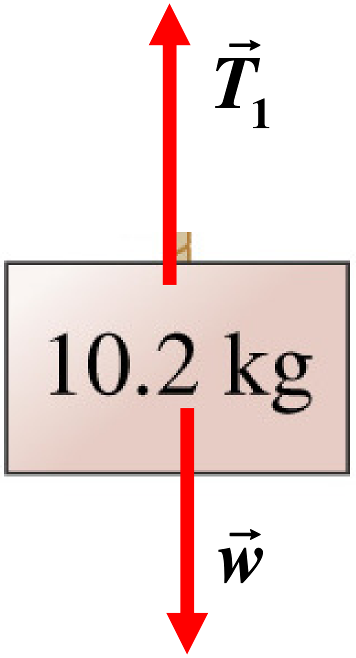
$$T_2 = \frac{m_B T_1}{m_A + m_B} = \frac{4 \cdot 100}{5} = 80N$$

Example: The 10.2 kg block is held in place by the massless rope passing over two massless, frictionless pulleys. Find the tensions T_1 to T_5 and the magnitude of force F.



The same rope:

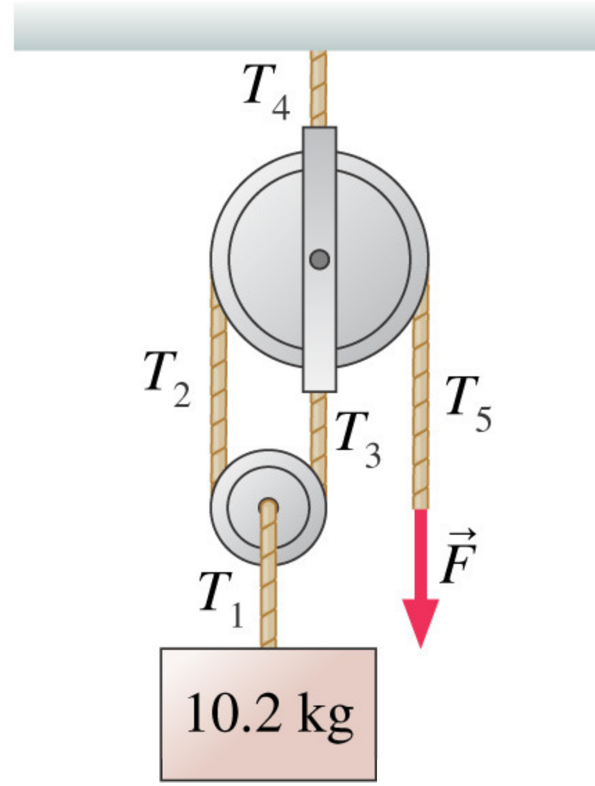
$$T_2 = T_3 = T_5$$

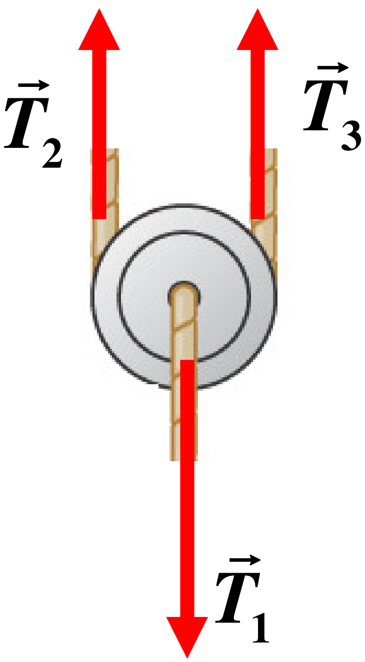


$$T_2 = T_3 = T_5$$

Equilibrium:

$$T_1 = w \approx 10.2 \times 10 = 102N$$

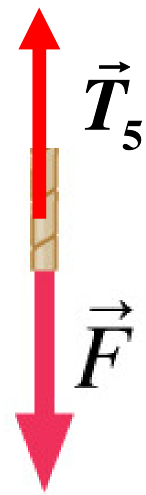




Equilibrium:

$$T_1 = T_3 + T_2 = 2T_2$$

$$T_2 = \frac{T_1}{2} = 51N = T_3 = T_5$$



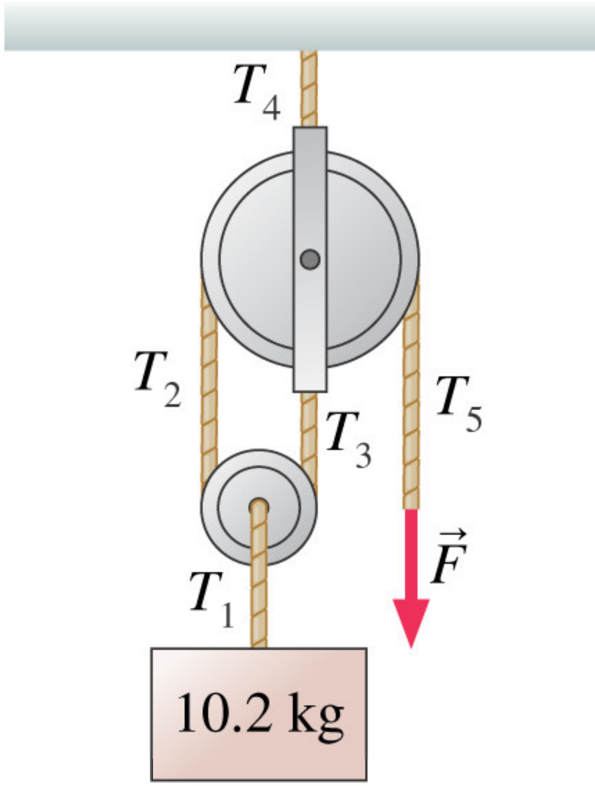
Equilibrium:

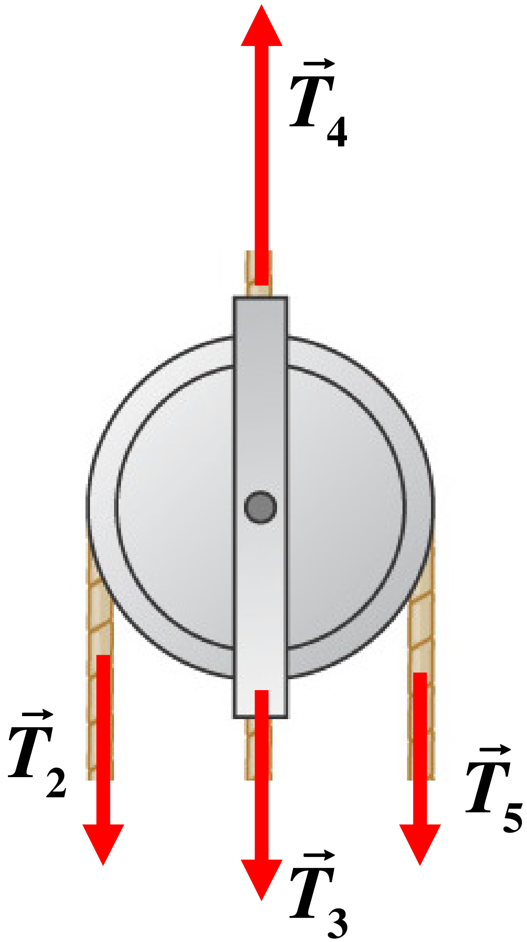
$$F = T_5 = 51N$$

$$T_2 = T_3 = T_5 = 51N$$

$$T_1 = 102N$$

$$F = 51N$$





Equilibrium:

$$T_4 = T_2 + T_3 + T_5 = 153N$$

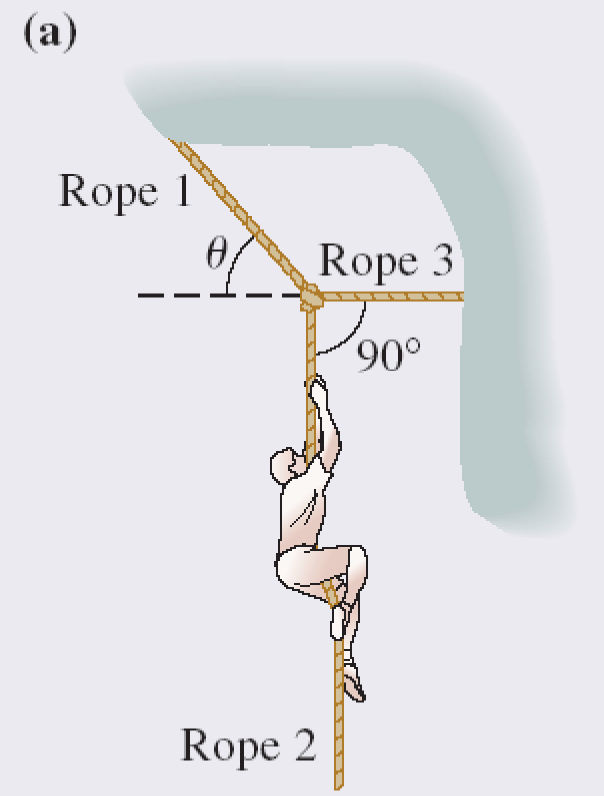
EXAMPLE 7.7 Mountain climbing

QUESTION:

FIGURE 7.27 A mountain climber hanging from ropes.

EXAMPLE 7.7 Mountain climbing

A 90 kg mountain climber is suspended from the ropes shown in FIGURE 7.27a. The maximum tension that rope 3 can withstand before breaking is 1500 N. What is the smallest that angle θ can become before the rope breaks and the climber falls into the gorge?

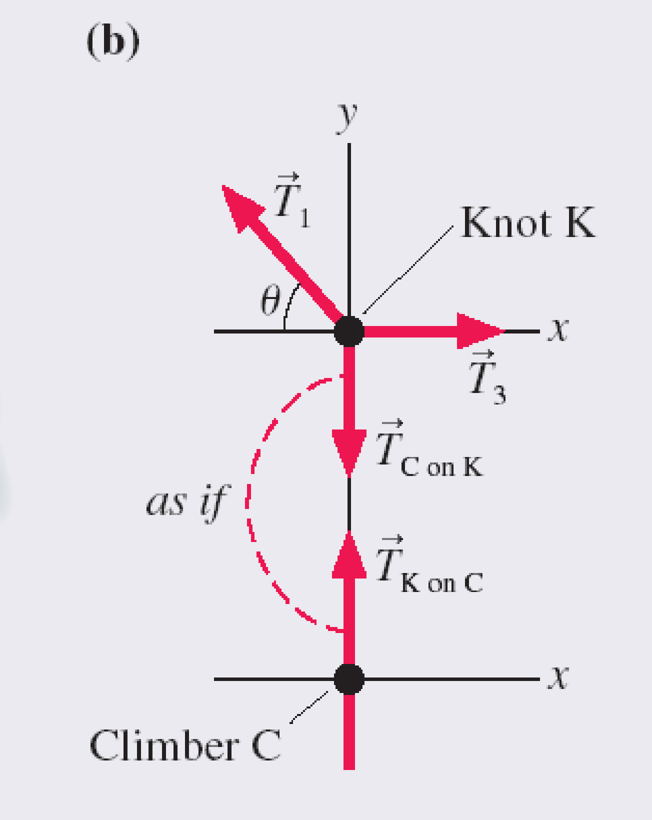


EXAMPLE 7.7 Mountain climbing

MODEL Climber C, who can be modeled as a particle, is one object. The other point where forces are exerted is the knot, where the three ropes are tied together. We'll consider knot K to be a second object. Both objects are in static equilibrium. We'll assume massless ropes.

EXAMPLE 7.7 Mountain climbing

VISUALIZE **FIGURE 7.27b** shows two free-body diagrams. Forces $\vec{T}_{\text{C on K}}$ and $\vec{T}_{\text{K on C}}$ are not, strictly speaking, an action/reaction pair because the climber is not in contact with the knot. But if the ropes are massless, $\vec{T}_{\text{C on K}}$ and $\vec{T}_{\text{K on C}}$ act *as if* they are an action/reaction pair.



EXAMPLE 7.7 Mountain climbing

SOLVE This is static equilibrium, so the net forces on the climber and on the knot are zero.

For the climber:

$$\sum (F_{\text{on C}})_y = T_{\text{K on C}} - mg = 0$$

And for the knot:

$$\sum (F_{\text{on K}})_x = T_3 - T_1 \cos\theta = 0$$

$$\sum (F_{\text{on K}})_y = T_1 \sin\theta - T_{\text{C on K}} = 0$$

From Newton's third law,

$$T_{\text{C on K}} = T_{\text{K on C}}$$

EXAMPLE 7.7 Mountain climbing

But $T_{\text{K on C}} = mg$ from the climber's equation, so $T_{\text{C on K}} = mg$. Using this gives us the knot's equations:

$$T_1 \cos\theta = T_3$$

$$T_1 \sin\theta = mg$$

Dividing the second of these by the first gives

$$\frac{T_1 \sin\theta}{T_1 \cos\theta} = \tan\theta = \frac{mg}{T_3}$$

If angle θ is too small, tension T_3 will exceed 1500 N. The smallest possible θ, at which T_3 reaches 1500 N, is

$$\theta_{\min} = \tan^{-1}\left(\frac{mg}{T_{3\,\max}}\right) = \tan^{-1}\left(\frac{(90\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N}}\right) = 30°$$

Chapter 7. Summary Slides

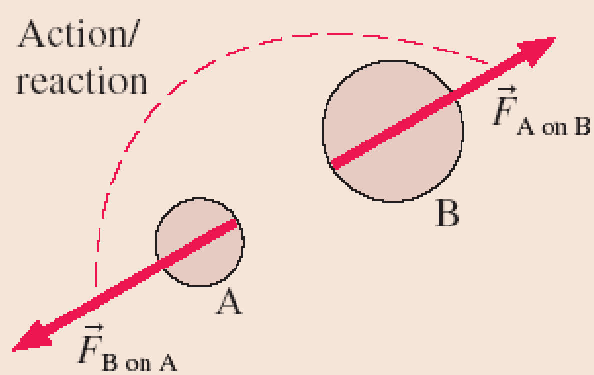
General Principles

Newton's Third Law

Every force occurs as one member of an **action/reaction pair** of forces. The two members of an action/reaction pair:

- Act on two *different* objects.
- Are equal in magnitude but opposite in direction:

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$



General Principles

Solving Interacting-Objects Problems

MODEL Choose the objects of interest.

VISUALIZE

Draw a pictorial representation.

- Sketch and define coordinates.
- Identify acceleration constraints.
- Draw an interaction diagram.
- Draw a separate free-body diagram for each object.
- Connect action/reaction pairs with dashed lines.

SOLVE Write Newton's second law for each object.

- Include *all* forces acting *on* each object.
- Use Newton's third law to equate the magnitudes of action/reaction pairs.
- Include acceleration constraints and friction.

ASSESS Is the result reasonable?

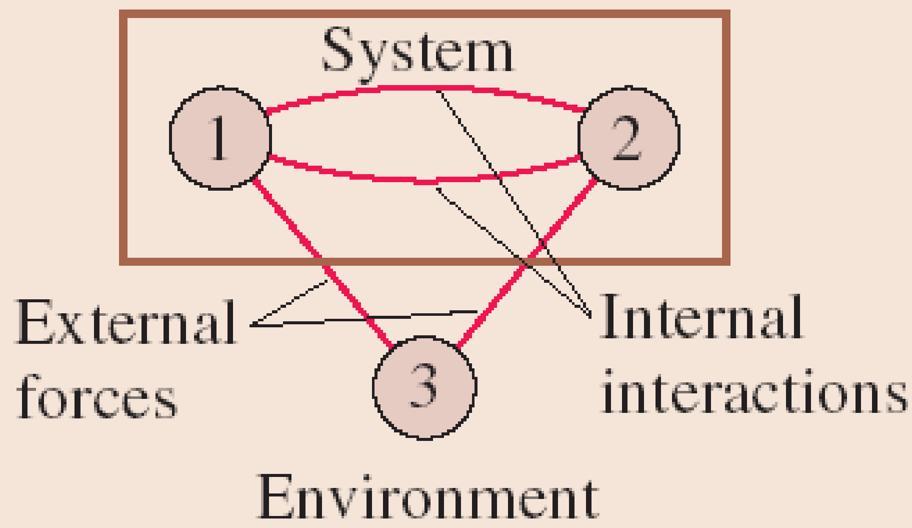
Important Concepts

Objects, systems, and the environment

Objects whose motion is of interest are the system.
Objects whose motion is not of interest form the environment.
The objects of interest interact with the environment, but those interactions can be considered external forces.

Important Concepts

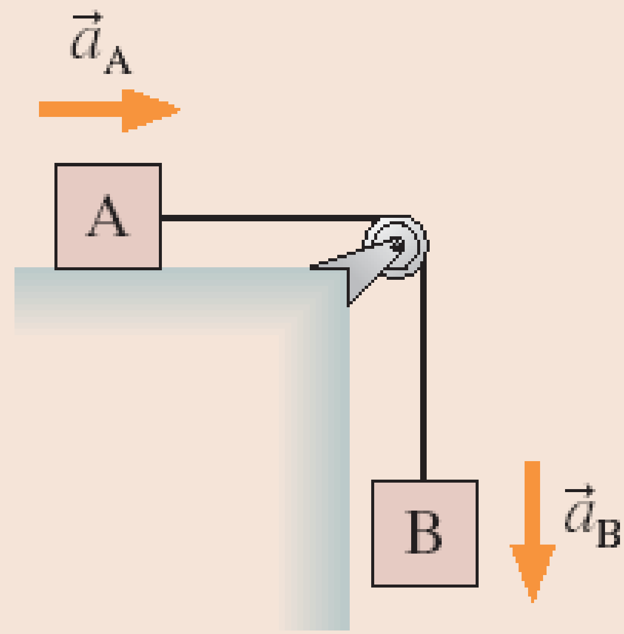
Interaction diagram



Applications

Acceleration constraints

Objects that are constrained to move together must have accelerations of equal magnitude: $a_A = a_B$. This must be expressed in terms of components, such as $a_{Ax} = -a_{By}$.

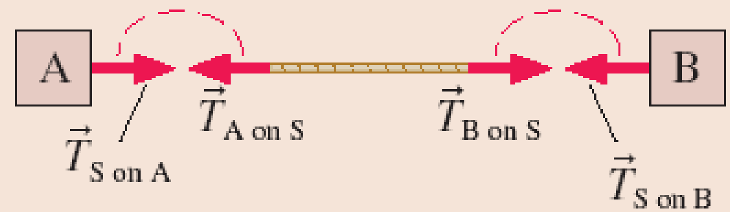


Applications

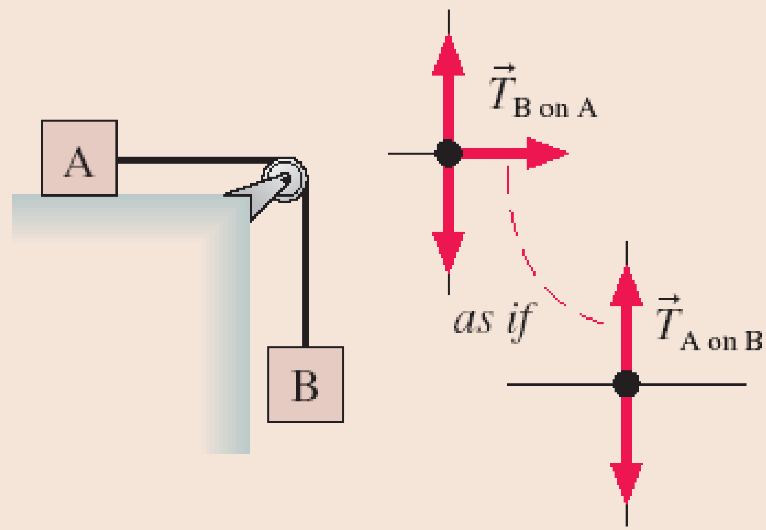
Strings and pulleys

The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:

- Massless, or
- In equilibrium



Objects connected by massless strings passing over massless, frictionless pulleys act *as if* they interact via an action/reaction pair of forces.



Chapter 7. Questions

A fishing line of negligible mass lifts a fish upward at constant speed. The line and the fish are the system, the fishing pole is part of the environment. What, if anything, is wrong with the free-body diagrams?

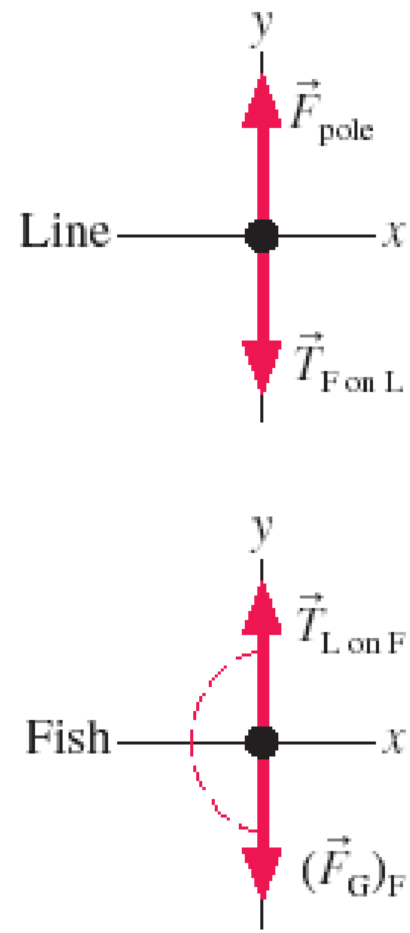
A. The force of the pole on the fish is missing.

B. The force of gravity on the line is missing.

C. The gravitational force and the tension force are incorrectly identified as an action/reaction pair.

D. There should be only one force on the fish.

E. There is nothing wrong with the free-body diagrams.



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✔ **C. The gravitational force and the tension force are incorrectly identified as an action/reaction pair.**

D. There should be only one force on the fish.

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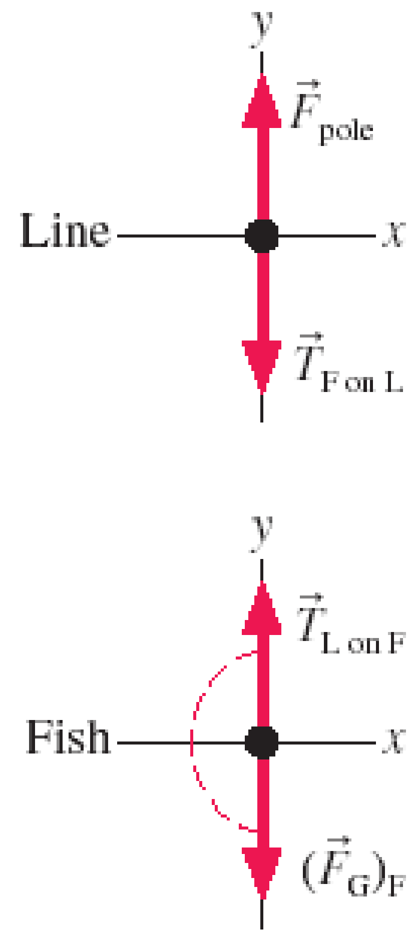
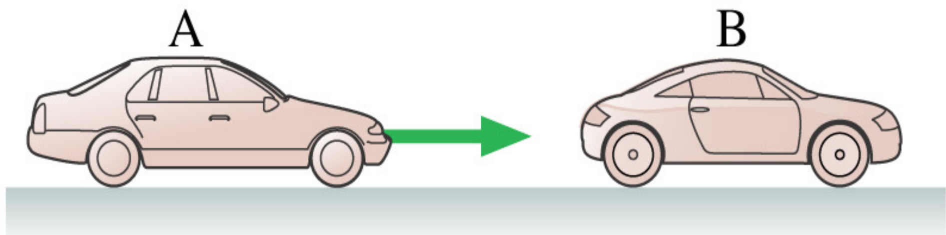


Car B is stopped for a red light. Car A, which has the same mass as car B, doesn't see the red light and runs into the back of B. Which of the following statements is true?



A. A exerts a larger force on B than B exerts on A.
B. A exerts a force on B but B doesn't exert a force on A.
C. B exerts a force on A but A doesn't exert a force on B.
D. B exerts a larger force on A than A exerts on B.
E. B exerts the same amount of force on A as A exerts on B.

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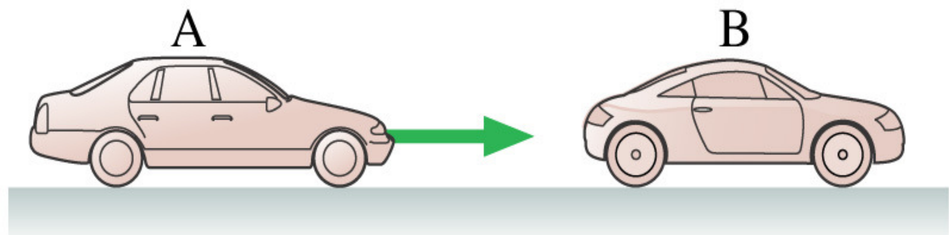
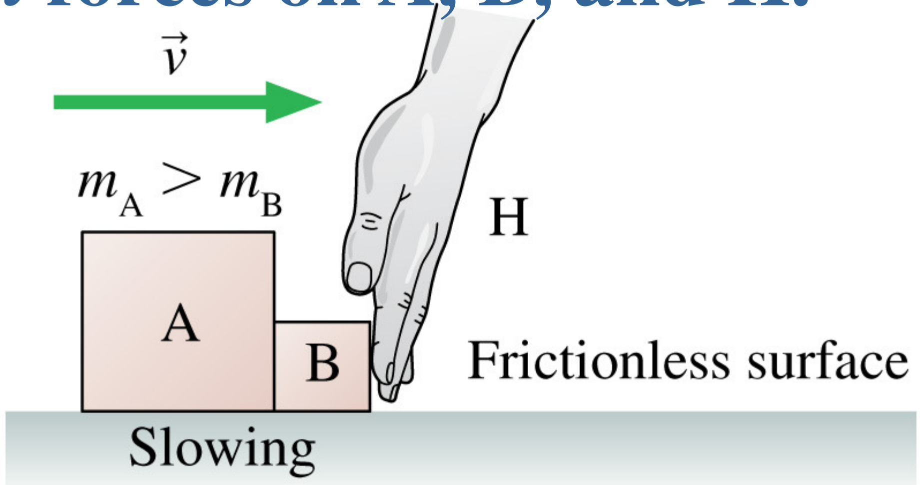


A. A exerts a larger force on B than B exerts on A.
B. A exerts a force on B but B doesn't exert a force on A.
C. B exerts a force on A but A doesn't exert a force on B.
D. B exerts a larger force on A than A exerts on B.
✔ **E. B exerts the same amount of force on A as A exerts on B.**

Boxes A and B are sliding to the right across a frictionless table. The hand H is slowing them down. The mass of A is larger than the mass of B. Rank in order, from largest to smallest, the *horizontal* forces on A, B, and H.



A. $F_{\text{H on B}} = F_{\text{H on A}} > F_{\text{A on B}}$

B. $F_{\text{B on H}} = F_{\text{H on B}} = F_{\text{A on B}} = F_{\text{B on A}}$

C. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$

D. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$

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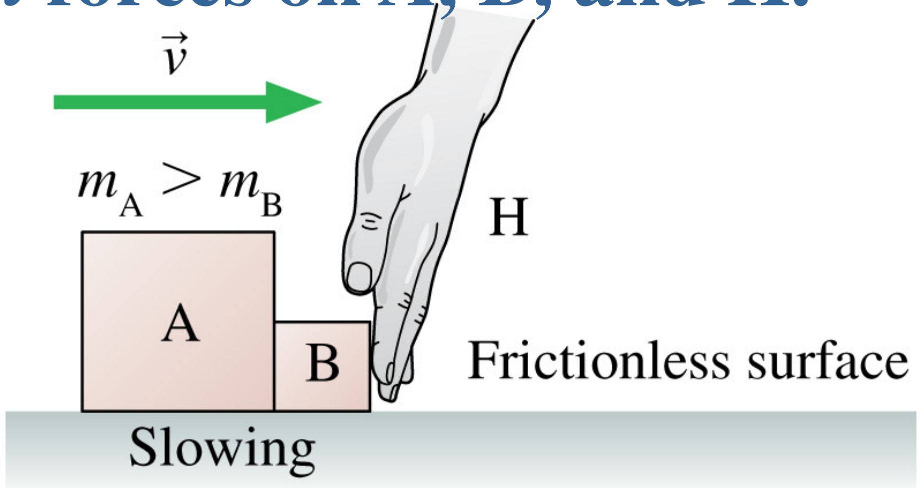


A. $F_{\text{H on B}} = F_{\text{H on A}} > F_{\text{A on B}}$

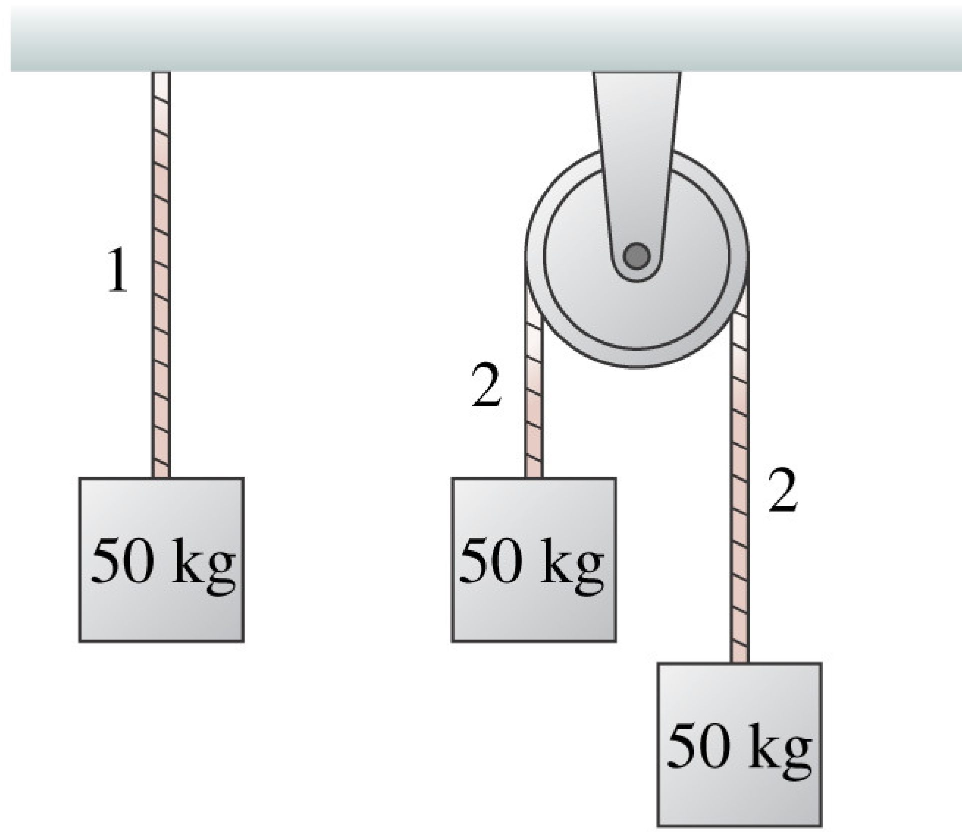
B. $F_{\text{B on H}} = F_{\text{H on B}} = F_{\text{A on B}} = F_{\text{B on A}}$

✔ **C. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$**

D. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$

All three 50 kg blocks are at rest. Is the tension in rope 2 greater than, less than, or equal to the tension in rope 1?

A. Equal to
B. Greater than
C. Less than

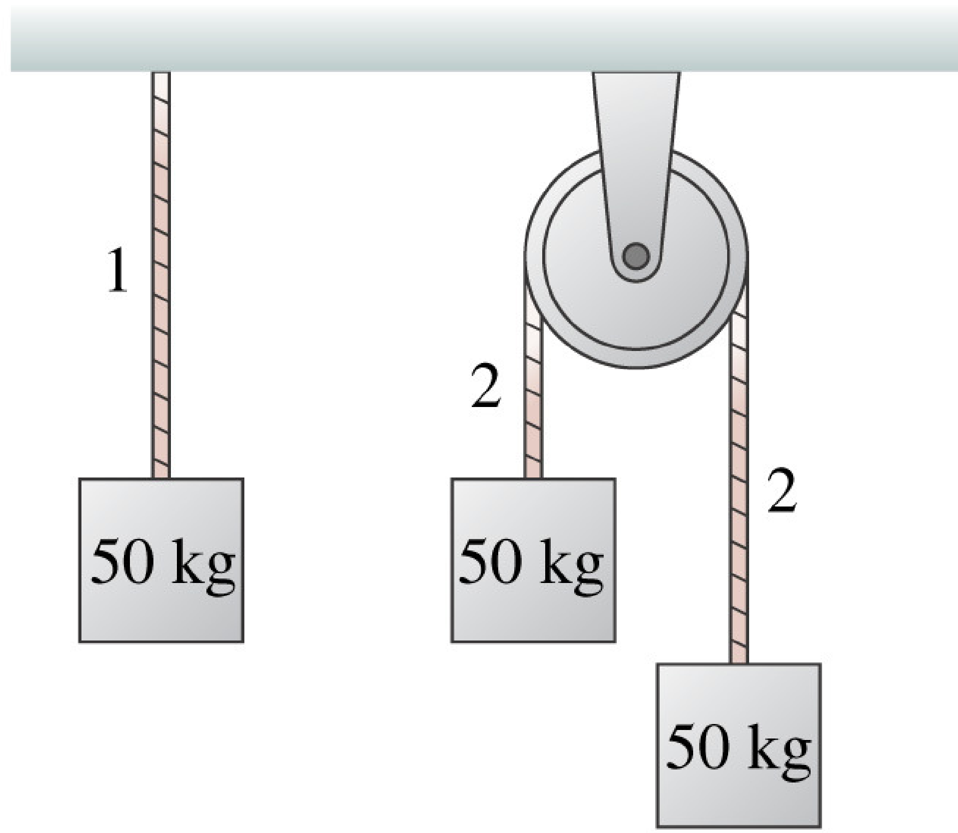


All three 50 kg blocks are at rest. Is the tension in rope 2 greater than, less than, or equal to the tension in rope 1?

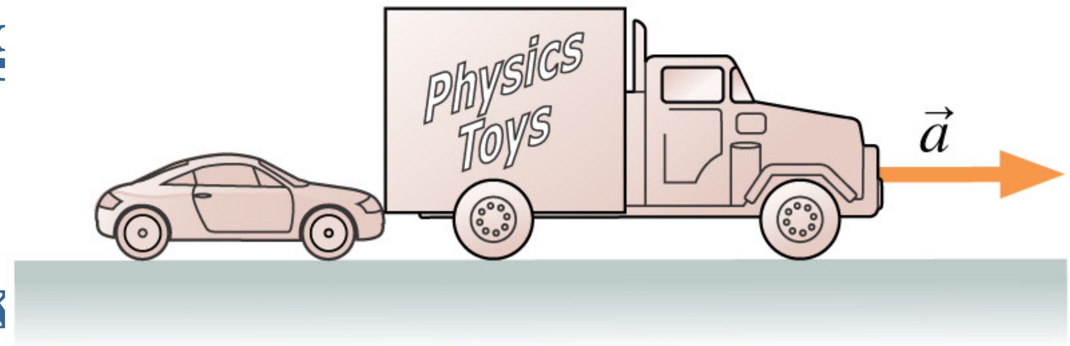
✔ **A. Equal to**

B. Greater than

C. Less than



A small car is pushing a larger truck that has a dead battery. The mass of the truck is larger than the mass of the car. Which of the following statements is true?



A. The truck exerts a larger force on the car than the car exerts on the truck.

B. The truck exerts a force on the car but the car doesn't exert a force on the truck.

C. The car exerts a force on the truck but the truck doesn't exert a force on the car.

D. The car exerts a larger force on the truck than the truck exerts on the car.

E. The car exerts the same amount of force on the truck as the truck exerts on the car.

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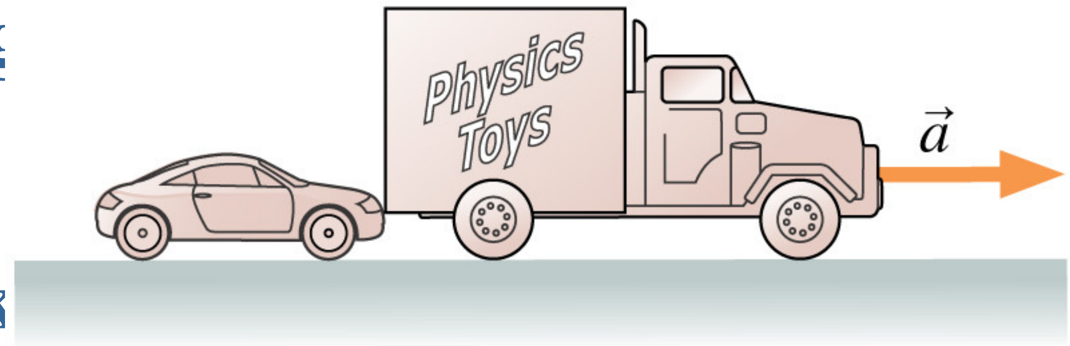

A. The truck exerts a larger force on the car than the car exerts on the truck.

B. The truck exerts a force on the car but the car doesn't exert a force on the truck.

C. The car exerts a force on the truck but the truck doesn't exert a force on the car.

D. The car exerts a larger force on the truck than the truck exerts on the car.

✔ **E. The car exerts the same amount of force on the truck as the truck exerts on the car.**