Chapter 32 Lecture

physics
FOR SCIENTISTS AND ENGINEERS
a strategic approach

THIRD EDITION
randall d. knight

Chapter 32 The Magnetic Field

Chapter Goal: To learn how to calculate and use the magnetic field.

Chapter 32 Preview

Magnetic Fields
Magnetism has been known since antiquity. Whereas electricity is understood in terms of electric charges, magnetism is based on magnetic poles. You will learn how to use the magnetic field, with symbol B, to work with the long-range interactions of magnetism. Iron filings, like little compasses, show the shape of the magnetic field.
Chapter 32 Preview

A loop of current also creates a dipole magnetic field.

One of our key tasks will be to understand the connection between electromagnets and permanent magnets.

Compasses work because the earth is a large magnet. It is an electromagnet with circulating currents in its molten iron core.

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Chapter 32 Preview

Magnetic Forces

Magnetic fields exert forces on moving charged particles. The force is perpendicular to the plane of \( \mathbf{I} \) and \( \mathbf{B} \).

Currents are moving charged particles. You’ll learn that currents create magnetic fields, and currents exert magnetic forces on each other. Opposite currents repel, parallel currents attract.

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Chapter 32 Preview

Motion of Charges

The magnetic force causes charged particles to move in circular orbits in a magnetic field. This cyclotron motion has many important applications, from particle accelerators to the aurora.

Magnetism is three-dimensional. You’ll learn how to represent vectors perpendicular to a plane. Here the arrows show a magnetic field into the page.
Chapter 32 Preview

Magnetic Torque
Magnetic forces exert a torque on a current traveling around a closed loop.

You'll learn that motors work because of magnetic torque.

Chapter 32 Preview

Magnetic Materials
Iron and a few other materials exhibit pronounced magnetic properties, including the ability to form permanent magnets. You'll learn that ferromagnetism arises because electrons have an inherent magnetic moment called electron spin.

This hard disk is made of nickel, a magnetic material. It stores digital data—1s and 0s—in the alignment of microscopic magnetic domains.

Chapter 32 Reading Quiz
Reading Question 32.1

What is the SI unit for the strength of the magnetic field?

A. Gauss.
B. Henry.
C. Tesla.
D. Becquerel.
E. Bohr magneton.

Reading Question 32.1

What is the SI unit for the strength of the magnetic field?

A. Gauss.
B. Henry.
C. Tesla. ✔
D. Becquerel.
E. Bohr magneton.

Reading Question 32.2

What is the shape of the trajectory that a charged particle follows in a uniform magnetic field?

A. Helix.
B. Parabola.
C. Circle.
D. Ellipse.
E. Hyperbola.
Reading Question 32.2

What is the shape of the trajectory that a charged particle follows in a uniform magnetic field?

A. Helix.
B. Parabola.
C. Circle.
D. Ellipse.
E. Hyperbola.

Reading Question 32.3

The magnetic field of a point charge is given by

A. Biot-Savart’s law.
B. Faraday’s law.
C. Gauss’s law.
D. Ampère’s law.
E. Einstein’s law.

A. Biot-Savart’s law.
The magnetic field of a straight, current-carrying wire is

A. Parallel to the wire.
B. Inside the wire.
C. Perpendicular to the wire.
D. Around the wire.
E. Zero.

The magnetic field of a straight, current-carrying wire is

A. Parallel to the wire.
B. Inside the wire.
C. Perpendicular to the wire.
D. **Around the wire.**
E. Zero.

Chapter 32 Content, Examples, and QuickCheck Questions
Discovering Magnetism: Experiment 1

- Tape a bar magnet to a piece of cork and allow it to float in a dish of water.
- It always turns to align itself in an approximate north-south direction.
- The end of a magnet that points north is called the north-seeking pole, or simply the north pole.
- The end of a magnet that points south is called the south pole.

Discovering Magnetism: Experiment 2

- If the north pole of one magnet is brought near the north pole of another magnet, they repel each other.
- Two south poles also repel each other, but the north pole of one magnet exerts an attractive force on the south pole of another magnet.

Discovering Magnetism: Experiment 3

- The north pole of a bar magnet attracts one end of a compass needle and repels the other.
- Apparently the compass needle itself is a little bar magnet with a north pole and a south pole.
Cutting a bar magnet in half produces two weaker but still complete magnets, each with a north pole and a south pole.

No matter how small the magnets are cut, even down to microscopic sizes, each piece remains a complete magnet with two poles.

Magnets can pick up some objects, such as paper clips, but not all.

If an object is attracted to one end of a magnet, it is also attracted to the other end.

Most materials, including copper (a penny), aluminum, glass, and plastic, experience no force from a magnet.

A magnet does not affect an electroscope.

A charged rod exerts a weak attractive force on both ends of a magnet.

However, the force is the same as the force on a metal bar that isn’t a magnet, so it is simply a polarization force like the ones we studied in Chapter 25.

Other than polarization forces, charges have no effects on magnets.
What Do These Experiments Tell Us?

1. Magnetism is not the same as electricity.
2. Magnetism is a long range force.
3. All magnets have two poles, called north and south poles. Two like poles exert repulsive forces on each other; two opposite poles attract.
4. The poles of a bar magnet can be identified by using it as a compass. The north pole tends to rotate to point approximately north.
5. Materials that are attracted to a magnet are called magnetic materials. The most common magnetic material is iron.

QuickCheck 32.1

If the bar magnet is flipped over and the south pole is brought near the hanging ball, the ball will be

A. Attracted to the magnet.
B. Repelled by the magnet.
C. Unaffected by the magnet.
D. I’m not sure.

QuickCheck 32.1

If the bar magnet is flipped over and the south pole is brought near the hanging ball, the ball will be

✓A. Attracted to the magnet.
B. Repelled by the magnet.
C. Unaffected by the magnet.
D. I’m not sure.
The compass needle can rotate on a pivot in a horizontal plane. If a positively charged rod is brought near, as shown, the compass needle will

A. Rotate clockwise.

B. Rotate counterclockwise.

C. Do nothing.

D. I’m not sure.

Magnetic poles are not the same as electric charges.

If a bar magnet is cut in half, you end up with

A. S N N S

B. N S S N

C. S N S N

D. S N

E. Unmagnetized
QuickCheck 32.3

If a bar magnet is cut in half, you end up with

A. S N N S
B. N S S N
C. S N S N
D. S N
E. S N Unmagnetized

Compasses and Geomagnetism

- Due to currents in the molten iron core, the earth itself acts as a large magnet.
- The poles are slightly offset from the poles of the rotation axis.
- The geographic north pole is actually a south magnetic pole!

Electric Current Causes a Magnetic Field

In 1819 Hans Christian Oersted discovered that an electric current in a wire causes a compass to turn.
**Electric Current Causes a Magnetic Field**

The right-hand rule determines the orientation of the compass needles to the direction of the current.

The compass needles are tangent to the circle with the north pole in the direction your fingers are pointing.

Right-hand rule: Point your right thumb in the direction of the current.

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**Electric Current Causes a Magnetic Field**

The magnetic field is revealed by the pattern of iron filings around a current-carrying wire.

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**Notation for Vectors and Currents Perpendicular to the Page**

- Magnetism requires a three-dimensional perspective, but two-dimensional figures are easier to draw.
- We will use the following notation:

  - **Vectors into page**
  - **Current into page**
  - **Vectors out of page**
  - **Current out of page**
The right-hand rule determines the orientation of the compass needles to the direction of the current.

QuickCheck 32.4

A long, straight wire extends into and out of the screen. The current in the wire is

A. Into the screen.
B. Out of the screen.
C. There is no current in the wire.
D. Not enough info to tell the direction.

Right-hand rule

QuickCheck 32.4

A long, straight wire extends into and out of the screen. The current in the wire is

A. Into the screen.
B. Out of the screen.  
C. There is no current in the wire.
D. Not enough info to tell the direction.

Right-hand rule
Magnetic Force on a Compass

- The figure shows a compass needle in a magnetic field.
- A magnetic force is exerted on each of the two poles of the compass, parallel to $\mathbf{B}$ for the north pole and opposite $\mathbf{B}$ for the south pole.
- This pair of opposite forces exerts a torque on the needle, rotating the needle until it is parallel to the magnetic field at that point.

Electric Current Causes a Magnetic Field

- Because compass needles align with the magnetic field, the magnetic field at each point must be tangent to a circle around the wire.
- The figure shows the magnetic field by drawing field vectors.
- Notice that the field is weaker (shorter vectors) at greater distances from the wire.

Electric Current Causes a Magnetic Field

Magnetic field lines are circles.

- A tangent to a field line is in the direction of the magnetic field.
- The field lines are closer together where the magnetic field strength is larger.
The magnetic field of a charged particle $q$ moving with velocity $v$ is given by the Biot-Savart law:

$$\mathbf{B}_{\text{point charge}} = \frac{\mu_0 q v \sin \theta}{4\pi r^2}, \text{ direction given by the right-hand rule}$$

The constant $\mu_0$ in the Biot-Savart law is called the permeability constant:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A} = 1.257 \times 10^{-6} \text{ T m/A}$$

The SI unit of magnetic field strength is the tesla, abbreviated as T:

1 tesla = 1 T = 1 N/A m
The right-hand rule for finding the direction of $\vec{B}$ due to a moving positive charge is similar to the rule used for a current carrying wire. Note that the component of $\vec{B}$ parallel to the line of motion is zero.

**Example 32.1 The Magnetic Field of a Proton**

**EXAMPLE 32.1 The magnetic field of a proton**

A proton moves with velocity $\vec{v} = 1.0 \times 10^7 \, \text{i m/s}$. As it passes the origin, what is the magnetic field at the $(x, y, z)$ positions (1 mm, 0 mm, 0 mm), (0 mm, 1 mm, 0 mm), and (1 mm, 1 mm, 0 mm)?

**MODEL** The magnetic field is that of a moving charged particle.
Magnetic fields, like electric fields, have been found experimentally to obey the principle of superposition.

If there are \( n \) moving point charges, the net magnetic field is given by the vector sum:

\[
\mathbf{B}_{\text{total}} = \mathbf{B}_1 + \mathbf{B}_2 + \cdots + \mathbf{B}_n
\]

The principle of superposition will be the basis for calculating the magnetic fields of several important current distributions.
The Cross Product

\[ \vec{C} \times \vec{D} = CD \sin \alpha, \text{ direction given by the right-hand rule} \]

Magnetic Field of a Moving Charge

The magnetic field of a charged particle \( q \) moving with velocity \( \vec{v} \) is given by the Biot-Savart law:

\[ \vec{B}_{\text{point charge}} = \frac{\mu_0 q \vec{v} \times \vec{r}}{4\pi r^2} \quad \text{(magnetic field of a point charge)} \]

QuickCheck 32.5

What is the direction of the magnetic field at the position of the dot?

A. Into the screen.
B. Out of the screen.
C. Up.
D. Down.
E. Left.
QuickCheck 32.5

What is the direction of the magnetic field at the position of the dot?

A. Into the screen.
B. Out of the screen.
C. Up. Direction of $\mathbf{\nabla} \times \mathbf{E}$ into screen
D. Down.
E. Left.

Example 32.2 The Magnetic Field Direction of a Moving Electron

The figure shows a current-carrying wire. The wire as a whole is electrically neutral, but current $I$ represents the motion of positive charge carriers through the wire.

- The figure shows a current-carrying wire.
- The magnetic field of a current.
The Magnetic Field of a Current

The magnetic field at the center of a coil of \( N \) turns and radius \( R \), carrying a current \( I \) is:

\[
\vec{B}_{\text{coil center}} = \frac{\mu_0 N I}{2R}
\]

The magnetic field of a long, straight wire carrying current \( I \) at a distance \( d \) from the wire is:

\[
\vec{B}_{\text{wire}} = \left( \frac{\mu_0 I}{2\pi d} \right) \text{tangent to a circle around the wire in the right-hand direction}
\]

QuickCheck 32.6

Compared to the magnetic field at point A, the magnetic field at point B is

A. Half as strong, same direction.
B. Half as strong, opposite direction.
C. One-quarter as strong, same direction.
D. One-quarter as strong, opposite direction.
E. Can't compare without knowing \( I \).
QuickCheck 32.6

Compared to the magnetic field at point A, the magnetic field at point B is

A. Half as strong, same direction.
B. Half as strong, opposite direction.  
C. One-quarter as strong, same direction.
D. One-quarter as strong, opposite direction.
E. Can't compare without knowing I.

Problem-Solving Strategy: The Magnetic Field of a Current

**MODEL:** Model the wire as a simple shape, such as a straight line or a loop.

**VISUALIZE:** For the pictorial representation:

- Draw a picture and establish a coordinate system.
- Identify the point P at which you want to calculate the magnetic field.
- Divide the current-carrying wire into small segments for which you already know how to determine $\mathbf{B}$. This is usually, though not always, a division into very short segments of length $\Delta s$.
- Draw the magnetic field vector for one or two segments. This will help you identify distances and angles that need to be calculated.
- Look for symmetries that simplify the field. You may conclude that some components of $\mathbf{B}$ are zero.

Problem-Solving Strategy: The Magnetic Field of a Current

**MODEL:** The mathematical representation is $\mathbf{B}_m = \sum \mathbf{B}$.

- Use superposition to form an algebraic expression for each of the three components of $\mathbf{B}$ (unless you are sure one or more is zero) at point P.
- Let the $(x, y, z)$-coordinates of the point remain as variables.
- Express all angles and distances in terms of the coordinates.
- Let $\Delta s \to ds$ and the sum become an integral. Think carefully about the integration limits for this variable: they will depend on the boundaries of the wire and on the coordinate system you have chosen to use. Carry out the integration and simplify the results as much as possible.

**ASSESS:** Check that your result is consistent with any limits for which you know what the field should be.
Example 32.4 The Magnetic Field Strength Near a Heater Wire

**Example 32.4** The magnetic field strength near a heater wire

A 1.0-m-long, 1.0-mm-diameter nichrome heater wire is connected to a 12 V battery. What is the magnetic field strength 1.0 cm away from the wire?

**MODEL** 1 cm is much less than the 1 m length of the wire, so model the wire as infinitely long.

**SOLVE**

The current through the wire is \( I = \Delta V / R \), where the wire's resistance \( R \) is

\[
R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = 1.91 \, \Omega.
\]

The nichrome resistivity \( \rho = 1.50 \times 10^6 \, \Omega \cdot \text{m} \) was taken from Table 30.2. Thus, the current is \( I = (12 \, V)(1.91 \, \Omega) = 6.24 \, \text{A} \). The magnetic field strength at distance \( d = 1.0 \, \text{cm} = 0.010 \, \text{m} \) from the wire is

\[
B_{\text{mag}} = \frac{\mu_0 I}{2\pi d} = \frac{(2.0 \times 10^{-7} \, \text{Tm/A})(6.28 \, \text{A})}{0.010 \, \text{m}} = 1.3 \times 10^{-4} \, \text{T}.
\]

**ASSESS** The magnetic field of the wire is slightly more than twice the strength of the earth's magnetic field.
Example 32.6 Matching the Earth’s Magnetic Field

**EXAMPLE 32.6** Matching the earth’s magnetic field

What current is needed in a 5-turn, 10-cm-diameter coil to cancel the earth’s magnetic field at the center of the coil?

**MODEL** One way to create a zero-field region of space is to generate a magnetic field equal to the earth’s field but pointing in the opposite direction. The vector sum of the two fields is zero.

**VISUALIZE** The figure below shows a five-turn coil of wire. The magnetic field is five times that of a single current loop.

\[ \vec{B} \]

\[ I \]

\[ \vec{B}_e \]

\[ I \]

SOLVE The earth’s magnetic field, from Table 32.1, is \( 5 \times 10^{-5} \) T. We can use Equation 32.7 to find that the current needed to generate a \( 5 \times 10^{-5} \) T field is

\[ I = \frac{2RB}{\mu_0 N} = \frac{2(0.050 \text{ m})(5.0 \times 10^{-3} \text{ T})}{5(4\pi \times 10^{-7} \text{ Tm/A})} = 0.80 \text{ A} \]

**ASSESS** A 0.80 A current is easily produced. Although there are better ways to cancel the earth’s field than using a simple coil, this illustrates the idea.
The magnetic field is revealed by the pattern of iron filings around a current-carrying loop of wire.

QuickCheck 32.7

The magnet field at point P is

A. Into the screen.
B. Out of the screen.
C. Zero.
QuickCheck 32.7

The magnet field at point P is

A. Into the screen.
B. Out of the screen.
C. Zero.

Tactics: Finding the Magnetic Field Direction of a Current Loop

Use either of the following methods to find the magnetic field direction:

1. Point your right thumb in the direction of the current at any point on the loop and let your fingers curl through the center of the loop. Your fingers are then pointing in the direction in which \( \mathbf{B} \) leaves the loop.
2. Curl the fingers of your right hand around the loop in the direction of the current. Your thumb is then pointing in the direction in which \( \mathbf{B} \) leaves the loop.

A Current Loop Is a Magnetic Dipole

Whether it’s a current loop or a permanent magnet, the magnetic field emerges from the north pole.
QuickCheck 32.8

Where is the north magnetic pole of this current loop?

A. Top side.
B. Bottom side.
C. Right side.
D. Left side.
E. Current loops don’t have north poles.

The Magnetic Dipole Moment

The magnetic dipole moment of a current loop enclosing an area $A$ is defined as:

$$\vec{\mu} = (AI, \text{from the south pole to the north pole})$$
The Magnetic Dipole Moment

\[ \vec{\mu} = (AI, \text{ from the south pole to the north pole}) \]

- The SI units of the magnetic dipole moment are \( A \text{ m}^2 \).
- The on-axis field of a magnetic dipole is:

\[
\vec{B}_{\text{dipole}} = \frac{\mu_0 2\vec{\mu}}{4\pi z^3} \quad \text{(on the axis of a magnetic dipole)}
\]

QuickCheck 32.9

What is the current direction in the loop?

A. Out at the top, in at the bottom.
B. In at the top, out at the bottom.
C. Either A or B would cause the current loop and the bar magnet to repel each other.

QuickCheck 32.9

What is the current direction in the loop?

A. Out at the top, in at the bottom.
B. In at the top, out at the bottom. **Correct Answer**
C. Either A or B would cause the current loop and the bar magnet to repel each other.
Figure (a) shows a curved line from \( i \) to \( f \).

The length \( l \) of this line can be found by doing a line integral:

\[
    l = \sum_{i} \Delta s_i \rightarrow \int_{i}^{f} ds
\]
Ampère's Law

- Consider a line integral of $\mathbf{B}$ evaluated along a circular path all the way around a wire carrying current $I$.
- This is the line integral around a closed curve, which is denoted:
  \[ \oint \mathbf{B} \cdot d\mathbf{s} \]

Because $\mathbf{B}$ is tangent to the circle and of constant magnitude at every point on the circle, we can write:
  \[ \oint \mathbf{B} \cdot d\mathbf{s} = BI = B(2\pi d) \]
- Here $B = \mu_0 I / (2\pi d)$, where $I$ is the current through this loop, hence:
  \[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \]

Whenever total current $I_{\text{through}}$ passes through an area bounded by a closed curve, the line integral of the magnetic field around the curve is given by Ampère's law:
  \[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \]
QuickCheck 32.10

The line integral of $\mathbf{B}$ around the loop is $\mu_0 \cdot 7.0 \, \text{A}$. Current $I_3$ is

A. 0 A.
B. 1 A out of the screen.
C. 1 A into the screen.
D. 5 A out of the screen.
E. 5 A into the screen.

QuickCheck 32.10

The line integral of $\mathbf{B}$ around the loop is $\mu_0 \cdot 7.0 \, \text{A}$. Current $I_3$ is

A. 0 A.
B. 1 A out of the screen.
C. 1 A into the screen. ✔
D. 5 A out of the screen.
E. 5 A into the screen.

QuickCheck 32.11

For the path shown, $\oint \mathbf{B} \cdot d\mathbf{r} =$

A. 0.
B. $\mu_0 (I_1 - I_2)$.
C. $\mu_0 (I_2 - I_1)$.
D. $\mu_0 (I_1 + I_2)$. 

Enclosed currents. $I_2$ is positive.
QuickCheck 32.11

For the path shown, \[ \oint \mathbf{B} \cdot d\mathbf{s} = \]

A. 0.
B. \( \mu_0 (I_1 - I_2) \).
C. \( \mu_0 (I_2 - I_1) \).
D. \( \mu_0 (I_1 + I_2) \).

Solenoids

- A uniform magnetic field can be generated with a solenoid.
- A solenoid is a helical coil of wire with the same current \( I \) passing through each loop in the coil.
- Solenoids may have hundreds or thousands of coils, often called turns, sometimes wrapped in several layers.
- The magnetic field is strongest and most uniform inside the solenoid.

The Magnetic Field of a Solenoid

With many current loops along the same axis, the field in the center is strong and roughly parallel to the axis, whereas the field outside the loops is very close to zero.
The Magnetic Field of a Solenoid

No real solenoid is ideal, but a very uniform magnetic field can be produced near the center of a tightly wound solenoid whose length is much larger than its diameter.

The figure shows a cross section through an infinitely long solenoid. The integration path that we’ll use is a rectangle. The current passing through this rectangle is \( I_{\text{through}} = NI \).

Ampère’s Law is thus:

\[
\mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{through}} = \mu_0 NI
\]

Along the top, the line integral is zero since \( \mathbf{B} = 0 \) outside the solenoid.

Along the sides, the line integral is zero since the field is perpendicular to the path.

Along the bottom, the line integral is simply \( BI \).

Solving for \( B \) inside the solenoid:

\[
B_{\text{internal}} = \frac{\mu_0 NI}{I} = \mu_0 nI
\]

where \( n = N/l \) is the number of turns per unit length.
QuickCheck 32.12

Solenoid 2 has twice the diameter, twice the length, and twice as many turns as solenoid 1. How does the field \( B_2 \) at the center of solenoid 2 compare to \( B_1 \) at the center of solenoid 1?

A. \( B_2 = \frac{B_1}{4} \)
B. \( B_2 = \frac{B_1}{2} \)
C. \( B_2 = B_1 \)
D. \( B_2 = 2B_1 \)
E. \( B_2 = 4B_1 \)

Same turns-per-length

QuickCheck 32.13

The current in this solenoid

A. Enters on the left, leaves on the right.
B. Enters on the right, leaves on the left.
C. Either A or B would produce this field.
QuickCheck 32.13

The current in this solenoid

A. Enters on the left, leaves on the right.
B. Enters on the right, leaves on the left.
C. Either A or B would produce this field.

Generating an MRI Magnetic Field

This patient is undergoing magnetic resonance imaging (MRI). The large cylinder surrounding the patient contains a solenoid that is wound with superconducting wire to generate a strong uniform magnetic field.

Example 32.9 Generating an MRI Magnetic Field

**EXAMPLE 32.9 Generating an MRI magnetic field**

A 1.0-m-long MRI solenoid generates a 1.2 T magnetic field. To produce such a large field, the solenoid is wrapped with superconducting wire that can carry a 100 A current. How many turns of wire does the solenoid need?

MODEL Assume that the solenoid is ideal.
The magnetic field outside a solenoid looks like that of a bar magnet. Thus a solenoid is an electromagnet, and you can use the right-hand rule to identify the north-pole end.
After the discovery that electric current produces a magnetic field, Ampère set up two parallel wires that could carry large currents either in the same direction or in opposite directions. 

Ampère’s experiment showed that a magnetic field exerts a force on a current.

The magnetic force turns out to depend not only on the charge and the charge’s velocity, but also on how the velocity vector is oriented relative to the magnetic field.

The magnetic force is perpendicular to \( \vec{v} \) and \( \vec{B} \). Its magnitude is \( qvB\sin\alpha \).
The magnetic force turns out to depend not only on the charge and the charge’s velocity, but also on how the velocity vector is oriented relative to the magnetic field.

The magnetic force on a charge \( q \) as it moves through a magnetic field \( \mathbf{B} \) with velocity \( \mathbf{v} \) is:

\[
\mathbf{F}_{\text{mag}} = q\mathbf{v} \times \mathbf{B} = (qvB \sin \alpha, \text{ direction of right-hand rule})
\]

where \( \alpha \) is the angle between \( \mathbf{v} \) and \( \mathbf{B} \).
QuickCheck 32.14

The direction of the magnetic force on the proton is

A. To the right.
B. To the left.
C. Into the screen.
D. Out of the screen.
E. The magnetic force is zero.

QuickCheck 32.15

The direction of the magnetic force on the electron is

A. Upward.
B. Downward.
C. Into the screen.
D. Out of the screen.
E. The magnetic force is zero.
QuickCheck 32.15

The direction of the magnetic force on the electron is

A. Upward.
B. Downward.
C. Into the screen.
D. Out of the screen.

E. The magnetic force is zero.

QuickCheck 32.16

Which magnetic field causes the observed force?
Example 32.10 The Magnetic Force on an Electron

**EXAMPLE 32.10** The magnetic force on an electron

A long wire carries a 10 A current from left to right. An electron 1.0 cm above the wire is traveling to the right at a speed of $1.0 \times 10^7$ m/s. What are the magnitude and the direction of the magnetic force on the electron?

**MODEL** The magnetic field is that of a long, straight wire.

**VISUALIZE** The figure shows the current and an electron moving to the right. The right-hand rule tells us that the wire’s magnetic field above the wire is out of the page, so the electron is moving perpendicular to the field.

**SOLUTION** The electron charge is negative, thus the direction of the force is opposite the direction of $\mathbf{\tau} \times \mathbf{B}$. The right-hand rule shows that $\mathbf{\tau} \times \mathbf{B}$ points down, toward the wire, so $\mathbf{F}$ points up, away from the wire. The magnitude of the force is $|F| = evB$. The field is that of a long, straight wire.

$$B = \frac{\mu_0 I}{2\pi r} = 2.0 \times 10^{-4} \text{ T}$$
Example 32.10 The Magnetic Force on an Electron

The magnetic force on an electron

Thus the magnitude of the force on the electron is

\[ F = e \times B = (1.60 \times 10^{-19} \text{C})(1.0 \times 10^2 \text{ m/s})(2.0 \times 10^{-1} \text{ T}) \]

\[ = 3.2 \times 10^{-18} \text{ N} \]

The force on the electron is \( F = 3.2 \times 10^{-18} \text{ N}, \) up.

**Assess** This force will cause the electron to curve away from the wire.

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QuickCheck 32.17

Which magnetic field (if it's the correct strength) allows the electron to pass through the charged electrodes without being deflected?

A.  
B.  
C.  
D.  
E.  

---

QuickCheck 32.17

Which magnetic field (if it's the correct strength) allows the electron to pass through the charged electrodes without being deflected?

A.  
B.  
C.  
D.  
E.  

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A proton is shot straight at the center of a long, straight wire carrying current into the screen. The proton will

A. Go straight into the wire.
B. Hit the wire in front of the screen.
C. Hit the wire behind the screen.
D. Be deflected over the wire.
E. Be deflected under the wire.

A proton is shot straight at the center of a long, straight wire carrying current into the screen. The proton will

A. Go straight into the wire.

B. Hit the wire in front of the screen. \( \vec{v} \times \vec{B} \) points out of the screen.
C. Hit the wire behind the screen.
D. Be deflected over the wire.
E. Be deflected under the wire.

The figure shows a positive charge moving in a plane that is perpendicular to a **uniform** magnetic field.

- Since \( \vec{F} \) is always perpendicular to \( \vec{v} \), the charge undergoes **uniform circular motion**.
- This motion is called the **cyclotron motion** of a charged particle in a magnetic field.
Electrons undergoing circular cyclotron motion in a magnetic field. You can see the electrons’ path because they collide with a low density gas that then emits light.

Consider a particle with mass $m$ and charge $q$ moving with a speed $v$ in a plane that is perpendicular to a uniform magnetic field of strength $B$.

- Newton’s second law for circular motion, which you learned in Chapter 8, is:
  $$ F = qvB = m\alpha = \frac{mv^2}{r} $$

- The radius of the cyclotron orbit is:
  $$ r_{\text{cy}} = \frac{mv}{qB} $$

- Recall that the frequency of revolution of circular motion is $f = \frac{v}{2\pi r}$, so the cyclotron frequency is:
  $$ f_{\text{cy}} = \frac{qB}{2\pi m} $$

The figure shows a more general situation in which the charged particle’s velocity is not exactly perpendicular to $\vec{B}$.

- The component of $\vec{v}$ parallel to $\vec{B}$ is not affected by the field, so the charged particle spirals around the magnetic field lines in a helical trajectory.

- The radius of the helix is determined by $v_{\perp}$, the component of $\vec{v}$ perpendicular to $\vec{B}$.
The first practical particle accelerator, invented in the 1930s, was the **cyclotron**. Cyclotrons remain important for many applications of nuclear physics, such as the creation of radioisotopes for medicine.

Consider a magnetic field perpendicular to a flat, current-carrying conductor. As the charge carriers move at the drift speed $v_d$, they will experience a magnetic force $F_B = ev_dB$ perpendicular to both $\vec{B}$ and the current $I$. The charge carriers are deflected to one surface.
The Hall Effect

- If the charge carriers are positive, the magnetic force pushes these positive charges down, creating an excess positive charge on the bottom surface, and leaving negative charge on the top.
- This creates a measureable Hall voltage $\Delta V_{H}$ which is higher on the bottom surface.

![Diagram showing the Hall Effect for positive charge carriers]

The Hall Effect

- If the charge carriers are negative, the magnetic force pushes these positive charges down, creating an excess negative charge on the bottom surface, and leaving positive charge on the top.
- This creates a measureable Hall voltage $\Delta V_{H}$ which is higher on the top surface.

![Diagram showing the Hall Effect for negative charge carriers]

The Hall Effect

- When charges are separated by a magnetic field in a rectangular conductor of thickness $t$ and width $w$, it creates an electric field $E = \Delta V_{H}/w$ inside the conductor.
- The steady-state condition is when the electric force balances the magnetic force, $F_{E} = F_{B}$:

$$F_{E} = e v_{d}B = e v_{d}E = e \frac{\Delta V}{w}$$

where $v_{d}$ is the drift speed, which is $v_{d} = I/(wtne)$.  
- From this we can find the Hall voltage:

$$\Delta V_{H} = \frac{IB}{tne}$$

where $n$ is the charge-carrier density (charge carriers per m$^3$).
Example 32.12 Measuring the Magnetic Field

**Example 32.12 Measuring the magnetic field**

A Hall probe consists of a strip of the metal bismuth that is 0.15 mm thick and 5.0 mm wide. Bismuth is a poor conductor with charge-carrier density $1.35 \times 10^{23}$ m$^{-3}$. The Hall voltage on the probe is 2.5 mV when the current through it is 1.5 A. What is the strength of the magnetic field, and what is the electric field strength inside the bismuth?

**SOLVE** Equation 32.24 gives the Hall voltage. We can rearrange the equation to find that the magnetic field is

$$B = \frac{be}{I} \Delta V_H$$

$$= \frac{(1.5 \times 10^{-4} \text{ m})(1.35 \times 10^{23} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})}{1.5 \text{ A}}$$

$$= 0.54 \text{ T}$$

The electric field created inside the bismuth by the excess charge on the surface is

$$E = \frac{\Delta V_H}{w} = \frac{0.0025 \text{ V}}{5.0 \times 10^{-3} \text{ m}} = 0.50 \text{ V/m}$$

**ASSESS** 0.54 T is a fairly typical strength for a laboratory magnet.
There's no force on a current-carrying wire parallel to a magnetic field.

A current perpendicular to the field experiences a force in the direction of the right-hand rule.

If a wire of length \( l \) contains a current \( I = q/\Delta t \), it means a charge \( q \) must move along its length in a time \( \Delta t = lv \).

Thus we have \( I = qv \).

Since \( \vec{F} = q\vec{v} \times \vec{B} \) the magnetic force on a current-carrying wire is:

\[
\vec{F}_{\text{mag}} = I\vec{v} \times \vec{B} = (IlB \sin \alpha, \text{direction of right-hand rule})
\]

The horizontal wire can be levitated – held up against the force of gravity – if the current in the wire is

A. Right to left.
B. Left to right.
C. It can’t be done with this magnetic field.
QuickCheck 32.19

The horizontal wire can be levitated – held up against the force of gravity – if the current in the wire is

A. Right to left.  
B. **Left to right.**  
C. It can’t be done with this magnetic field.

$\vec{F} \times \vec{B}$ points upward

Example 32.13 Magnetic Levitation

**Example 32.13 Magnetic levitation**

The 0.10 T uniform magnetic field of the figure below is horizontal, parallel to the floor. A straight segment of 1.0-mm-diameter copper wire, also parallel to the floor, is perpendicular to the magnetic field. What current through the wire, and in which direction, will allow the wire to “float” in the magnetic field?

MODEL The wire will float in the magnetic field if the magnetic force on the wire points upward and has magnitude $mg$, allowing it to balance the downward gravitational force.

Example 32.13 Magnetic Levitation
Example 32.13 Magnetic Levitation

**Example 32.13**

**Magnetic levitation**

**SOLVE** We can use the right-hand rule to determine which current direction experiences an upward force. With \( \vec{B} \) pointing away from us, the direction of the current needs to be from left to right. The forces will balance when

\[
F = ilB = mg = \rho(\pi r^2)g
\]

where \( \rho = 8920 \text{ kg/m}^3 \) is the density of copper. The length of the wire cancels, leading to

\[
I = \frac{\pi r^2 g}{B} = \frac{(8920 \text{ kg/m}^3)(\pi)(0.00050 \text{ m}^2)(9.80 \text{ m/s}^2)}{0.10 \text{ T}}
\]

\( = 0.69 \text{ A} \)

A 0.69 A current from left to right will levitate the wire in the magnetic field.

---

**Example 32.13 Magnetic Levitation**

**Example 32.13**

**Magnetic levitation**

**ASSESS** A 0.69 A current is quite reasonable, but this idea is useful only if we can get the current into and out of this segment of wire. In practice, we could do so with wires that come in from below the page. These input and output wires would be parallel to \( \vec{B} \) and not experience a magnetic force. Although this example is very simple, it is the basis for applications such as magnetic levitation trains.

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**Magnetic Forces Between Parallel Current-Carrying Wires: Current in Same Direction**

**Magnetic field \( \vec{B}_2 \) created by current \( I_2 \)**

**Magnetic field \( \vec{B}_1 \) created by current \( I_1 \)**

\[
F_{\text{magnet wires}} = I_1 I_2 \frac{\mu_0 B_2}{2\pi d} \]
Magnetic Forces Between Parallel Current-Carrying Wires: Current in Opposite Directions

\[ F_{\text{parallel}} = I_1 B_2 = \frac{\mu_0 I_1 I_2}{2 \pi d^2} \]

The two figures show alternative but equivalent ways to view magnetic forces between two current loops.

- Parallel currents attract, opposite currents repel.
- Opposite poles attract, like poles repel.

Forces on Current Loops

A Uniform Magnetic Field Exerts a Torque on a Square Current Loop

- \( \vec{F}_{\text{top}} \) and \( \vec{F}_{\text{bottom}} \) are opposite to each other and cancel.
- Both \( \vec{F}_{\text{top}} \) and \( \vec{F}_{\text{bottom}} \) exert a force of magnitude \( F = \mu_0 I B \) around a moment arm \( d = \frac{1}{2} l \sin \theta \).
The total torque is:
\[ \tau = 2Fd = (Il^2)B \sin \theta = \mu B \sin \theta \]
where \( \mu = \frac{l^2}{A} \) is the loop's magnetic dipole moment.

Although derived for a square loop, the result is valid for a loop of any shape:
\[ \tau = \vec{\mu} \times \vec{B} \]

QuickCheck 32.20
If released from rest, the current loop will
A. Move upward.
B. Move downward.
C. Rotate clockwise.
D. Rotate counterclockwise. **Correct**
E. Do something not listed here.

QuickCheck 32.20
If released from rest, the current loop will
A. Move upward.
B. Move downward.
C. Rotate clockwise.
D. Rotate counterclockwise. **Correct**  Net torque but no net force
E. Do something not listed here.
A plausible explanation for the magnetic properties of materials is the orbital motion of the atomic electrons. The figure shows a simple, classical model of an atom in which a negative electron orbits a positive nucleus.

- In this picture of the atom, the electron's motion is that of a current loop!
- An orbiting electron acts as a tiny magnetic dipole, with a north pole and a south pole.

Atomic Magnets

- An electron's inherent magnetic moment is often called the electron spin because, in a classical picture, a spinning ball of charge would have a magnetic moment.
- While it may not be spinning in a literal sense, an electron really is a microscopic magnet.
Magnetic Properties of Matter

- For most elements, the magnetic moments of the atoms are randomly arranged when the atoms join together to form a solid.
- As the figure shows, this random arrangement produces a solid whose net magnetic moment is very close to zero.

Ferromagnetism

- In iron, and a few other substances, the atomic magnetic moments tend to all line up in the same direction, as shown in the figure.
- Materials that behave in this fashion are called ferromagnetic, with the prefix ferro meaning “iron-like.”

Ferromagnetism

- A typical piece of iron is divided into small regions, typically less than 100 µm in size, called magnetic domains.
- The magnetic moments of all the iron atoms within each domain are perfectly aligned, so each individual domain is a strong magnet.
- However, the various magnetic domains that form a larger solid are randomly arranged.
If a ferromagnetic substance is subjected to an external magnetic field, the external field exerts a torque on the magnetic dipole of each domain. The torque causes many of the domains to rotate and become aligned with the external field.

The induced magnetic dipole always has an opposite pole facing the solenoid. Consequently the magnetic force between the poles pulls the ferromagnetic object to the electromagnet.

Now we can explain how a magnet attracts and picks up ferromagnetic objects:
1. Electrons are microscopic magnets due to their spin.
2. A ferromagnetic material in which the spins are aligned is organized into magnetic domains.
3. The individual domains align with an external magnetic field to produce an induced magnetic dipole moment for the entire object.
An object’s magnetic dipole may not return to zero when the external field is removed because some domains remain “frozen” in the alignment they had in the external field.

Thus a ferromagnetic object that has been in an external field may be left with a net magnetic dipole moment after the field is removed.

In other words, the object has become a permanent magnet.

At its most fundamental level, magnetism is an interaction between moving charges. The magnetic field of one moving charge exerts a force on another moving charge.
**Magnetic Fields**

The Biot-Savart law:

- A point charge, \( \mathbf{B} = \frac{\mu_0}{4\pi} \frac{q r^2 \times \mathbf{r}}{r^3} \)

- A short current element, \( \mathbf{B} = \frac{\mu_0}{4\pi} \frac{l I \times \mathbf{r}}{r^3} \)

To find the magnetic field of a current:
- Divide the wire into many short segments.
- Find the field of each segment \( \Delta s \).
- Find \( \mathbf{B} \) by summing the fields of all \( \Delta s \), usually as an integral.

An alternative method for fields with a high degree of symmetry is Ampere’s law:

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}
\]

where \( I_{\text{enc}} \) is the current through the area bounded by the integration path.

---

**Magnetic Forces**

- The magnetic force on a moving charge is \( \mathbf{F} = q \mathbf{v} \times \mathbf{B} \).
- The force is perpendicular to \( \mathbf{v} \) and \( \mathbf{B} \).
- The magnetic force on a current-carrying wire is \( \mathbf{F} = I \mathbf{l} \times \mathbf{B} \).
- \( \mathbf{F} = 0 \) for a charge at current moving parallel to \( \mathbf{B} \).
- The magnetic torque on a magnetic dipole is \( \mathbf{T} = \mathbf{p} \times \mathbf{B} \).