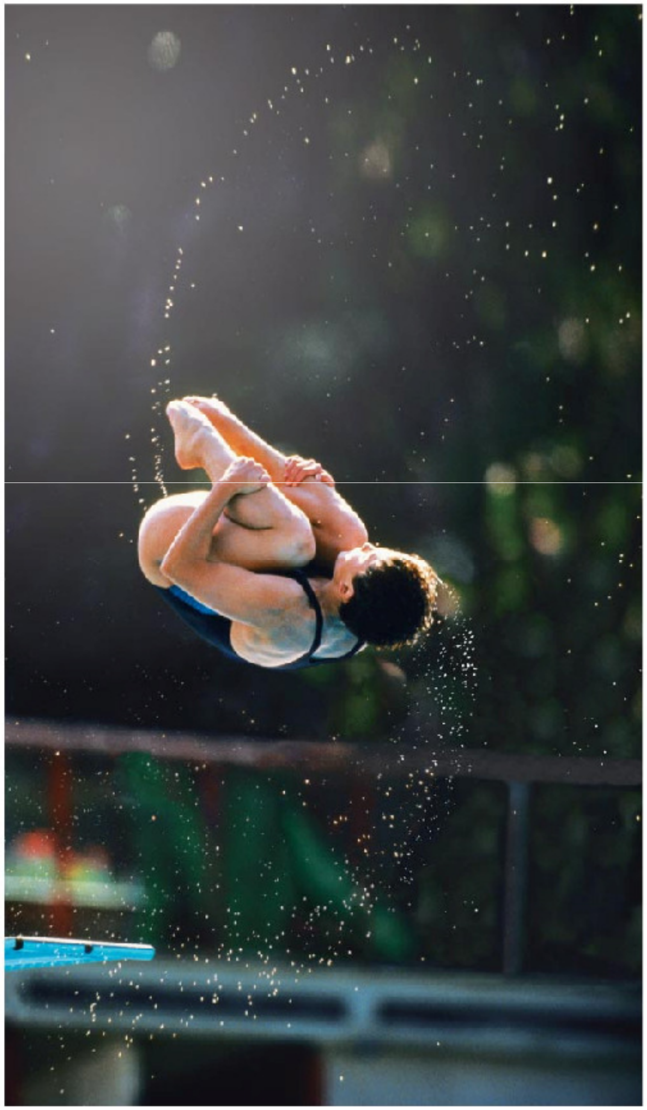
Chapter 12. Rotation of a Rigid Body

Not all motion can be described as that of a particle. Rotation requires the idea of an extended object. This diver is moving toward the water along a parabolic trajectory, and she's rotating rapidly around her center of mass.

Chapter Goal: To understand the physics of rotating objects.



Chapter 12. Rotation of a Rigid Body

Topics:

- Rotational Motion
- Rotation About the Center of Mass
- Rotational Energy
- Calculating Moment of Inertia
- Torque
- Rotational Dynamics
- Rotation About a Fixed Axis
- Static Equilibrium
- Rolling Motion
- The Vector Description of Rotational Motion
- Angular Momentum of a Rigid Body

Chapter 12. Reading Quizzes

A new way of multiplying two vectors is introduced in this chapter. What is it called:

A. Dot Product
B. Scalar Product
C. Tensor Product
D. Cross Product
E. Angular Product

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C. Tensor Product
✔ **D. Cross Product**
E. Angular Product

Moment of inertia is

A. the rotational equivalent of mass.
B. the point at which all forces appear to act.
C. the time at which inertia occurs.
D. an alternative term for *moment arm*.

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✔ **A. the rotational equivalent of mass.**

B. the point at which all forces appear to act.

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A rigid body is in equilibrium if

A. $\vec{F}_{\text{net}} = 0.$

B. $\vec{\tau}_{\text{net}} = 0.$

C. neither A nor B.

D. either A or B.

E. both A and B.

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Chapter 12. Basic Content and Examples

Rotational Motion

The figure shows a wheel rotating on an axle. Its angular velocity is

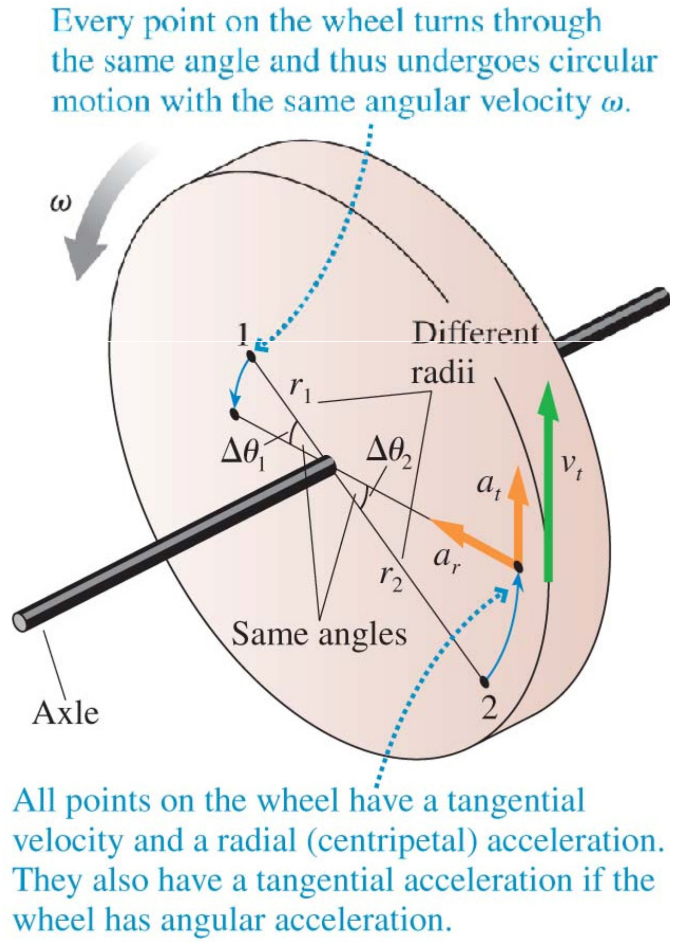
$$\omega = \frac{d\theta}{dt}$$

The units of ω are rad/s. If the wheel is speeding up or slowing down, its angular acceleration is

$$\alpha = \frac{d\omega}{dt}$$

The units of α are rad/s^2.

FIGURE 12.3 Two points on a wheel rotate with the same angular velocity.



Rotational Motion

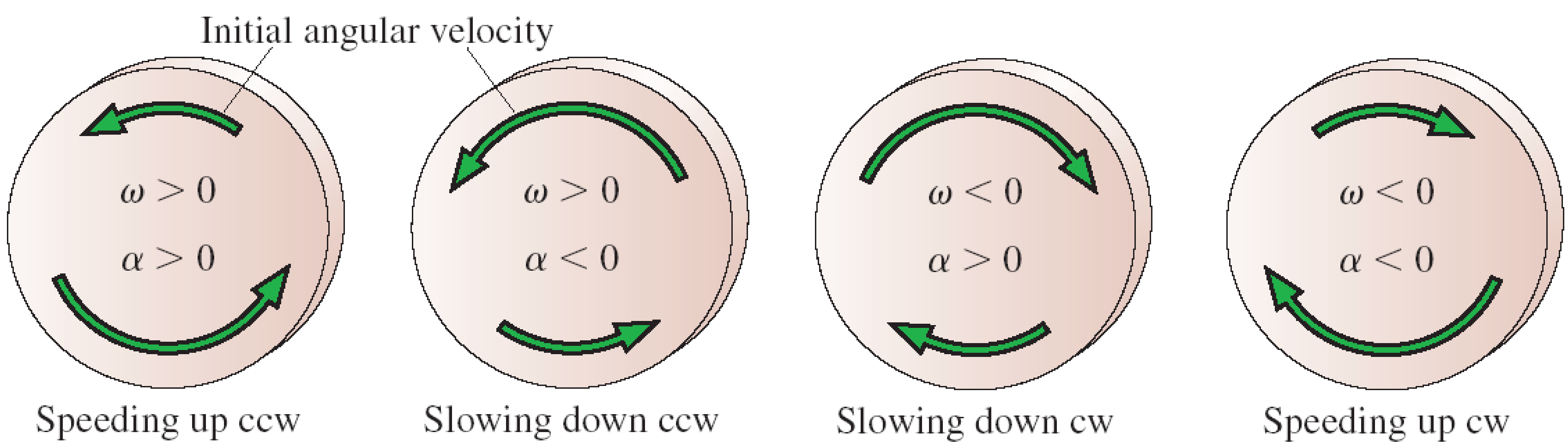
TABLE 12.1 Rotational kinematics for constant angular acceleration

$$\omega_f = \omega_i + \alpha\,\Delta t$$

$$\theta_f = \theta_i + \omega_i\,\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$

FIGURE 12.4 The signs of angular velocity and angular acceleration.



EXAMPLE 12.1 A rotating crankshaft

QUESTION:

EXAMPLE 12.1 A rotating crankshaft

A car engine is idling at 500 rpm. When the light turns green, the crankshaft rotation speeds up at a constant rate to 2500 rpm over an interval of 3.0 s. How many revolutions does the crankshaft make during these 3.0 s?

EXAMPLE 12.1 A rotating crankshaft

MODEL The crankshaft is a rotating rigid body with constant angular acceleration.

EXAMPLE 12.1 A rotating crankshaft

SOLVE Imagine painting a dot on the crankshaft. Let the dot be at $\theta_i = 0$ rad at $t = 0$ s. Three seconds later the dot will have turned to angle

$$\theta_f = \omega_i \, \Delta t + \frac{1}{2}\alpha(\Delta t)^2$$

where $\Delta t = 3.0$ s. We can find the angular acceleration from the initial and final angular velocities, but first they must be converted to SI units:

$$\omega_i = 500 \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 52.4 \text{ rad/s}$$

$$\omega_f = 2500 \frac{\text{rev}}{\text{min}} = 5\omega_i = 262.0 \text{ rad/s}$$

EXAMPLE 12.1 A rotating crankshaft

The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{262.0 \text{ rad/s} - 52.4 \text{ rad/s}}{3.0 \text{ s}} = 69.9 \text{ rad/s}^2$$

During these 3.0 s, the dot turns through an angle

$$\Delta\theta = (52.4 \text{ rad/s})(3.0 \text{ s}) + \frac{1}{2}(69.9 \text{ rad/s}^2)(3.0 \text{ s})^2 = 472 \text{ rad}$$

Because $472/2\pi = 75$, the crankshaft completes 75 revolutions as it spins up to 2500 rpm.

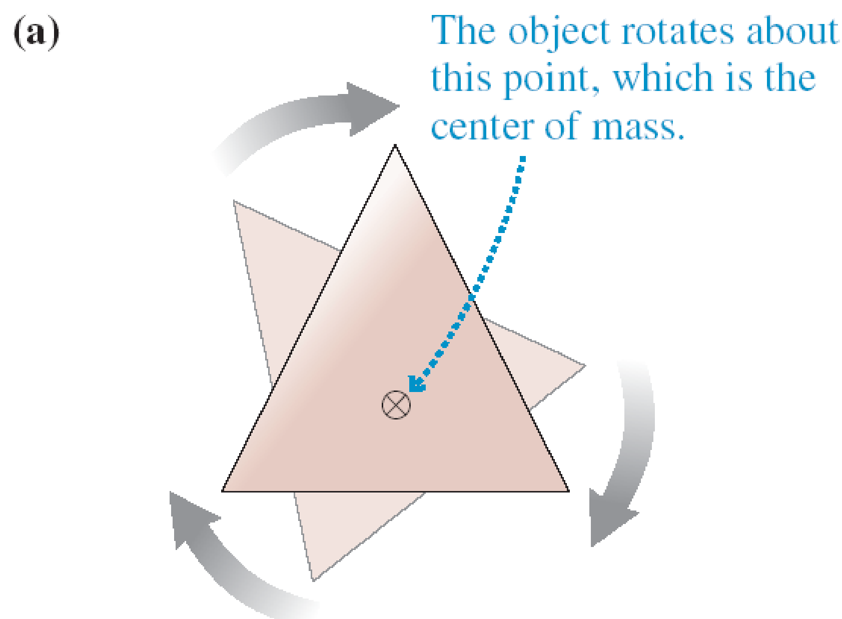
EXAMPLE 12.1 A rotating crankshaft

ASSESS This problem is solved just like the linear kinematics problems you learned to solve in Chapter 2.

Rotation About the Center of Mass

An unconstrained object (i.e., one not on an axle or a pivot) on which there is no net force rotates about a point called the center of mass. The center of mass remains motionless while every other point in the object undergoes circular motion around it.

FIGURE 12.5 Rotation about the center of mass.

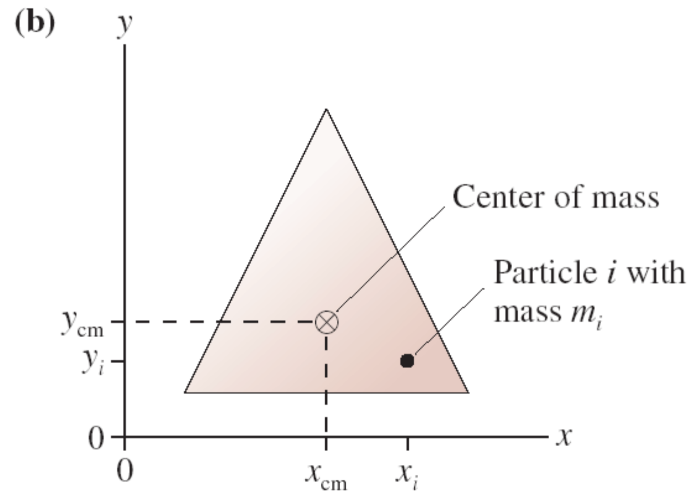


Rotation About the Center of Mass

The center of mass is the mass-weighted center of the object.

$$x_{\text{cm}} = \frac{1}{M}\int x\,dm \qquad \text{and} \qquad y_{\text{cm}} = \frac{1}{M}\int y\,dm$$

FIGURE 12.5 Rotation about the center of mass.



Rotational Energy

A rotating rigid body has kinetic energy because all atoms in the object are in motion. The kinetic energy due to rotation is called **rotational kinetic energy**.

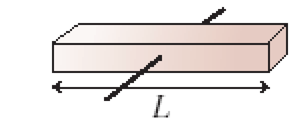
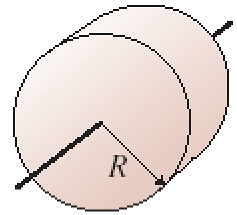
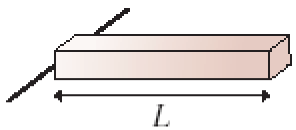
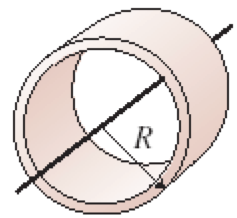
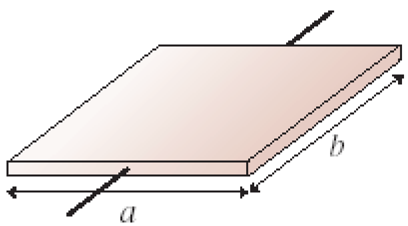
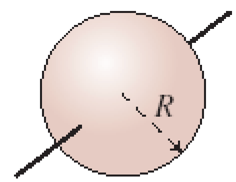
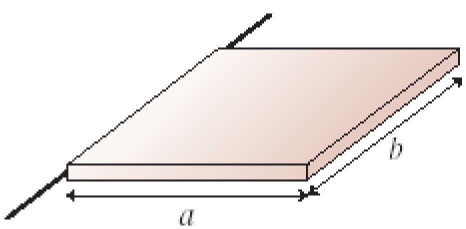
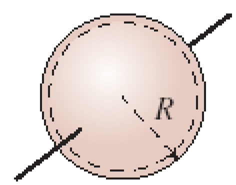
$$K_{\text{rot}} = \frac{1}{2} I\omega^2$$

Here the quantity *I* is called the object's moment of inertia.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

The units of moment of inertia are kg m^2. An object's moment of inertia depends on the axis of rotation.

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

EXAMPLE 12.5 The speed of a rotating rod

QUESTION:

EXAMPLE 12.5 The speed of a rotating rod

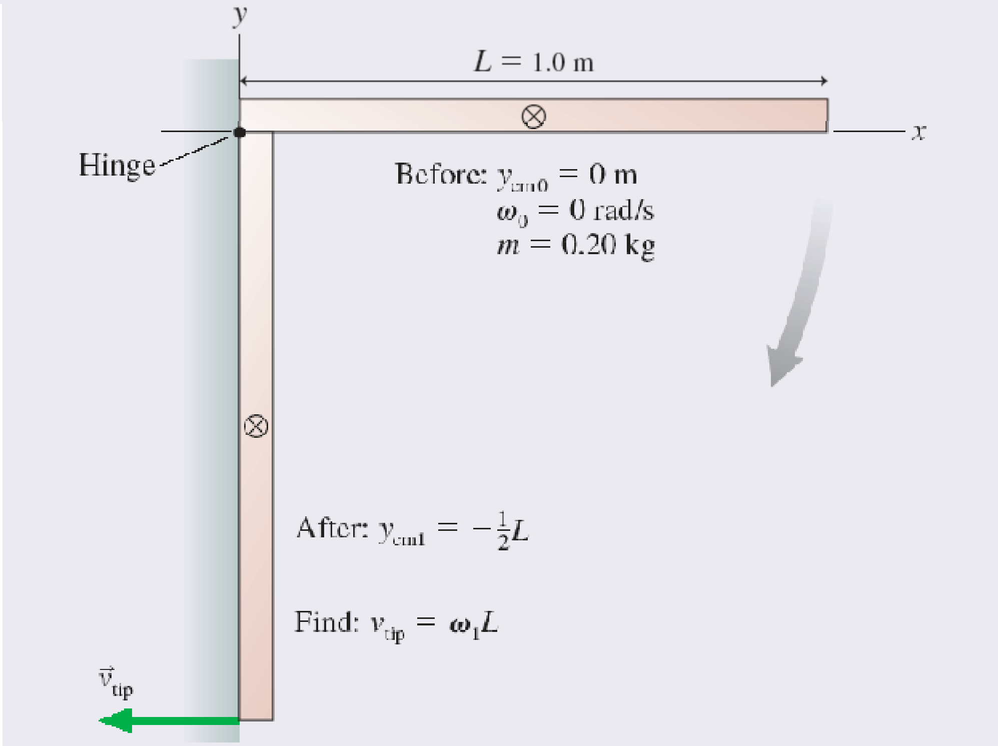
A 1.0-m-long, 200 g rod is hinged at one end and connected to a wall. It is held out horizontally, then released. What is the speed of the tip of the rod as it hits the wall?

EXAMPLE 12.5 The speed of a rotating rod

MODEL The mechanical energy is conserved if we assume the hinge is frictionless. The rod's gravitational potential energy is transformed into rotational kinetic energy as it "falls."

EXAMPLE 12.5 The speed of a rotating rod

VISUALIZE **FIGURE 12.13** is a familiar before-and-after pictorial representation of the rod. We've placed the origin of the coordinate system at the pivot point.



EXAMPLE 12.5 The speed of a rotating rod

SOLVE Mechanical energy is conserved, so we can equate the rod's final mechanical energy to its initial mechanical energy:

$$\frac{1}{2}I\omega_1^2 + Mgy_{cm1} = \frac{1}{2}I\omega_0^2 + Mgy_{cm0}$$

The initial conditions are $\omega_0 = 0$ and $y_{cm0} = 0$. The center of mass moves to $y_{cm1} = -\frac{1}{2}L$ as the rod hits the wall. From Table 12.2 we find $I = \frac{1}{3}ML^2$ for a rod rotating about one end. Thus

$$\frac{1}{2}I\omega_1^2 + Mgy_{cm1} = \frac{1}{6}ML^2\omega_1^2 - \frac{1}{2}MgL = 0$$

EXAMPLE 12.5 The speed of a rotating rod

We can solve this for the rod's angular velocity as it hits the wall:

$$\omega_1 = \sqrt{\frac{3g}{L}}$$

The tip of the rod is moving in a circle with radius $r = L$. Its final speed is

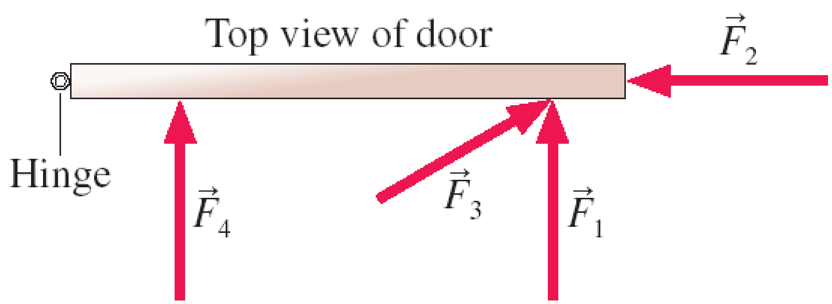
$$v_{\text{tip}} = \omega_1 L = \sqrt{3gL} = 5.4 \text{ m/s}$$

EXAMPLE 12.5 The speed of a rotating rod

ASSESS Energy conservation is a powerful tool for rotational motion, just as it was for translational motion.

Torque

Consider the common experience of pushing open a door. Shown is a top view of a door hinged on the left. Four pushing forces are shown, all of equal strength. Which of these will be most effective at opening the door?

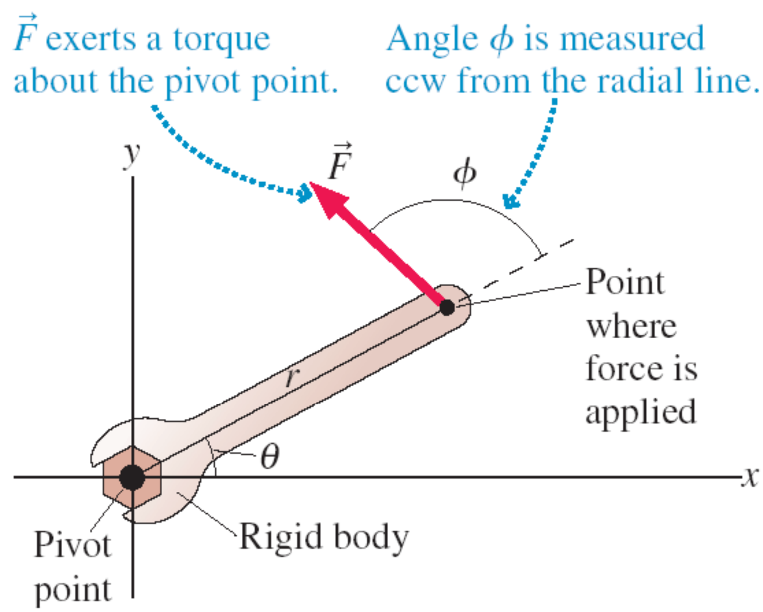


The ability of a force to cause a rotation depends on three factors:

1. the magnitude F of the force.
2. the distance r from the point of application to the pivot.
3. the angle at which the force is applied.

Torque

FIGURE 12.19 Force $\vec{F}$ exerts a torque about the pivot point.



Let's define a new quantity called torque τ (Greek tau) as

$$\boxed{\tau \equiv rF\sin\phi}$$

EXAMPLE 12.9 Applying a torque

QUESTION:

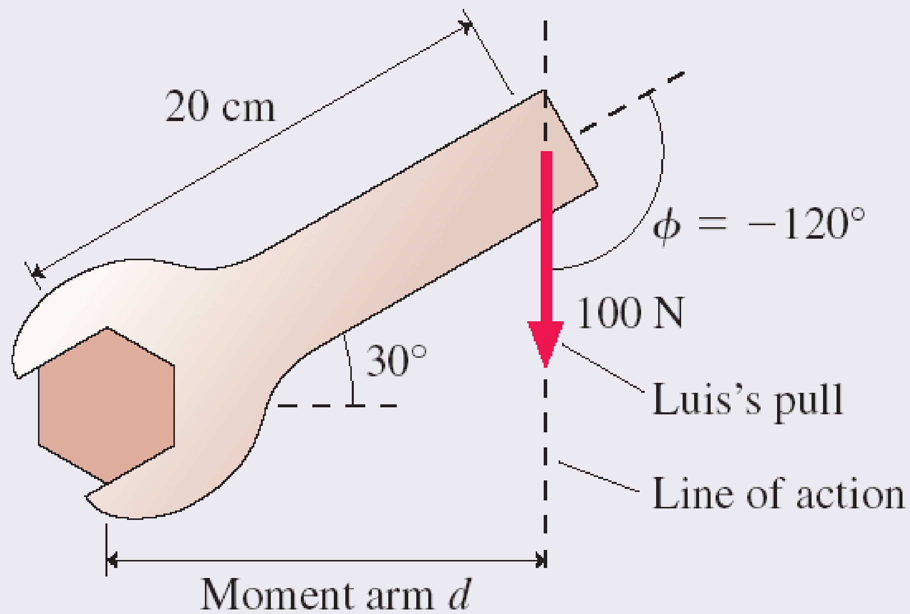
EXAMPLE 12.9 Applying a torque

Luis uses a 20-cm-long wrench to turn a nut. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?

EXAMPLE 12.9 Applying a torque

VISUALIZE **FIGURE 12.22** shows the situation. The angle is a negative $\phi = -120°$ because it is *clockwise* from the radial line.

FIGURE 12.22 A wrench being used to turn a nut.



EXAMPLE 12.9 Applying a torque

SOLVE The tangential component of the force is

$$F_t = F\sin\phi = -86.6\text{ N}$$

According to our sign convention, F_t is negative because it points in a cw direction. The torque, from Equation 12.21, is

$$\tau = rF_t = (0.20\text{ m})(-86.6\text{ N}) = -17\text{ N m}$$

Alternatively, Figure 12.22 shows that the moment arm from the pivot to the line of action is

$$d = r\sin(60°) = 0.17\text{ m}$$

Inserting the moment arm in Equation 12.22 gives

$$|\tau| = dF = (0.17\text{ m})(100\text{ N}) = 17\text{ N m}$$

The torque acts to give a cw rotation, so we insert a minus sign to end up with

$$\tau = -17\text{ N m}$$

EXAMPLE 12.9 Applying a torque

ASSESS Luis could increase the torque by changing the angle so that his pull is perpendicular to the wrench ($\phi = -90°$).

Analogies between Linear and Rotational Dynamics

TABLE 12.3 Rotational and linear dynamics

Rotational dynamics		Linear dynamics	
torque	τ_{net}	force	$\vec{F}_{net}$
moment of inertia	I	mass	m
angular acceleration	α	acceleration	$\vec{a}$
second law	$\alpha = \tau_{net}/I$	second law	$\vec{a} = \vec{F}_{net}/m$

In the absence of a net torque ($\tau_{net} = 0$), the object either does not rotate ($\omega = 0$) or rotates with *constant* angular velocity ($\omega =$ constant).

Problem-Solving Strategy: Rotational Dynamics Problems

PROBLEM-SOLVING STRATEGY 12.1 **Rotational dynamics problems**

MP

MODEL Model the object as a simple shape.

Problem-Solving Strategy: Rotational Dynamics Problems

VISUALIZE Draw a pictorial representation to clarify the situation, define coordinates and symbols, and list known information.

- Identify the axis about which the object rotates.
- Identify forces and determine their distances from the axis. For most problems it will be useful to draw a free-body diagram.
- Identify any torques caused by the forces and the signs of the torques.

Problem-Solving Strategy: Rotational Dynamics Problems

SOLVE The mathematical representation is based on Newton's second law for rotational motion:

$$\tau_{\text{net}} = I\alpha \qquad \text{or} \qquad \alpha = \frac{\tau_{\text{net}}}{I}$$

- Find the moment of inertia in Table 12.2 or, if needed, calculate it as an integral or by using the parallel-axis theorem.
- Use rotational kinematics to find angles and angular velocities.

Problem-Solving Strategy: Rotational Dynamics Problems

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

EXAMPLE 12.12 Starting an airplane engine

QUESTION:

EXAMPLE 12.12 Starting an airplane engine

The engine in a small airplane is specified to have a torque of 60 Nm. This engine drives a 2.0-m-long, 40 kg propeller. On start-up, how long does it take the propeller to reach 200 rpm?

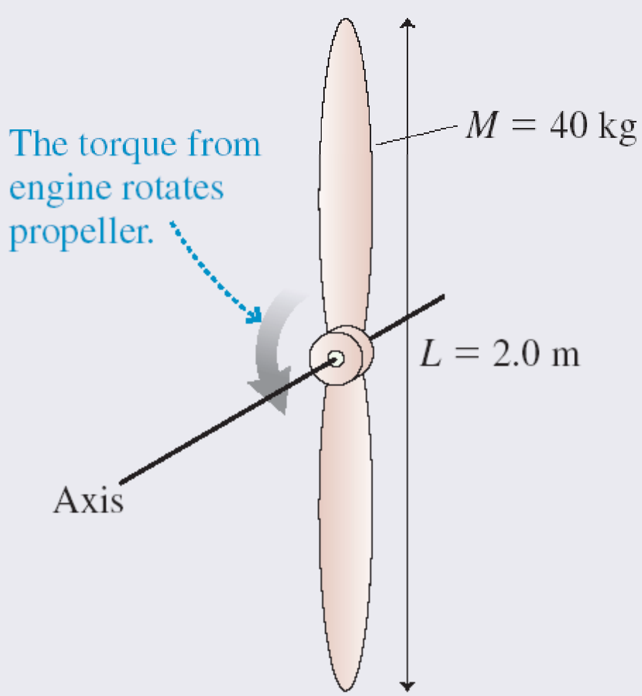
EXAMPLE 12.12 Starting an airplane engine

MODEL The propeller can be modeled as a rod that rotates about its center. The engine exerts a torque on the propeller.

EXAMPLE 12.12 Starting an airplane engine

VISUALIZE **FIGURE 12.31** shows the propeller and the rotation axis.

FIGURE 12.31 A rotating airplane propeller.



EXAMPLE 12.12 Starting an airplane engine

SOLVE The moment of inertia of a rod rotating about its center is found from Table 12.2:

$$I = \frac{1}{12}ML^2 = \frac{1}{12}(40\text{ kg})(2.0\text{ m})^2 = 13.33\text{ kg m}^2$$

The 60 Nm torque of the engine causes an angular acceleration

$$\alpha = \frac{\tau}{I} = \frac{60\text{ N m}}{13.33\text{ kg m}^2} = 4.50\text{ rad/s}^2$$

The time needed to reach $\omega_f = 200\text{ rpm} = 3.33\text{ rev/s} = 20.9\text{ rad/s}$ is

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{\omega_f - \omega_i}{\alpha} = \frac{20.9\text{ rad/s} - 0\text{ rad/s}}{4.5\text{ rad/s}^2} = 4.6\text{ s}$$

EXAMPLE 12.12 Starting an airplane engine

ASSESS We've assumed a constant angular acceleration, which is reasonable for the first few seconds while the propeller is still turning slowly. Eventually, air resistance and friction will cause opposing torques and the angular acceleration will decrease. At full speed, the negative torque due to air resistance and friction cancels the torque of the engine. Then $\tau_{net} = 0$ and the propeller turns at *constant* angular velocity with no angular acceleration.

Static Equilibrium

- The condition for a rigid body to be in *static equilibrium* is that there is **no net force** and **no net torque**.
- An important branch of engineering called *statics* analyzes buildings, dams, bridges, and other structures in total static equilibrium.
- No matter which pivot point you choose, an object that is not rotating is not rotating about that point.
- **For a rigid body in total equilibrium, there is no net torque about any point.**
- This is the basis of a problem-solving strategy.

Problem-Solving Strategy: Static Equilibrium Problems

PROBLEM-SOLVING STRATEGY 12.2 **Static equilibrium problems**

MODEL Model the object as a simple shape.

Problem-Solving Strategy: Static Equilibrium Problems

VISUALIZE Draw a pictorial representation showing all forces and distances. List known information.

- Pick any point you wish as a pivot point. The net torque about this point is zero.
- Determine the moment arms of all forces about this pivot point.
- Determine the sign of each torque about this pivot point.

Problem-Solving Strategy: Static Equilibrium Problems

SOLVE The mathematical representation is based on the fact that an object in total equilibrium has no net force and no net torque:

$$\vec{F}_{net} = \vec{0} \qquad \text{and} \qquad \tau_{net} = 0$$

- Write equations for $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau = 0$.
- Solve the three simultaneous equations.

Problem-Solving Strategy: Static Equilibrium Problems

ASSESS Check that your result is reasonable and answers the question.

EXAMPLE 12.17 Will the ladder slip?

QUESTION:

EXAMPLE 12.17 Will the ladder slip?

A 3.0-m-long ladder leans against a frictionless wall at an angle of 60°. What is the minimum value of μ_s, the coefficient of static friction with the ground, that prevents the ladder from slipping?

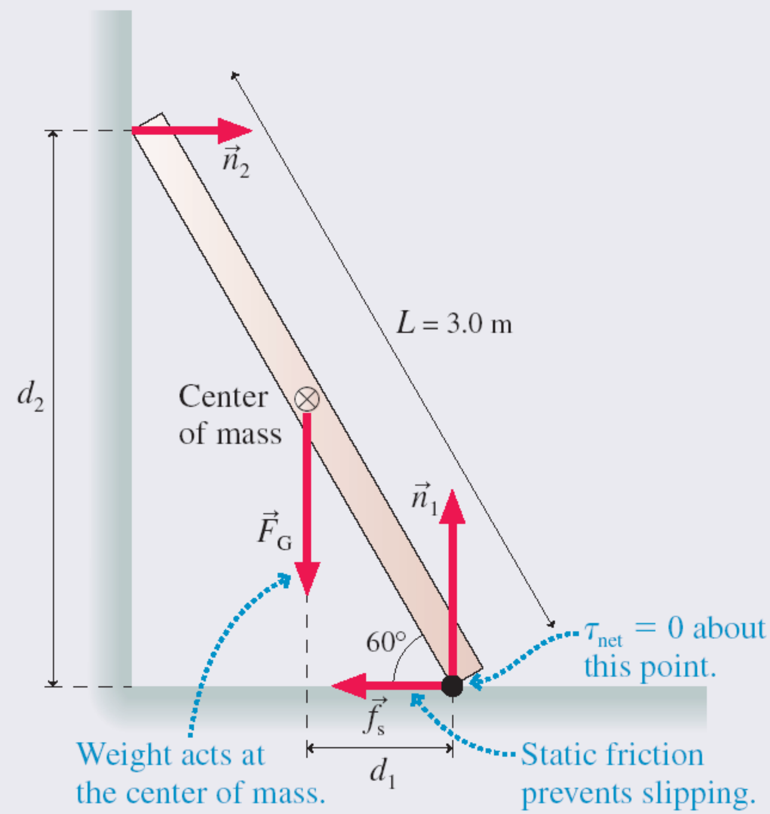
EXAMPLE 12.17 Will the ladder slip?

MODEL The ladder is a rigid rod of length L. To not slip, it must be in both translational equilibrium ($\vec{F}_{net} = \vec{0}$) and rotational equilibrium ($\tau_{net} = 0$).

EXAMPLE 12.17 Will the ladder slip?

VISUALIZE **FIGURE 12.39** shows the ladder and the forces acting on it.

FIGURE 12.39 A ladder in total equilibrium.



EXAMPLE 12.17 Will the ladder slip?

SOLVE The x- and y-components of $\vec{F}_{net} = \vec{0}$ are

$$\sum F_x = n_2 - f_s = 0$$

$$\sum F_y = n_1 - Mg = 0$$

The net torque is zero about *any* point, so which should we choose? The bottom corner of the ladder is a good choice because two forces pass through this point and have no torque about it. The torque about the bottom corner is

$$\tau_{net} = d_1 F_G - d_2 n_2 = \frac{1}{2}(L\cos 60°)Mg - (L\sin 60°)n_2 = 0$$

EXAMPLE 12.17 Will the ladder slip?

The signs are based on the observation that $\vec{F}_G$ would cause the ladder to rotate ccw while $\vec{n}_2$ would cause it to rotate cw. All together, we have three equations in the three unknowns n_1, n_2, and f_s. If we solve the third for n_2,

$$n_2 = \frac{\frac{1}{2}(L\cos 60°)Mg}{L\sin 60°} = \frac{Mg}{2\tan 60°}$$

we can then substitute this into the first to find

$$f_s = \frac{Mg}{2\tan 60°}$$

EXAMPLE 12.17 Will the ladder slip?

Our model of friction is $f_s \leq f_{s\,max} = \mu_s n_1$. We can find n_1 from the second equation: $n_1 = Mg$. Using this, the model of static friction tells us that

$$f_s \leq \mu_s Mg$$

Comparing these two expressions for f_s, we see that μ_s must obey

$$\mu_s \geq \frac{1}{2\tan 60°} = 0.29$$

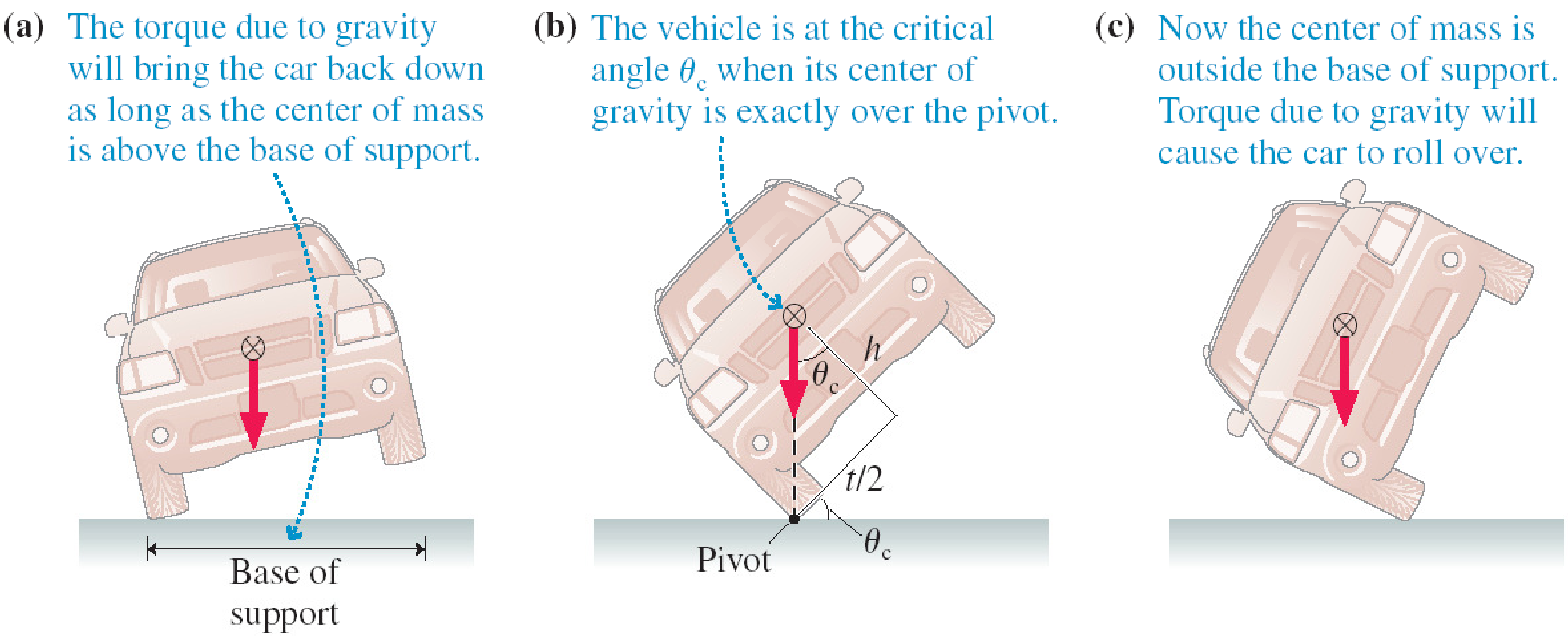
Thus the minimum value of the coefficient of static friction is 0.29.

EXAMPLE 12.17 Will the ladder slip?

ASSESS You know from experience that you can lean a ladder or other object against a wall if the ground is "rough," but it slips if the surface is too smooth. 0.29 is a "medium" value for the coefficient of static friction, which is reasonable.

Balance and Stability

FIGURE 12.40 Stability depends on the position of the center of mass.

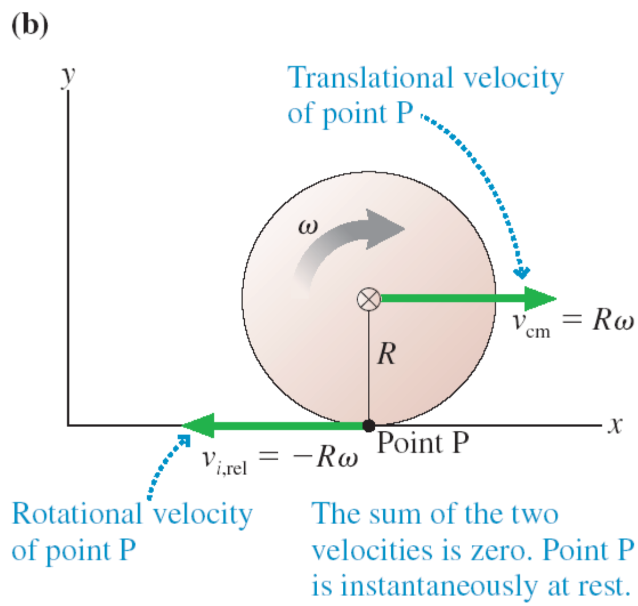


Rolling Without Slipping

For an object that is rolling without slipping, there is a **rolling constraint** that links translation and rotation:

$$v_{\text{cm}} = R\omega$$

FIGURE 12.44 The motion of a particle in the rolling object.



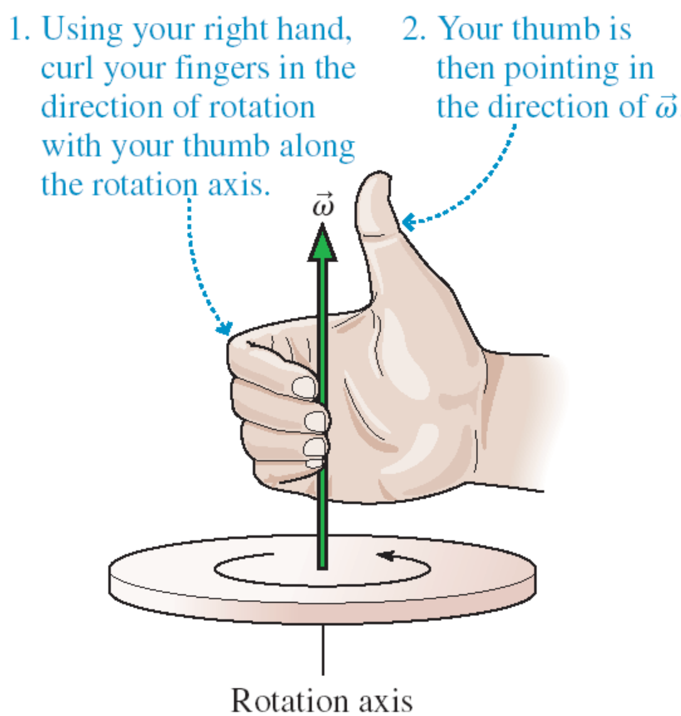
Rolling Without Slipping

We know from the rolling constraint that $R\omega$ is the center-of-mass velocity v_{cm}. Thus the kinetic energy of a rolling object is

$$K_{rolling} = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}{}^2 = K_{rot} + K_{cm}$$

In other words, the rolling motion of a rigid body can be described as a translation of the center of mass (with kinetic energy K_{cm}) plus a rotation about the center of mass (with kinetic energy K_{rot}).

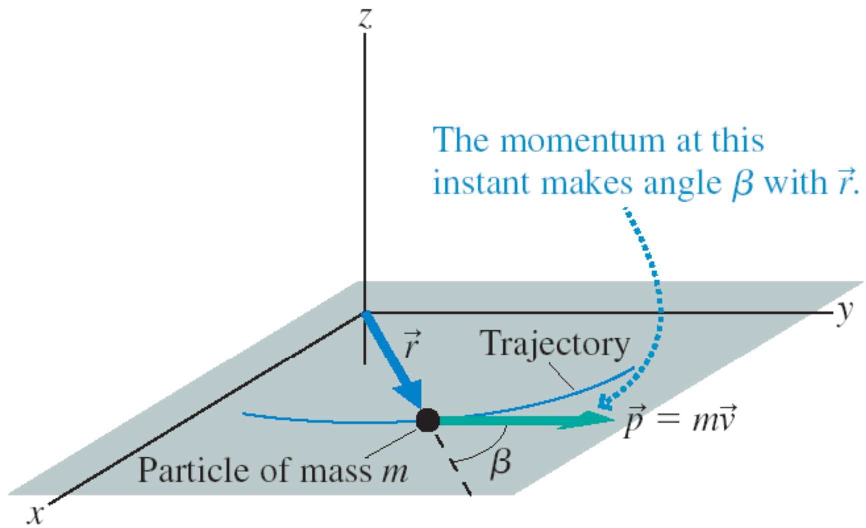
The Angular Velocity Vector



- The magnitude of the angular velocity vector is ω.
- The angular velocity vector points along the axis of rotation in the direction given by the right-hand rule as illustrated above.

Angular Momentum of a Particle

FIGURE 12.56 The angular momentum vector $\vec{L}$.

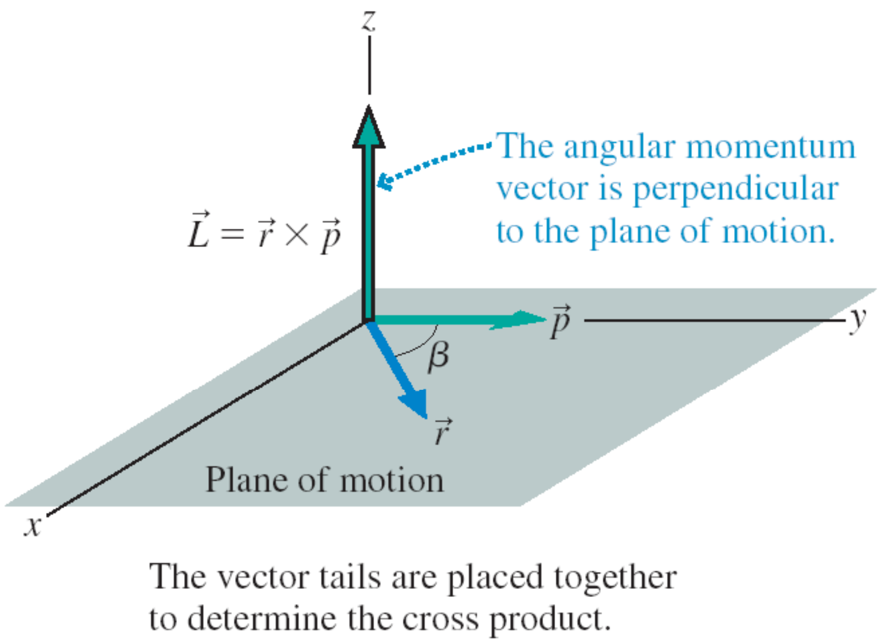


The particle is moving along a trajectory.

A particle is moving along a trajectory as shown. At this instant of time, the particle's momentum vector, tangent to the trajectory, makes an angle β with the position vector.

Angular Momentum of a Particle

FIGURE 12.56 The angular momentum vector $\vec{L}$.



We define the particle's angular momentum vector relative to the origin to be

$$\vec{L} \equiv \vec{r} \times \vec{p} = (mrv\sin\beta, \text{ direction of right-hand rule})$$

Analogies between Linear and Angular Momentum and Energy

TABLE 12.4 Angular and linear momentum and energy

Angular momentum	Linear momentum
$K_{\text{rot}} = \frac{1}{2}I\omega^2$	$K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}{}^2$
$\vec{L} = I\vec{\omega}$ *	$\vec{P} = M\vec{v}_{\text{cm}}$
$d\vec{L}/dt = \vec{\tau}_{\text{net}}$	$d\vec{P}/dt = \vec{F}_{\text{net}}$
The angular momentum of a system is conserved if there is no net torque.	The linear momentum of a system is conserved if there is no net force.

*Rotation about an axis of symmetry.

Law of conservation of angular momentum The angular momentum $\vec{L}$ of an isolated system ($\vec{\tau}_{\text{net}} = \vec{0}$) is conserved. The final angular momentum $\vec{L}_{\text{f}}$ is equal to the initial angular momentum $\vec{L}_{\text{i}}$.

Chapter 12. Summary Slides

General Principles

Rotational Dynamics

Every point on a **rigid body** rotating about a fixed axis has the same angular velocity ω and angular acceleration α.

Newton's second law for rotational motion is

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

Use rotational kinematics to find angles and angular velocities.

General Principles

Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation $E = K_{rot} + U_g = \frac{1}{2}I\omega^2 + Mgy_{cm}$
- Rolling $E = K_{rot} + K_{cm} + U_g = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{cm}{}^2 + Mgy_{cm}$

Angular momentum is conserved if $\vec{\tau}_{net} = \vec{0}$.

- Particle $\vec{L} = \vec{r} \times \vec{p}$
- Rigid body rotating about axis of symmetry $\vec{L} = I\vec{\omega}$

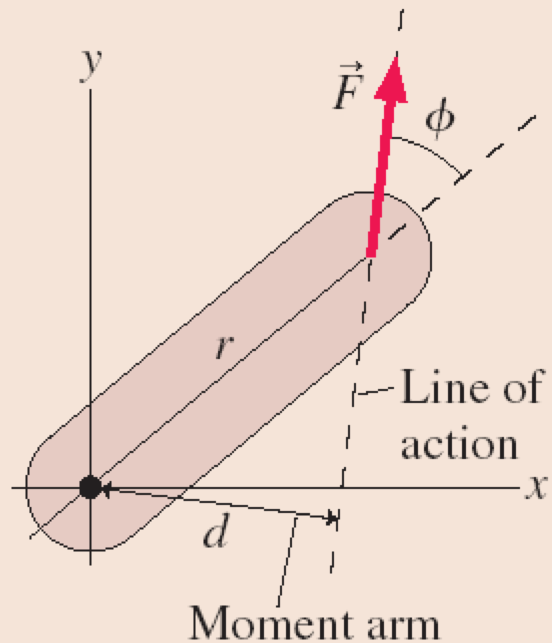
Important Concepts

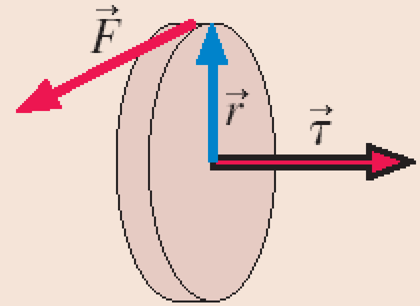
Torque is the rotational equivalent of force:

$$\tau = rF\sin\phi = rF_t = dF$$

The vector description of torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$





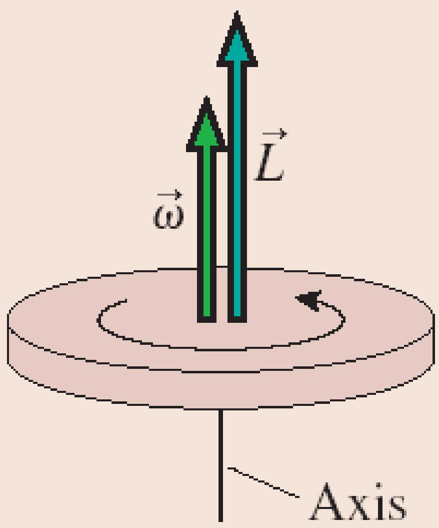
Important Concepts

Vector description of rotation

Angular velocity $\vec{\omega}$ points along the rotation axis in the direction of the right-hand rule.

For a rigid body rotating about an axis of symmetry, the angular momentum is $\vec{L} = I\vec{\omega}$.

Newton's second law is $\dfrac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$.



Important Concepts

A system of particles on which there is no net force undergoes unconstrained rotation about the center of mass:

$$x_{cm} = \frac{1}{M}\int x\,dm \qquad y_{cm} = \frac{1}{M}\int y\,dm$$

The gravitational torque on a body can be found by treating the body as a particle with all the mass M concentrated at the center of mass.

Important Concepts

The moment of inertia

$$I = \sum_i m_i r_i^2 = \int r^2 \, dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, the I about a parallel axis distance d away is given by the **parallel-axis theorem:** $I = I_{cm} + Md^2$.

Applications

Rotational kinematics

$$\omega_{\mathrm{f}} = \omega_{\mathrm{i}} + \alpha \Delta t$$

$$\theta_{\mathrm{f}} = \theta_{\mathrm{i}} + \omega_{\mathrm{i}} \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

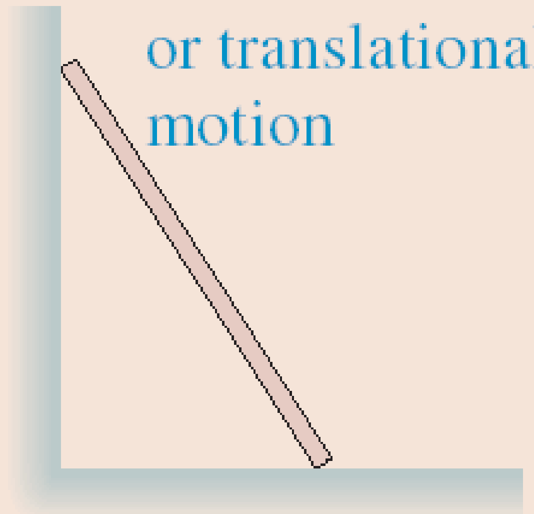
$$v_t = r\omega \qquad a_t = r\alpha$$

Applications

Rigid-body equilibrium

An object is in total equilibrium only if both $\vec{F}_{net} = \vec{0}$ and $\vec{\tau}_{net} = \vec{0}$.

No rotational or translational motion



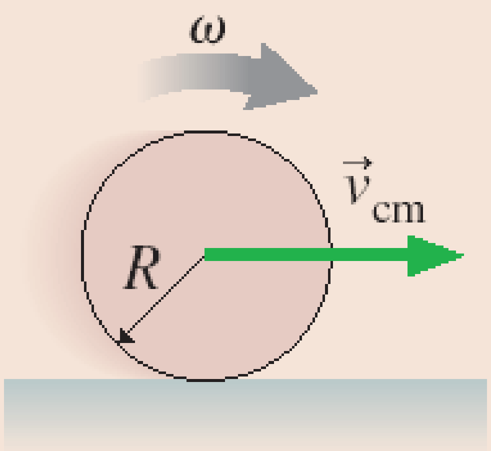
Applications

Rolling motion

For an object that rolls without slipping

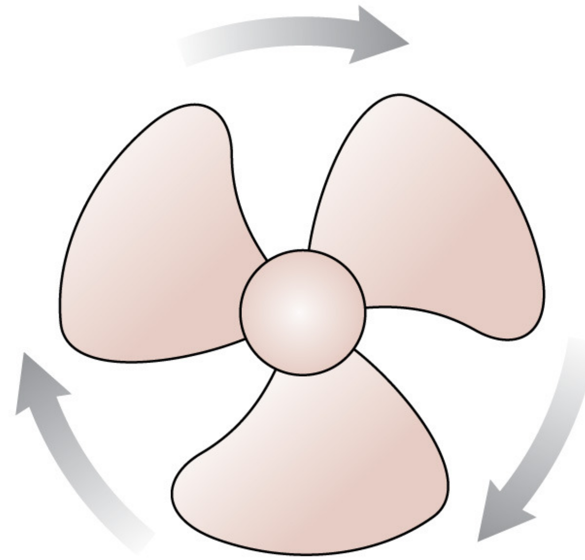
$$v_{\mathrm{cm}} = R\omega$$

$$K = K_{\mathrm{rot}} + K_{\mathrm{cm}}$$



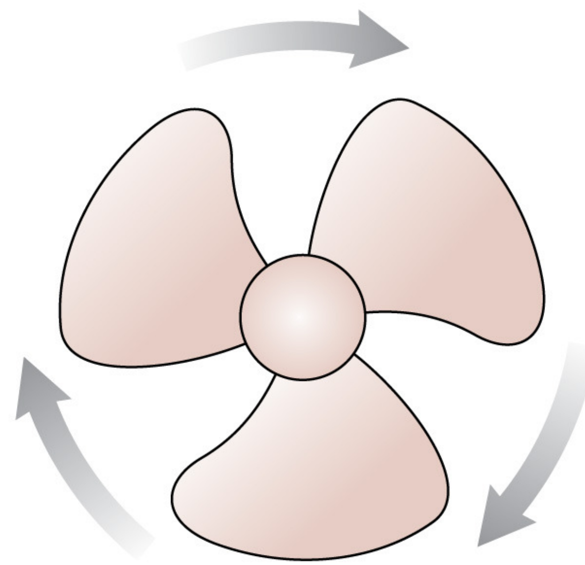
Chapter 12. Questions

The fan blade is speeding up. What are the signs of ω and α?



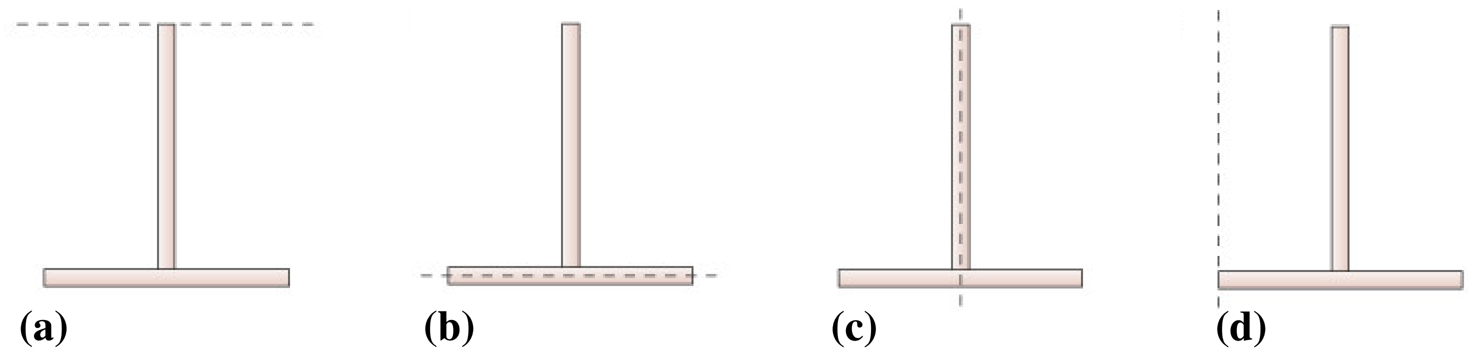
A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
D. ω is negative and α is negative.

The fan blade is speeding up. What are the signs of ω and α?



A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
✔ **D. ω is negative and α is negative.**

Four Ts are made from two identical rods of equal mass and length. Rank in order, from largest to smallest, the moments of inertia I_a to I_d for rotation about the dotted line.



A. $I_a > I_d > I_b > I_c$

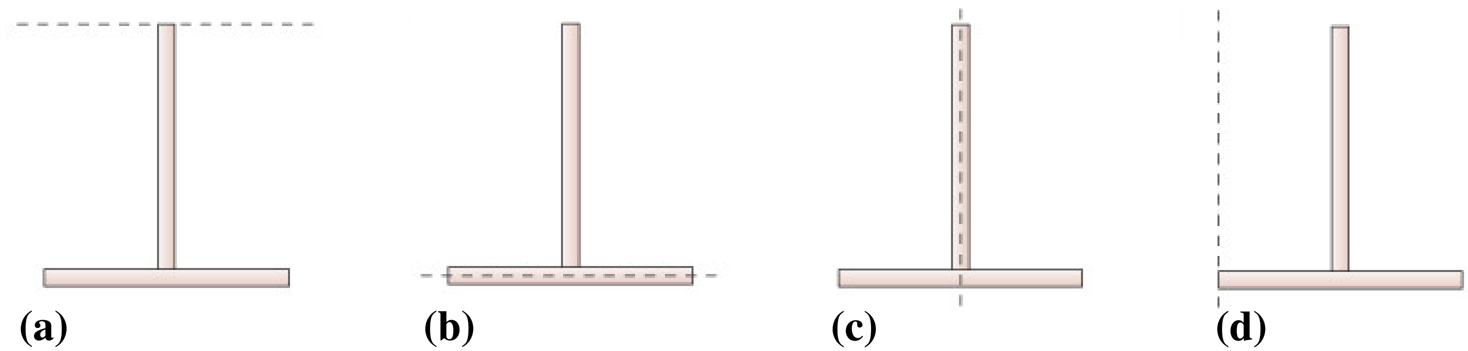
B. $I_c = I_d > I_a = I_b$

C. $I_a = I_b > I_c = I_d$

D. $I_a > I_b > I_d > I_c$

E. $I_c > I_b > I_d > I_a$

Four Ts are made from two identical rods of equal mass and length. Rank in order, from largest to smallest, the moments of inertia I_a to I_d for rotation about the dotted line.



✔ **A.** $\boldsymbol{I_a > I_d > I_b > I_c}$

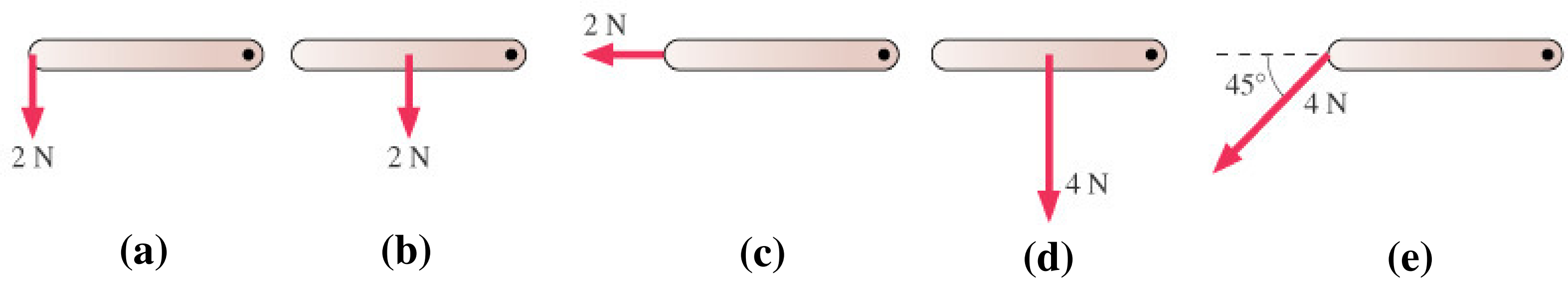
B. $I_c = I_d > I_a = I_b$

C. $I_a = I_b > I_c = I_d$

D. $I_a > I_b > I_d > I_c$

E. $I_c > I_b > I_d > I_a$

Rank in order, from largest to smallest, the five torques $\tau_a - \tau_e$. The rods all have the same length and are pivoted at the dot.



A. $\tau_e > \tau_a = \tau_d > \tau_b > \tau_c$

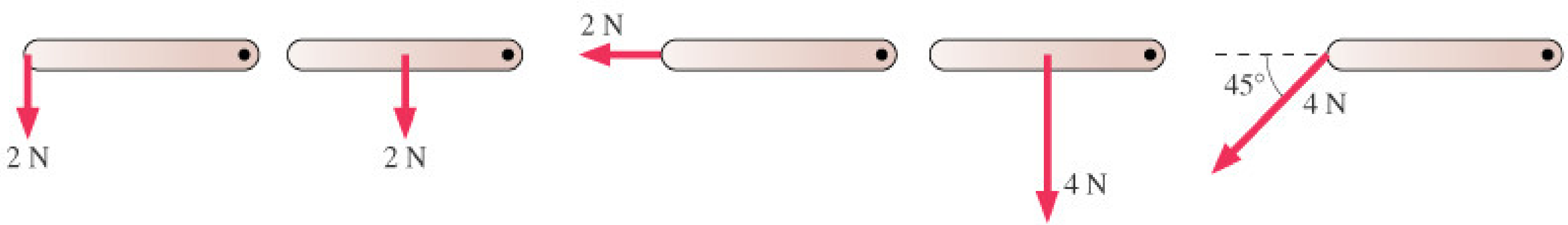
B. $\tau_d = \tau_e > \tau_a = \tau_b = \tau_c$

C. $\tau_d > \tau_e > \tau_a = \tau_b > \tau_c$

D. $\tau_d = \tau_e > \tau_d = \tau_b > \tau_c$

E. $\tau_e > \tau_a > \tau_d > \tau_b > \tau_c$

Rank in order, from largest to smallest, the five torques $\tau_a - \tau_e$. The rods all have the same length and are pivoted at the dot.



(a) (b) (c) (d) (e)

A. $\tau_e > \tau_a = \tau_d > \tau_b > \tau_c$

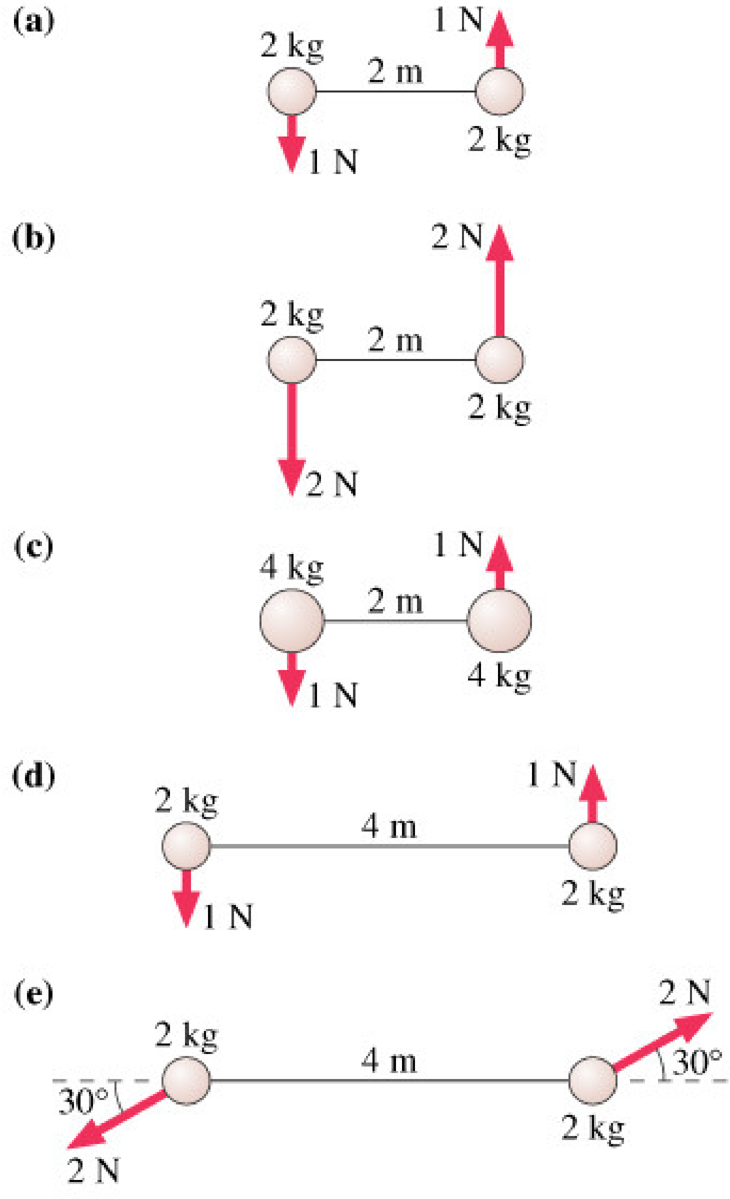
B. $\tau_d = \tau_e > \tau_a = \tau_b = \tau_c$

C. $\tau_d > \tau_e > \tau_a = \tau_b > \tau_c$

D. $\tau_d = \tau_e > \tau_d = \tau_b > \tau_c$

E. $\tau_e > \tau_a > \tau_d > \tau_b > \tau_c$

Rank in order, from largest to smallest, the angular accelerations α_a to α_e.



A. $\alpha_b = \alpha_e > \alpha_a = \alpha_c > \alpha_d$

B. $\alpha_b > \alpha_a > \alpha_c > \alpha_d > \alpha_e$

C. $\alpha_b > \alpha_a > \alpha_c = \alpha_d = \alpha_e$

D. $\alpha_a = \alpha_b = \alpha_c > \alpha_d = \alpha_e$

E. $\alpha_a = \alpha_b = \alpha_c > \alpha_d > \alpha_e$

Rank in order, from largest to smallest, the angular accelerations α_a to α_e.

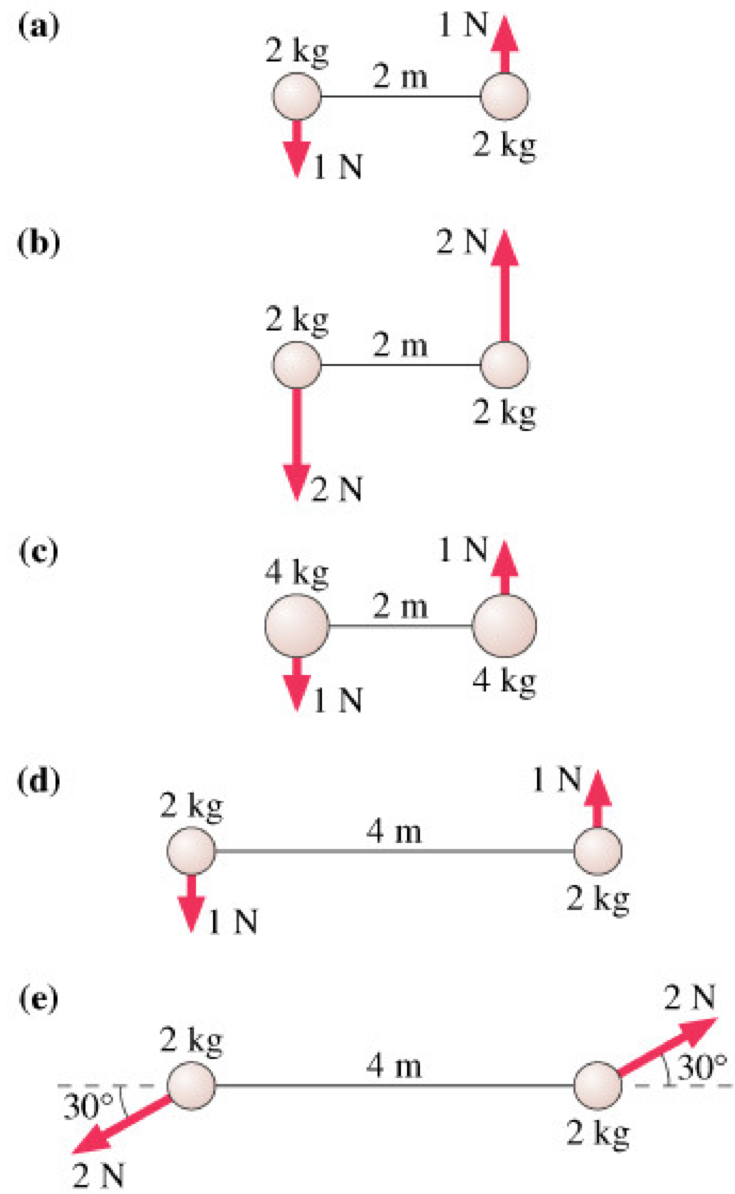
A. $\alpha_b = \alpha_e > \alpha_a = \alpha_c > \alpha_d$

B. $\alpha_b > \alpha_a > \alpha_c > \alpha_d > \alpha_e$

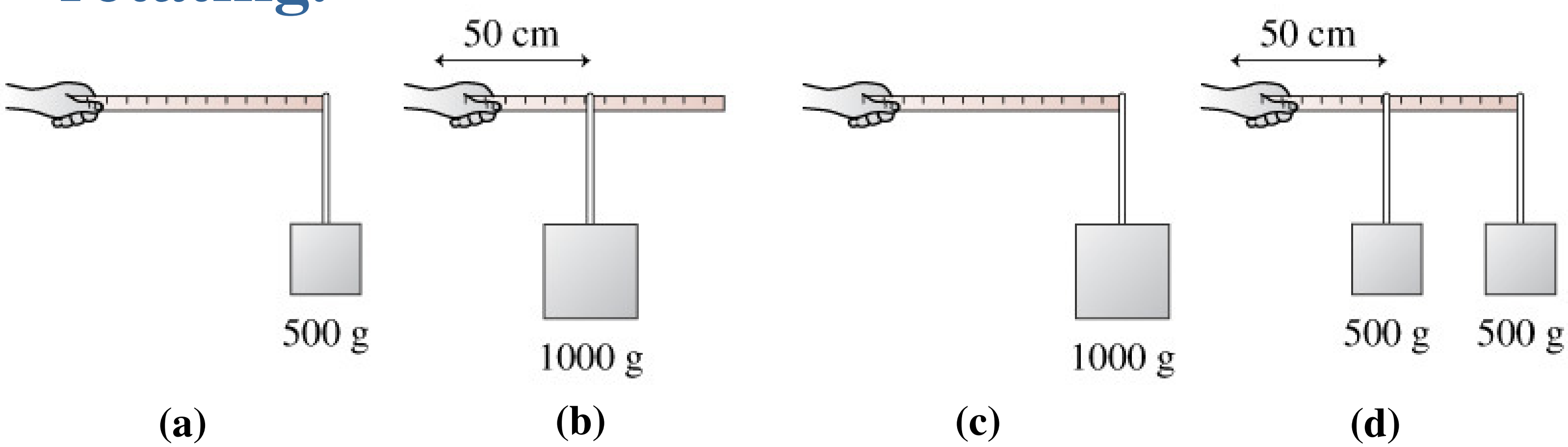
✔ **C.** $\boldsymbol{\alpha_b > \alpha_a > \alpha_c = \alpha_d = \alpha}$

D. $\alpha_a = \alpha_b = \alpha_c > \alpha_d = \alpha_e$

E. $\alpha_a = \alpha_b = \alpha_c > \alpha_d > \alpha_e$



A student holds a meter stick straight out with one or more masses dangling from it. Rank in order, from most difficult to least difficult, how hard it will be for the student to keep the meter stick from rotating.



A. c > b > d > a
B. b = c = d > a
C. c > d > b > a
D. c > d > a = b
E. b > d > c > a

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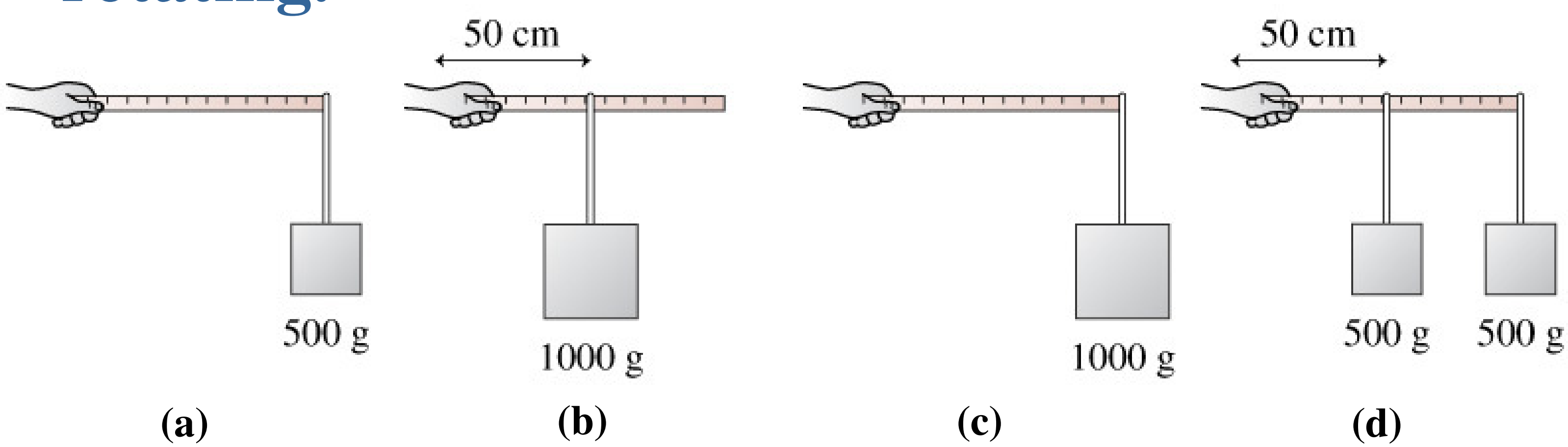


A. c > b > d > a

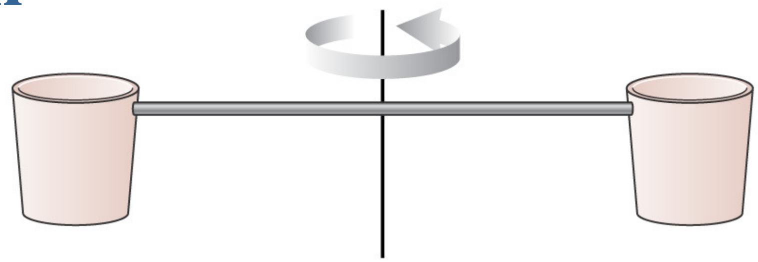
B. b = c = d > a

C. c > d > b > a

D. c > d > a = b

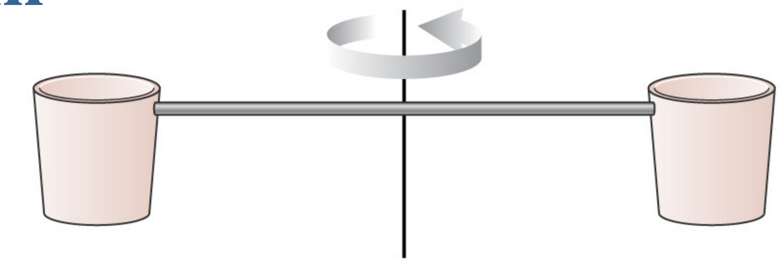
E. b > d > c > a

Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,



A. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.
B. The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.
C. The buckets slow down because the angular momentum of the bucket + rain system is conserved.
D. The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved.
E. None of the above.

Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,



A. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.

B. The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.

✔ **C. The buckets slow down because the angular momentum of the bucket + rain system is conserved.**

D. The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved.

E. None of the above.