Chapter 35 Lecture

physics
FOR SCIENTISTS AND ENGINEERS
a strategic approach
randall d. knight

Chapter 35 AC Circuits

Chapter Goal: To understand and apply basic techniques of AC circuit analysis.

Chapter 35 Preview

AC Electricity
The wires that transport electricity across the country—the grid—use alternating current, called AC.

Transformers allow an oscillating voltage to be “stepped up” to a higher voltage so that power can be delivered using lower currents that don’t overheat the wires. Smaller transformers bring the voltage down to 120 V.
Chapter 35 Preview

Capacitors and Inductors
You’ll learn that capacitors and inductors are much more useful in AC circuits than they were in DC circuits.

The peak current and peak voltage of a capacitor or an inductor are related by a resistance-like quantity called reactance, also measured in ohms. An inductor’s reactance increases with frequency, that of a capacitor decreases.

Chapter 35 Preview

RLC Circuits
A circuit that is especially important in communication electronics is the series RLC circuit, consisting of a resistor, capacitor, and inductor.

You’ll learn that an RLC circuit exhibits resonance, allowing it to be tuned to a specific frequency.

Chapter 35 Preview

Phasors
Voltages and currents oscillate, so the mathematics of AC circuits is similar to that of simple harmonic motion.

You’ll learn a new way to represent oscillating quantities with rotating vectors called phasors. The instantaneous value of a phasor is its horizontal projection.
Chapter 35 Preview

**Filter Circuits**
Simple circuits consisting of resistors and capacitors can act as filters.

You'll see how this circuit transmits low frequencies to the output—the capacitor voltage—but blocks high frequencies. It is called a low-pass filter.

Chapter 35 Preview

**Phase and Power**
The emf and the current of an AC circuit oscillate with the same frequency but usually not in phase with each other.

You'll find that the phase difference limits an emf's ability to deliver power because the current and voltage aren't pushing and pulling together.

The power delivered to, say, a motor is reduced by a quantity called the power factor.

Chapter 35 Reading Quiz
In Chapter 35, “AC” stands for

A. Air cooling.
B. Air conditioning.
C. All current.
D. Alternating current. **Correct Answer**
E. Analog current.

The analysis of AC circuits uses a rotating vector called a

A. Rotor.
B. Wiggler.
C. Phasor.
D. Motor.
E. Variator.
**Reading Question 35.2**

The analysis of AC circuits uses a rotating vector called a

A. Rotor.
B. Wiggler.
C. **Phasor.**
D. Motor.
E. Variator.

**Reading Question 35.3**

In a capacitor, the peak current and peak voltage are related by the

A. Capacitive resistance.
B. **Capacitive reactance.**
C. Capacitive impedance.
D. Capacitive inductance.
In a series RLC circuit, what quantity is maximum at resonance?

A. The voltage.
B. The current.
C. The impedance.
D. The phase.

Reading Question 35.5

In the United States a typical electrical outlet has a “line voltage” of 120 V. This is actually the

A. Average voltage.
B. Maximum voltage.
C. Maximum voltage minus the minimum voltage.
D. Minimum voltage.
E. rms voltage.
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A. Average voltage.
B. Maximum voltage.
C. Maximum voltage minus the minimum voltage.
D. Minimum voltage.
E. rms voltage.

Circuits powered by a sinusoidal emf are called AC circuits, where AC stands for alternating current. Steady-current circuits studied in Chapter 31 are called DC circuits, for direct current. The instantaneous emf of an AC generator or oscillator can be written:

\[ E = E_0 \cos \omega t \]
An alternative way to represent the emf and other oscillatory quantities is with a phasor diagram, as shown.

- A phasor is a vector that rotates counterclockwise (ccw) around the origin at angular frequency \( \omega \).

- The quantity’s value at time \( t \) is the projection of the phasor onto the horizontal axis.

The figure below helps you visualize the phasor rotation by showing how the phasor corresponds to the more familiar graph at several specific points in the cycle.

QuickCheck 35.1

This is a current phasor. The magnitude of the instantaneous value of the current is

A. Increasing.
B. Decreasing.
C. Constant.
D. Can’t tell without knowing which way it is rotating.
QuickCheck 35.1

This is a current phasor. The magnitude of the instantaneous value of the current is

A. Increasing.
B. Decreasing.
C. Constant.
D. Can't tell without knowing which way it is rotating.

In Chapter 31 we used the symbols $I$ and $V$ for DC current and voltage.

Now, because the current and voltage are oscillating, we will use lowercase $i$ to represent the instantaneous current and $v$ for the instantaneous voltage.

The figure shows a resistor $R$ connected across an AC generator of peak emf equal to $V_R$.

The current through the resistor is:

$$i_R = \frac{V_R}{R} = \frac{V_R \cos \omega t}{R} = I_p \cos \omega t$$

where $I_p = V_p/R$ is the peak current.
Resistor Circuits

The resistor’s instantaneous current and voltage are in phase, both oscillating as \( \cos \omega t \).

\[
\begin{align*}
\text{Instantaneous current and voltage} & \quad \text{Phase} \\
V_R & \quad I_R \\
0 & \quad \omega t \\
\text{Period} & = T
\end{align*}
\]

\[
\begin{align*}
\frac{V_R}{R} & = \frac{V_R \cos \omega t}{R} = I_R \cos \omega t
\end{align*}
\]

Resistor Circuits

- Below is the phasor diagram for the resistor circuit.
- \( V_R \) and \( I_R \) point in the same direction, indicating that resistor voltage and current oscillate in phase.

Capacitor Circuits

The figure shows current \( i_C \) charging a capacitor with capacitance \( C \).
The figure shows a capacitor \( C \) connected across an AC generator of peak emf equal to \( V_C \).

The charge sitting on the positive plate of the capacitor at a particular instant is:

\[
q = CV_C = CV_C \cos \omega t
\]

The current is the rate at which charge flows through the wires, \( i_C = dq/dt \), thus:

\[
i_C = \frac{dq}{dt} = \frac{d}{dt}(CV_C \cos \omega t) = -\omega CV_C \sin \omega t
\]

We can most easily see the relationship between the capacitor voltage and current if we use the trigonometric identity:

\[-\sin (x) = \cos (x + \pi/2)\]

to write:

\[
i_C = \omega CV_C \cos (\omega t + \pi/2)
\]

A capacitor’s current and voltage are not in phase.

The current peaks one-quarter of a period before the voltage peaks.

\[
i_C = \omega CV_C \cos (\omega t + \pi/2)
\]
Below is the phasor diagram for the capacitor circuit.

- The AC current of a capacitor leads the capacitor voltage by $\frac{\pi}{2}$ rad, or 90°.

QuickCheck 35.2

In the circuit represented by these phasors, the current ____ the voltage

A. leads  
B. lags  
C. is perpendicular to  
D. is out of phase with

A. leads  
B. lags  
C. is perpendicular to  
D. is out of phase with
QuickCheck 35.3

In the circuit represented by these graphs, the current ____ the voltage

A. leads
B. lags
C. is less than
D. is out of phase with

The peak current to and from a capacitor is \( I_C = \omega CV_C \).

We can find a relationship that looks similar to Ohm’s Law if we define the capacitive reactance to be:

\[ X_C = \frac{1}{\omega C} \]

\[ I_C = \frac{V_C}{X_C} \] or \[ V_C = I_C X_C \]
If the value of the capacitance is doubled, the capacitive reactance

A. Is quartered.
B. Is halved.
C. Is doubled.
D. Is quadrupled.
E. Can’t tell without knowing $\omega$.  

\[ X_C = \frac{1}{\omega C} \]

QuickCheck 35.5

If the value of the capacitance is doubled, the peak current

A. Is quartered.
B. Is halved.
C. Is doubled.
D. Is quadrupled.
E. Can’t tell without knowing $C$.  

\[ i_C = \varepsilon \cos \omega t \]
QuickCheck 35.5

If the value of the capacitance is doubled, the peak current

A. Is quartered.
B. Is halved.
C. Is doubled.
D. Is quadrupled.
E. Can’t tell without knowing \( C \).

Example 35.2 Capacitive Reactance

Example 35.3 Capacitor Current
Example 35.3 Capacitor Current

**SOLVE** The capacitive reactance at \( \omega = 2\pi f = 6280 \text{ rad/s} \) is

\[ X_C = \frac{1}{\omega C} = \frac{1}{(6280 \text{ rad/s})(10 \times 10^{-9} \text{ F})} = 16 \Omega \]

The peak voltage across the capacitor is \( V_C = E_C = 5.0 \text{ V} \), hence the peak current is

\[ I_C = \frac{V_C}{X_C} = \frac{5.0 \text{ V}}{16 \Omega} = 0.31 \text{ A} \]

**ASSUM** Using reactance is just like using Ohm’s law, but don’t forget it applies to only the peak current and voltage, not the instantaneous values.

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**RC Filter Circuits**

- The figure shows a circuit in which a resistor \( R \) and capacitor \( C \) are in series with an emf oscillating at angular frequency \( \omega \).
- If the frequency is very low, the capacitive reactance will be very large, and thus the peak current \( I_C \) will be very small.
- If the frequency is very high, the capacitive reactance approaches zero and the peak current, determined by the resistance alone, will be \( I_R = \frac{E}{R} \).

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**Using Phasors to Analyze an RC Circuit**

**Step 1 of 4**

- Begin by drawing a current phasor of length \( I \).
- This is the starting point because the series circuit elements have the same current \( i \).
- The angle at which the phasor is drawn is not relevant.
Using Phasors to Analyze an RC Circuit

**Step 2 of 4**

- The current and voltage of a resistor are in phase, so draw a resistor voltage phasor of length \( V_R \) parallel to the current phasor \( I \).
- The capacitor current leads the capacitor voltage by 90°, so draw a capacitor voltage phasor of length \( V_C \) that is 90° behind the current phasor.

**Step 3 of 4**

- The series resistor and capacitor are in parallel with the emf, so their instantaneous voltages satisfy \( V_R + V_C = E \).
- This is a vector addition of phasors.
- The emf is \( E = E_0 \cos \omega t \), hence the emf phasor is at angle \( \omega t \).

**Step 4 of 4**

- The length of the emf phasor, \( E_0 \), is the hypotenuse of a right triangle formed by the resistor and capacitor phasors.
- Thus \( E_0^2 = V_R^2 + V_C^2 \).
The relationship $E_0^2 = V_R^2 + V_C^2$ is based on the peak values.

The peak voltages are related to the peak current by $V_R = IR$ and $V_C = IX_C$, so:

$$E_0^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 = (R^2 + X_C^2)^2$$

This can be solved for the peak current, which in turn gives us the two peak voltages:

$$V_R = IR = \frac{E_0 R}{\sqrt{R^2 + X_C^2}} = \frac{E_0 R}{\sqrt{R^2 + 1/ωC^2}}$$

$$V_C = IX_C = \frac{E_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{E_0 /ωC}{\sqrt{R^2 + 1/ωC^2}}$$

The figure shows a graph of the resistor and capacitor peak voltages as functions of the emf angular frequency $ω$.

The frequency at which $V_R = V_C$ is called the crossover frequency:

$$ω_c = \frac{1}{RC}$$

QuickCheck 35.6

Does $V_R + V_C = E_0$?

A. Yes.
B. No.
C. Can’t tell without knowing $ω$.
QuickCheck 35.6

Does \( V_R + V_C = \mathcal{E} \)?

A. Yes.

✓ B. No.

C. Can’t tell without knowing \( \omega \).

Instantaneous voltages add.
Peak voltages don’t because the voltages are not in phase.

RC Filter Circuits

- The figure below shows an RC circuit in which \( v_C \) is the output voltage.
- This circuit is called a low-pass filter.

Transmits frequencies \( \omega < \omega_L \) and blocks frequencies \( \omega > \omega_L \).

RC Filter Circuits

- The figure below shows an RC circuit in which \( v_R \) is the output voltage.
- This circuit is called a high-pass filter.

Transmits frequencies \( \omega > \omega_H \) and blocks frequencies \( \omega < \omega_H \).
The figure shows the instantaneous current \( i_L \) through an inductor.

If the current is changing, the instantaneous inductor voltage is:

\[ v_L = L \frac{di_L}{dt} \]

The potential decreases in the direction of the current if the current is increasing, and increases if the current is decreasing.

\[ \text{The instantaneous current through the inductor} \]

\[ \text{The instantaneous inductor voltage is} \ v_L = L(\frac{di_L}{dt}). \]

The figure shows an inductor \( L \) connected across an AC generator of peak emf equal to \( V_L \).

The instantaneous inductor voltage is equal to the emf:

\[ v_L = V_L \cos \omega t \]

Combining the two previous equations for \( v_L \):

\[ di_L = \frac{v_L}{L} \, dt = \frac{V_L}{L} \cos \omega t \, dt \]

Integrating gives:

\[ i_L = \frac{V_L}{L} \int \cos \omega t \, dt = \frac{V_L}{\omega L} \sin \omega t = \frac{V_L}{\omega L} \cos \left( \omega t - \frac{\pi}{2} \right) \]

\[ = I_L \cos \left( \omega t - \frac{\pi}{2} \right) \]

where \( I_L = \frac{V_L}{\omega L} \) is the peak or maximum inductor current.
An inductor's current and voltage are not in phase.
- The current peaks one-quarter of a period after the voltage peaks.

\[ i(t) = i_L \cos\left(\omega t - \frac{\pi}{2}\right) \]

Below is the phasor diagram for the inductor circuit.
- The AC current through an inductor lags the inductor voltage by \( \pi/2 \) rad, or 90°.

We define the inductive reactance, analogous to the capacitive reactance, to be:

\[ X_L = \omega L \]

\[ i_L = \frac{V_L}{X_L} \quad \text{or} \quad V_L = i_L X_L \]
Example 35.5 Current and Voltage of an Inductor

**EXAMPLE 35.5** Current and voltage of an inductor

A 25 μH inductor is used in a circuit that oscillates at 100 kHz. The current through the inductor reaches a peak value of 20 mA at \( t = 5.0 \mu s \). What is the peak inductor voltage, and when, closest to \( t = 5.0 \mu s \), does it occur?

**MODEL** The inductor current lags the voltage by 90°, or, equivalently, the voltage reaches its peak value one-quarter period before the current.

\[ E = E_0 \cos \omega t \]

\[ i_L \]

\[ V_L \]

**VISUALIZE** The circuit looks like the figure below.

\[ X_L = \omega L = 2\pi(1.0 \times 10^5 \text{ Hz})(25 \times 10^{-6} \text{ H}) = 16 \Omega \]

Thus the peak voltage is \( V_L = i_L X_L = (20 \text{ mA})(16 \Omega) = 320 \text{ mV} \). The voltage peak occurs one-quarter period before the current peaks, and we know that the current peaks at \( t = 5.0 \mu s \). The period of a 100 kHz oscillation is 10.0 μs, so the voltage peaks at \( t = 5.0 \mu s - \frac{10.0 \mu s}{4} = 2.5 \mu s \).
The circuit shown, where a resistor, inductor, and capacitor are in series, is called a series RLC circuit.

- The instantaneous current of all three elements is the same: \( i = i_R = i_L = i_C \).
- The sum of the instantaneous voltages matches the emf: \( \mathcal{E} = v_R + v_L + v_C \).

Using Phasors to Analyze an RLC Circuit

Step 1 of 4

- Begin by drawing a current phasor of length \( I \).
- This is the starting point because the series circuit elements have the same current \( I \).
- The angle at which the phasor is drawn is not relevant.

Using Phasors to Analyze an RLC Circuit

Step 2 of 4

- The current and voltage of a resistor are in phase, so draw a resistor voltage phasor parallel to the current phasor \( I \).
- The capacitor current leads the capacitor voltage, so draw a capacitor voltage phasor that is 90° behind the current phasor.
- The inductor current lags the voltage, so draw an inductor voltage phasor that is 90° ahead of the current phasor.
Using Phasors to Analyze an RLC Circuit
Step 3 of 4

- The instantaneous voltages satisfy \( \mathcal{E} = v_R + v_L + v_C \).
- This is a vector addition of phasors.
- Because the capacitor and inductor phasors are in opposite directions, their vector sum has length \( v_L - v_C \).
- Adding the resistor phasor, at right angles, then gives the emf phasor \( \mathcal{E} \) at angle \( \omega t \).

Using Phasors to Analyze an RLC Circuit
Step 4 of 4

- The length of the emf phasor, \( \mathcal{E} \), is the hypotenuse of a right triangle.
- Thus \( \mathcal{E}^2 = v_R^2 + (v_L - v_C)^2 \).

The Series RLC Circuit

- If \( V_L < V_C \), which we've assumed, then the instantaneous current \( i \) lags the emf by a phase angle \( \phi \).

\[
i = I \cos(\omega t - \phi)
\]

- Based on the right-triangle, the square of the peak voltage is:

\[
\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = \left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]I^2
\]

where we wrote each of the peak voltages in terms of the peak current \( I \) and a resistance or a reactance.

- Consequently, the peak current in the RLC circuit is:

\[
i = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}
\]
Phase Angle in a Series RLC Circuit

The current lags the emf by
\[ \phi = \tan^{-1}\left(\frac{V_L - V_C}{V_E}\right) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \]

- It is often useful to know the phase angle \( \phi \) between the emf and the current in an RLC circuit:

\[ \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \]

Resonance in a Series RLC Circuit

- Suppose we vary the emf frequency \( \omega \) while keeping everything else constant.
- There is very little current at very low or very high frequencies.
- \( I \) is maximum when \( X_L = X_C \), which occurs at the resonance frequency:

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

QuickCheck 35.7

If the value of \( R \) is increased, the resonance frequency of this circuit

A. Increases.
B. Decreases.
C. Stays the same.
QuickCheck 35.7

If the value of \( R \) is increased, the resonance frequency of this circuit

A. Increases.
B. Decreases.
\( \checkmark \) C. Stays the same.

The resonance frequency depends on \( C \) and \( L \) but not on \( R \).

QuickCheck 35.8

The resonance frequency of this circuit is 1000 Hz. To change the resonance frequency to 2000 Hz, replace the capacitor with one having capacitance

A. \( C/4 \).
B. \( C/2 \).
C. \( 2C \).
D. \( 4C \).
E. It’s impossible to change the resonance frequency by changing only the capacitor.
Below is a graph of the instantaneous emf and current in a series RLC circuit driven **below** the resonance frequency: $\omega < \omega_0$.
- In this case, $X_L < X_C$, and $\phi$ is negative.

Below is a graph of the instantaneous emf and current in a series RLC circuit driven **at** the resonance frequency: $\omega = \omega_0$.
- In this case, $X_L = X_C$, and $\phi = 0$.

Below is a graph of the instantaneous emf and current in a series RLC circuit driven **above** the resonance frequency: $\omega > \omega_0$.
- In this case, $X_L > X_C$, and $\phi$ is positive.
The graphs show the instantaneous power loss in a resistor $R$ carrying a current $i_R$:

$$P_R = i_R^2 R = I_R^2 R \cos^2 \theta$$

- The average power $P_R$ is the total energy dissipated per second:

$$P_R = \frac{1}{2} I_R^2 R$$

- We can define the root-mean-square current and voltage as:

$$I_{\text{rms}} = \frac{I_R}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V_R}{\sqrt{2}}$$

- The resistor's average power loss in terms of the rms quantities is:

$$P_R = (I_{\text{rms}})^2 R = \frac{(V_{\text{rms}})^2}{R} = I_{\text{rms}} V_{\text{rms}}$$

- The average power supplied by the emf is:

$$P_{\text{source}} = I_{\text{rms}} \mathcal{E}_{\text{rms}}$$

**Example 35.7 Lighting a Bulb**

**EXAMPLE 35.7 Lighting a bulb**
A 100 W incandescent light bulb is plugged into a 120 V/60 Hz outlet. What is the resistance of the bulb’s filament? What is the peak current through the bulb?

**MODEL** The filament in a light bulb acts as a resistor.

**VISUALIZE**
Example 35.7 Lighting a Bulb

**Example 35.7 Lighting a bulb**

**Solve** A bulb labeled 100 W is designed to dissipate an average 100 W at $V_{rms} = 120 \text{ V}$. We can use Equation 35.39 to find

$$R = \frac{(V_{rms})^2}{P} = \frac{(120 \text{ V})^2}{100 \text{ W}} = 144 \Omega$$

The rms current is then found from

$$I_{rms} = \frac{P}{V_{rms}} = \frac{100 \text{ W}}{120 \text{ V}} = 0.833 \text{ A}$$

The peak current is $I_p = \sqrt{2}I_{rms} = 1.18 \text{ A}$.

**Assess** Calculations with rms values are just like the calculations for DC circuits.

---

**Capacitors in AC Circuits**

- Energy flows into and out of a capacitor as it is charged and discharged.
- The energy is not dissipated, as it would be by a resistor.
- The energy is stored as potential energy in the capacitor’s electric field.

---

**Capacitors in AC Circuits**

The instantaneous power flowing into a capacitor is:

$$P_C = V_C I_C = (V_C \cos \omega t)(-\frac{1}{\omega}CV_C \sin \omega t) = -\frac{1}{2}\omega CV_C^2 \sin 2\omega t$$
In an RLC circuit, energy is supplied by the emf and dissipated by the resistor. The average power supplied by the emf is:

\[ P_{\text{avg}} = \frac{1}{2} E_0 \cos \phi = I_{\text{rms}} E_{\text{rms}} \cos \phi \]

The term \( \cos \phi \), called the power factor, arises because the current and the emf are not in phase. Large industrial motors, such as the one shown, operate most efficiently, doing the maximum work per second, when the power factor is as close to 1 as possible.
Important Concepts

### Basic circuit elements

<table>
<thead>
<tr>
<th>Element</th>
<th>( i ) and ( r )</th>
<th>Resistance/reactance</th>
<th>( I ) and ( V )</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>In phase</td>
<td>( R ) is fixed</td>
<td>( V = IR )</td>
<td>( I_{\text{rms}}V_{\text{rms}} )</td>
</tr>
<tr>
<td>Capacitor</td>
<td>Leads ( \phi ) by ( 90^\circ )</td>
<td>( X_C = \frac{1}{2\pi fC} )</td>
<td>( V = IX )</td>
<td>0</td>
</tr>
<tr>
<td>Inductor</td>
<td>Lags ( \phi ) by ( 90^\circ )</td>
<td>( X_L = \omega L )</td>
<td>( V = IX )</td>
<td>0</td>
</tr>
</tbody>
</table>

For many purposes, especially calculating power, the **root-mean-square** (rms) quantities

\[
V_{\text{rms}} = V\sqrt{2} \quad I_{\text{rms}} = I\sqrt{2} \quad E_{\text{rms}} = E\sqrt{2}
\]

are equivalent to the corresponding DC quantities.

Key Skills

### Using phasor diagrams

- Start with a phasor \((v \text{ or } i)\) common to two or more circuit elements.
- The sum of instantaneous quantities is vector addition.
- Use the Pythagorean theorem to relate peak quantities.

For an RC circuit, shown here,

\[
v_R + v_C = E \\
v_R^2 + v_C^2 = E^2
\]

Key Skills

### Kirchhoff's laws

**Loop law** The sum of the potential differences around a loop is zero.

**Junction law** The sum of currents entering a junction equals the sum leaving the junction.

### Instantaneous and peak quantities

Instantaneous quantities \(v\) and \(i\) generally obey different relationships than peak quantities \(V\) and \(I\).