IN THIS CHAPTER, you will learn about and apply the ray model of light
Chapter 34 Preview

What is the law of reflection?
Light rays bounce, or reflect, off a surface.
- Specular reflection is mirror like.
- Diffuse reflection is like light reflecting from the page of this book.

The law of reflection says that the angle of reflection equals the angle of incidence. You’ll learn how reflection allows images to be seen in both flat and curved mirrors.

Chapter 34 Preview

What is refraction?
Light rays change direction at the boundary when they move from one medium to another. This is called refraction, and it is the basis for image formation by lenses.

Snell's law will allow you to find the angles on both sides of the boundary.

LOOKING BACK Section 16.5 Index of refraction

Chapter 34 Preview

How do lenses form images?
Lenses form images by refraction.
- We’ll start with ray tracing, a graphical method of seeing how and where images are formed.
- We’ll then develop the thin-lens equation for more quantitative results.

The same methods apply to image formation by curved mirrors.
Chapter 34 Preview

Why is optics important?
Optics is everywhere, from your smart phone camera and your car headlights to laser pointers and the optical scanners that read bar codes. Our knowledge of the microscopic world and of the cosmos comes through optical instruments. And, of course, your eye is one of the most marvelous optical devices of all. Modern optical engineering is called photonics. Photonics does draw on all three models of light, as needed, but ray optics is usually the foundation on which optical instruments are designed.

Chapter 34 Reading Questions

Reading Question 34.1

What is specular reflection?

A. The image of a specimen
B. A reflection that separates different colors
C. Reflection by a flat smooth object
D. Reflection in which the image is virtual and special
E. This topic is not covered in Chapter 34.
Reading Question 34.1

What is specular reflection?

A. The image of a specimen
B. A reflection that separates different colors
C. Reflection by a flat smooth object
D. Reflection in which the image is virtual and special
E. This topic is not covered in Chapter 34.

Reading Question 34.2

What is diffuse reflection?

A. A reflection that separates different colors
B. Reflection by a surface with tiny irregularities that cause the reflected rays to leave in many random directions
C. Reflection that increases in size linearly with distance from the mirror
D. Reflection in which the image is virtual
E. This topic is not covered in Chapter 34.
Reading Question 34.3

A paraxial ray

A. Moves in a parabolic path.
B. Is a ray that has been reflected from a parabolic mirror.
C. Is a ray that moves nearly parallel to the optical axis.
D. Is a ray that moves exactly parallel to the optical axis.

Reading Question 34.3

A paraxial ray

A. Moves in a parabolic path.
B. Is a ray that has been reflected from a parabolic mirror.
C. **Is a ray that moves nearly parallel to the optical axis.**
D. Is a ray that moves exactly parallel to the optical axis.

Reading Question 34.4

A virtual image is

A. The cause of optical illusions.
B. A point from which rays appear to diverge.
C. An image that only seems to exist.
D. The image that is left in space after you remove a viewing screen.
Reading Question 34.4

A virtual image is

A. The cause of optical illusions.
B. A point from which rays appear to diverge.
C. An image that only seems to exist.
D. The image that is left in space after you remove a viewing screen.

Reading Question 34.5

The focal length of a converging lens is

A. The distance at which an image is formed.
B. The distance at which an object must be placed to form an image.
C. The distance at which parallel light rays are focused.
D. The distance from the front surface to the back surface.

Reading Question 34.5

The focal length of a converging lens is

A. The distance at which an image is formed.
B. The distance at which an object must be placed to form an image.
C. The distance at which parallel light rays are focused.
D. The distance from the front surface to the back surface.
Let us define a light ray as a line in the direction along which light energy is flowing. Any narrow beam of light, such as a laser beam, is actually a bundle of many parallel light rays. You can think of a single light ray as the limiting case of a laser beam whose diameter approaches zero.
Objects can be either self-luminous, such as the sun, flames, and lightbulbs, or reflective. Most objects are reflective.

- The diverging rays from a **point source** are emitted in all directions.
- Each point on an object is a point source of light rays.
- A **parallel bundle** of rays could be a laser beam or light from a **distant object**.
Rays originate from every point on an object and travel outward in all directions, but a diagram trying to show all these rays would be messy and confusing. To simplify the picture, we use a ray diagram showing only a few rays.

These are just a few of the infinitely many rays leaving the object.

A camera obscura is a darkened room with a single, small hole, called an aperture. The geometry of the rays causes the image to be upside down. The object and image heights are related by

\[ \frac{h_1}{h_0} = \frac{d_1}{d_0} \]

We can apply the ray model to more complex apertures, such as the L-shaped aperture below.
QuickCheck 34.1

The dark screen has a small hole, ≈2 mm in diameter. The lightbulb is the only source of light. What do you see on the viewing screen?

A.  
B.  
C.  

QuickCheck 34.2

Two point sources of light illuminate a narrow vertical aperture in a dark screen. What do you see on the viewing screen?

A.  
B.  
C.  
D.  
E.  
QuickCheck 34.2

Two point sources of light illuminate a narrow vertical aperture in a dark screen. What do you see on the viewing screen?

A. [Diagram of A]  
B. [Diagram of B]  
C. [Diagram of C]  
D. [Diagram of D]  
E. [Diagram of E]

Specular Reflection of Light

• Reflection from a flat, smooth surface, such as a mirror or a piece of polished metal, is called **specular reflection**.

  Both the incident and reflected rays lie in a plane that is perpendicular to the surface.

Reflection

• The **law of reflection** states that
  1. The incident ray and the reflected ray are in the same plane normal to the surface, and
  2. The angle of reflection equals the angle of incidence:

\[
\theta_i = \theta_r
\]
Example 34.1 Light Reflecting from a Mirror

**EXAMPLE 34.1** Light reflecting from a mirror

A dressing mirror on a closet door is 1.50 m tall. The bottom is 0.50 m above the floor. A bare lightbulb hangs 1.00 m from the closet door, 2.50 m above the floor. How long is the streak of reflected light across the floor?

**MODEL** Treat the lightbulb as a point source and use the ray model of light.

**VISUALIZE** Figure 34.8 is a pictorial representation of the light rays. We need to consider only the two rays that strike the edges of the mirror. All other reflected rays will fall between these two.

![Diagram of light reflecting from a mirror](image)

The angles are the same by the law of reflection.

**SOLVE** Figure 34.8 has used the law of reflection to set the angles of reflection equal to the angles of incidence. Other angles have been identified with simple geometry. The two angles of incidence are

\[
\theta_i = \tan^{-1}\left(\frac{2.50 \text{ m}}{1.00 \text{ m}}\right) = 63.4^\circ
\]

\[
\theta_i = \tan^{-1}\left(\frac{1.50 \text{ m}}{2.50 \text{ m}}\right) = 36.6^\circ
\]
Most objects are seen by virtue of their reflected light.

For a “rough” surface, the law of reflection is obeyed at each point but the irregularities of the surface cause the reflected rays to leave in many random directions.

This situation is called diffuse reflection.

It is how you see this slide, the wall, your hand, your friend, and so on.

The Plane Mirror

Consider \( P \), a source of rays that reflect from a mirror.

The reflected rays appear to emanate from \( P' \), the same distance behind the mirror as \( P \) is in front of the mirror.

That is, \( s' = s \).
QuickCheck 34.3

You are looking at the image of a pencil in a mirror. What do you see in the mirror if the top half of the mirror is covered with a piece of dark paper?

A. The full image of the pencil
B. The top half only of the pencil
C. The bottom half only of the pencil
D. No pencil, only the paper

✓ A. The full image of the pencil

B. The top half only of the pencil
C. The bottom half only of the pencil
D. No pencil, only the paper
Example 34.2 How High Is the Mirror?

**EXAMPLE 34.2** How high is the mirror?

If your height is \( h \), what is the shortest mirror on the wall in which you can see your full image? Where must the top of the mirror be hung?

**MODEL** Use the ray model of light.

**VISUALIZE** Figure 34.13 is a pictorial representation of the light rays. We need to consider only the two rays that leave your head and feet and reflect into your eye.

[Diagram of light rays reflecting off a mirror]

**SOLUTION**

1. Let the distance from your eye to the top of your head be \( h_1 \), and the distance from your toe to \( f \). Your height is \( h \).
2. A light ray from the tip of your head that shines from the mirror is a distance \( h \) below your eyes.
3. From your feet is a distance \( f \) below your eyes.

By geometric principles, we can solve for the distance \( h \) between your eye and the mirror.
Two things happen when a light ray is incident on a smooth boundary between two transparent materials:

1. Part of the light reflects from the boundary, obeying the law of reflection.

2. Part of the light continues into the second medium. The transmission of light from one medium to another, but with a change in direction, is called refraction.
Refraction

Refraction from a lower-index medium to a higher-index medium

Angle of incidence

Incident ray

Medium 1

Normal

Medium 2

Assume $n_2 > n_1$

Weak reflected ray

The ray has a kink at the boundary.

Refracted ray

Angle of refraction

$n_1 \sin \theta_1 = n_2 \sin \theta_2$ (Snell’s law of refraction)

Indices of Refraction

<table>
<thead>
<tr>
<th>Medium</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.00 exactly</td>
</tr>
<tr>
<td>Air (actual)</td>
<td>1.0003</td>
</tr>
<tr>
<td>Air (accepted)</td>
<td>1.00</td>
</tr>
<tr>
<td>Water</td>
<td>1.33</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>1.36</td>
</tr>
<tr>
<td>Oil</td>
<td>1.46</td>
</tr>
<tr>
<td>Glass (typical)</td>
<td>1.50</td>
</tr>
<tr>
<td>Polystyrene plastic</td>
<td>1.59</td>
</tr>
<tr>
<td>Cubic zirconia</td>
<td>2.18</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.41</td>
</tr>
<tr>
<td>Silicon (infrared)</td>
<td>3.50</td>
</tr>
</tbody>
</table>
Refraction

- The figure shows a wave crossing the boundary between two media, where we're assuming \( n_2 > n_1 \).
- Because the wavelengths differ on opposite sides of the boundary, the wave fronts can stay lined up only if the waves in the two media are traveling in different directions.

QuickCheck 34.4

A laser beam passing from medium 1 to medium 2 is refracted as shown. Which is true?

A. \( n_1 < n_2 \)
B. \( n_1 > n_2 \)
C. There's not enough information to compare \( n_1 \) and \( n_2 \).
### Tactics: Analyzing Refraction

**TACTICS BOX 34.1**

**Analyzing refraction**

1. Draw a ray diagram. Represent the light beam with one ray.
2. Draw a line normal to the boundary. Do this at each point where the ray intersects a boundary.
3. Show the ray bending in the correct direction. The angle is larger on the side with the smaller index of refraction. This is the qualitative application of Snell’s law.
4. Label angles of incidence and refraction. Measure all angles from the normal.
5. Use Snell’s law. Calculate the unknown angle or unknown index of refraction.

*Exercise 11-15*

---

### Example 34.4 Measuring the Index of Refraction

**EXAMPLE 34.4** Measuring the index of refraction

*Figure 34.18* shows a laser beam deflected by a 30°-60°-90° prism. What is the prism’s index of refraction?

**MODEL:** Represent the laser beam with a single ray and use the ray model of light.

![Diagram of laser beam deflection](image)

---

### Example 34.4 Measuring the Index of Refraction

**EXAMPLE 34.4** Measuring the index of refraction

*Figure 34.19* uses the steps of Tactics Box 34.1 to draw a ray diagram. The ray is incident perpendicular to the front face of the prism (θ1 = 0°), thus it is transmitted through the first boundary without deflection. At the second boundary it is especially important to draw the normal to the surface at the point of incidence and to measure angles from the normal.

![Diagram of ray diagram](image)
Example 34.4 Measuring the Index of Refraction

\[ n_1 = 1.59 \]

**Example 34.4 Measuring the Index of Refraction**

**Solve:** From the geometry of the triangle, you can find that the base angle of incidence is \( \phi = 30^\circ \), the same as the apex angle of the prism. The ray exits the prism at angle \( \theta \), such that the deflection is \( \theta = \theta_1 - \theta_2 = 22.6^\circ \). Thus, \( \theta = 52.6^\circ \). Knowing both angles and \( n_1 = 1.00 \) for air, we can use Snell’s law to find \( n_2 \):

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2} = \frac{1.00 \sin 52.6^\circ}{\sin 30^\circ} = 1.59 \]

**Total Internal Reflection**

- When a ray crosses a boundary into a material with a lower index of refraction, it bends away from the normal.
- As the angle \( \theta_1 \) increases, the refraction angle \( \theta_2 \) approaches \( 90^\circ \), and the fraction of the light energy transmitted decreases while the fraction reflected increases.
- The critical angle of incidence occurs when \( \theta_2 = 90^\circ \):

\[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]

- The refracted light vanishes at the critical angle and the reflection becomes 100% for any angle \( \theta_1 > \theta_c \).
A laser beam undergoes two refractions plus total internal reflection at the interface between medium 2 and medium 3. Which is true?

A. \(n_1 < n_3\)
B. \(n_1 > n_3\)
C. There's not enough information to compare \(n_1\) and \(n_3\).
**Example 34.5 Total Internal Reflection**

**EXAMPLE 34.5** Total internal reflection

A small light bulb is set in the bottom of a 3.0 m deep swimming pool. What is the diameter of the circle of light seen on the water’s surface from above?

**MODEL** Use the ray model of light.

\[ n_1 = 1.00 \quad \text{Air} \]
\[ n_2 = 1.33 \quad \text{Water} \]

\[ h = 3.0 \text{ m} \]

Rays at the critical angle \( \theta_c \) form the edge of the circle of light seen from above.

**SOLVE** From trigonometry, the circle diameter is

\[ D = 2h \tan \theta_c \]

where \( h \) is the depth of the water. The critical angle for a water-air boundary is \( \theta_c = \sin^{-1}(1.00/1.33) = 48.7^\circ \). Thus

\[ D = 2(3.0 \text{ m}) \tan 48.7^\circ = 6.8 \text{ m} \]
The most important modern application of **total internal reflection** (TIR) is optical fibers.
- Light rays enter the glass fiber, then impinge on the inside wall of the glass at an angle above the critical angle, so they undergo TIR and remain inside the glass.
- The light continues to "bounce" its way down the tube as if it were inside a pipe.

In a practical optical fiber, a small-diameter glass core is surrounded by a layer of glass cladding.
- The glasses used for the core and the cladding have
  \[ n_{\text{core}} > n_{\text{cladding}} \]

If you see a fish that appears to be swimming close to the front window of the aquarium, but then look through the side of the aquarium, you'll find that the fish is actually farther from the window than you thought.
- Refraction causes the rays to bend at the boundary.
- Now the rays that reach the eye are diverging from this point, the image.
Rays emerge from a material with $n_1 > n_2$.

Consider only paraxial rays, for which $\theta_1$ and $\theta_2$ are quite small.

In this case:

$$s' = \frac{n_2}{n_1} s$$

where $s$ is the object distance and $s'$ is the image distance.

The minus sign tells us that we have a virtual image.

---

**QuickCheck 34.6**

A fish in an aquarium with flat sides looks out at a hungry cat. To the fish, the distance to the cat appears to be

A. Less than the actual distance.
B. Equal to the actual distance.
C. More than the actual distance.

---

**QuickCheck 34.6**

A fish in an aquarium with flat sides looks out at a hungry cat. To the fish, the distance to the cat appears to be

A. Less than the actual distance.
B. Equal to the actual distance.

✔ C. More than the actual distance.
Example 34.6 An Air Bubble in a Window

**EXAMPLE 34.6** An air bubble in a window

A fish and a sailor look at each other through a 3.0-cm-thick glass window. There happens to be an air bubble right in the center of the glass. How far behind the surface of the glass does the air bubble appear to the fish? To the sailor?

**MODEL** Represent the air bubble as a point source and use the ray model of light.

---

**EXAMPLE 34.6** An air bubble in a window

**VISUALIZE** Parallel light rays from the bubble refract into the air on one side and into the water on the other. The ray diagram looks like Figure 34.26.

---

**EXAMPLE 34.6** An air bubble in a window

**SOLVE** The index of refraction of the glass is \( n_1 = 1.50 \). The bubble is in the center of the window, so the object distance from either side of the window is \( s = 7.5 \text{ cm} \). On the water side, the image distance is

\[
\frac{s'}{s} = \frac{n_1}{n_2} = \frac{1.50}{1.33} = 1.13
\]

The minus sign indicates a virtual image. Physically, the fish sees the bubble 2.2 cm behind the surface. The image distance on the water side is

\[
\frac{s'}{s} = \frac{n_1}{n_2} = \frac{1.50}{1.00} = 1.50
\]

So the fish sees the bubble 1.7 cm behind the surface.

**ASSUME** The image distance is less for the sailor because of the larger difference between the two indices of refraction.
The photos below show parallel light rays entering two different lenses.

- The left lens, called a **converging lens**, causes the rays to refract **toward** the optical axis.
- The right lens, called a **diverging lens**, refracts parallel rays **away from** the optical axis.

**Converging Lenses**

- A **converging lens** is thicker in the center than at the edges.
- The focal length $f$ is the distance from the lens at which rays parallel to the optical axis converge.
- The focal length is a **property of the lens**, independent of how the lens is used.

**Diverging Lenses**

- A **diverging lens** is thicker at the edges than in the center.
- The focal length $f$ is the distance from the lens at which rays parallel to the optical axis appear to diverge.
- The focal length is a **property of the lens**, independent of how the lens is used.
QuickCheck 34.7

You can use the sun’s rays and a lens to start a fire. To do so, you should use

A. A converging lens.
B. A diverging lens.
C. Either a converging or a diverging lens will work if you use it correctly.

✓ A. A converging lens.
B. A diverging lens.
C. Either a converging or a diverging lens will work if you use it correctly.

Thin Lenses: Ray Tracing

- Three situations form the basis for ray tracing through a thin converging lens.
- Situation 1:
  A ray initially parallel to the optic axis will go through the far focal point after passing through the lens.

- Diagram showing ray tracing through a lens with parallel rays and focal point.
Three situations form the basis for ray tracing through a thin converging lens.

Situation 2:
A ray through the near focal point of a thin lens becomes parallel to the optic axis after passing through the lens.

Situation 3:
A ray through the center of a thin lens is neither bent nor displaced but travels in a straight line.

Rays from an object point \( P \) are refracted by the lens and converge to a real image at point \( P' \).
A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if the lens is removed?

A. An inverted but blurry image
B. An image that is dimmer but otherwise unchanged
C. A sharp, upright image
D. A blurry, upright image
E. No image at all

Correct answer: E. No image at all

A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if a piece of dark paper is lowered to cover the top half of the lens?

A. An inverted but blurry image
B. An image that is dimmer but otherwise unchanged
C. Only the top half of the image
D. Only the bottom half of the image
E. No image at all

Correct answer: E. No image at all
QuickCheck 34.9

A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if a piece of dark paper is lowered to cover the top half of the lens?

A. An inverted but blurry image
B. An image that is dimmer but otherwise unchanged
C. Only the top half of the image
D. Only the bottom half of the image
E. No image at all

QuickCheck 34.10

A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if the lens is covered by a dark mask having only a small hole in the center?

A. An inverted but blurry image
B. An image that is dimmer but otherwise unchanged
C. Only the middle piece of the image
D. A circular diffraction pattern
E. No image at all
The figure is a close-up view of the rays very near the image plane. To focus an image, you must either move the screen to coincide with the image plane or move the lens or object to make the image plane coincide with the screen.
QuickCheck 34.11

A lens creates an image as shown. In this situation, the object distance \( s \) is

A. Larger than the focal length \( f \).
B. Equal to the focal length \( f \).
C. Smaller than focal length \( f \).

QuickCheck 34.11

A lens creates an image as shown. In this situation, the object distance \( s \) is

✓ A. Larger than the focal length \( f \).
B. Equal to the focal length \( f \).
C. Smaller than focal length \( f \).

QuickCheck 34.12

A lens creates an image as shown. In this situation, the image distance \( s' \) is

A. Larger than the focal length \( f \).
B. Equal to the focal length \( f \).
C. Smaller than focal length \( f \).
QuickCheck 34.12

A lens creates an image as shown. In this situation, the image distance $s'$ is

- A. Larger than the focal length $f$.
- B. Equal to the focal length $f$.
- C. Smaller than focal length $f$.

Lateral Magnification

- The image can be either larger or smaller than the object, depending on the location and focal length of the lens.
- The lateral magnification $m$ is defined as
  \[
  m = \frac{s'}{s}
  \]
- A positive value of $m$ indicates that the image is upright relative to the object.
- A negative value of $m$ indicates that the image is inverted relative to the object.
- The absolute value of $m$ gives the size ratio of the image and object: $h'/h = |m|

Virtual Images

- Consider a converging lens for which the object is inside the focal point, at distance $s < f$.
- You can see all three rays appear to diverge from point $P'$.
- Point $P'$ is an upright, virtual image of the object point $P$.
Virtual Images

- You can see a virtual image by looking through the lens.
- This is exactly what you do with a magnifying glass, microscope, or binoculars.

Example 34.8 Magnifying a Flower

**EXAMPLE 34.8** Magnifying a flower
To see a flower better, a naturalist holds a 8.0-cm-focal-length magnifying glass 4.0 cm from the flower. What is the magnification?

**MODEL** The flower is in the object plane. Use ray tracing to locate the image.

**EXAMPLE 34.8** Magnifying a flower

**VISUALIZE FIGURE 34.35** shows the ray-tracing diagram. The three special rays diverge from the lens, but we can use a straightedge to extend the rays backward to the point from which they diverge. This point, the image point, is seen to be 12 cm to the left of the lens.
Example 34.8 Magnifying a Flower

**Example 34.8** Magnifying a flower

**Visualize**: Because this is a virtual image, the image distance is a negative \( s' = -12 \text{ cm} \). Thus the magnification is

\[
m = \frac{s'}{s} = \frac{-12 \text{ cm}}{4.0 \text{ cm}} = 3.0
\]

The image is three times as large as the object and, because \( m \) is positive, upright.

Thin Lenses: Ray Tracing

- Three situations form the basis for ray tracing through a thin **diverging lens**.
- **Situation 1:**
  A ray initially parallel to the optic axis will appear to diverge from the near focal point after passing through the lens.

Thin Lenses: Ray Tracing

- Three situations form the basis for ray tracing through a thin **diverging lens**.
- **Situation 2:**
  A ray directed along a line toward the far focal point becomes parallel to the optic axis after passing through the lens.
Three situations form the basis for ray tracing through a thin diverging lens.

Situation 3:
A ray through the center of a thin lens is neither bent nor displaced but travels in a straight line.

QuickCheck 34.13

Light rays are converging to point 1. The lens is inserted into the rays with its focal point at point 1. Which picture shows the rays leaving the lens?
Tactics: Ray Tracing for a Diverging Lens

**TACTICS BOX 34.3**

Ray tracing for a diverging lens

1. Follow steps 1 through 3 of Tactic Box 34.2.
2. Draw the three "special rays" from the tip of the arrow. Use a straightedge.
   a. A ray parallel to the axis diverges along a line through the near focal point.
   b. A ray along a line toward the far focal point emerges parallel to the axis.
   c. A ray through the center of the lens does not bend.
3. Trace the diverging rays backward. The point from which they are diverging is the image point, which is always a virtual image.
4. Measure the image distance $s'$. This will be a negative number.

Example 34.9 Demagnifying a Flower

**EXAMPLE 34.9** Demagnifying a flower

A diverging lens with a focal length of 50 cm is placed 100 cm from a flower. Where is the image? What is its magnification?

**MODEL** The flower is in the object plane. Use ray tracing to locate the image.

![Ray tracing diagram for Example 34.9](image)
Example 34.9 Demagnifying a Flower

**EXAMPLE 34.9** Demagnifying a Flower

**VISUALIZE** A virtual image is formed at \( s' = -33 \text{ cm} \) with a magnification of

\[
m = \frac{s'}{s} = \frac{-33 \text{ cm}}{100 \text{ cm}} = 0.33
\]

The image, which can be seen by looking through the lens, is one-third the size of the object and upright.

---

Thin Lenses: Refraction Theory

- Consider a spherical boundary between two transparent media with indices of refraction \( n_1 \) and \( n_2 \).
- The sphere has radius of curvature \( R \) and is centered at point \( C \).

---

Thin Lenses: Refraction Theory

- If an object is located at distance \( s \) from a spherical refracting surface, an image will be formed at distance \( s' \) given by

\[
s' = \frac{n_1 s - n_2 s' - n_1}{n_2}
\]

**TABLE 34.2** Sign convention for refracting surfaces

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>Convex toward the object</td>
</tr>
<tr>
<td>( s' )</td>
<td>Real image, opposite side from object</td>
</tr>
</tbody>
</table>
Example 34.11 A Goldfish in a Bowl

**EXAMPLE 34.11** A goldfish in a bowl

A goldfish lives in a spherical fish bowl 30 cm in diameter. If the fish is 10 cm from the inner edge of the bowl, where does the fish appear when viewed from the outside?

**MODE**: Model the fish as a point source and consider the paraxial rays that refract from the water into the air. The thin glass wall has little effect and will be ignored.

**VISUALIZE**: Figure 34.49 shows the rays refracting away from the normal as they move from the water into the air. We expect to find a virtual image at a distance less than 10 cm.

\[
\begin{align*}
\eta_1 &= 1.33 \\
R &= -25 \text{ cm} \\
\eta_2 &= 1.00 \\
\end{align*}
\]

**SOLVE**: The object is in the water, so \( \eta_1 = 1.33 \) and \( \eta_2 = 1.00 \). The inner surface is concave (you can remember “concave” because it's like looking into a cave), so \( R = -25 \text{ cm} \). The object distance is \( s = 10 \text{ cm} \). Thus, Equation 34.20 is

\[
\frac{1.33}{10 \text{ cm}} \cdot \frac{1.00}{s} = \frac{1.00 - 1.33}{-25 \text{ cm}} = \frac{0.33}{25 \text{ cm}}
\]

Solving for the image distance \( s' \) gives:

\[
\begin{align*}
\frac{0.33}{25 \text{ cm}} &\cdot \frac{1.33}{10 \text{ cm}} = -0.12 \text{ cm}^{-1} \\
\frac{1.00}{s'} &= -0.12 \text{ cm}^{-1} \\
s' &= -\frac{10 \text{ cm}}{-0.12 \text{ cm}^{-1}} = 83 \text{ cm}
\end{align*}
\]
Example 34.11 A Goldfish in a Bowl

**Example 34.11** A goldfish in a bowl

*Assess* The image is virtual, located to the left of the boundary. A person looking into the bowl will see a fish that appears to be 8.3 cm from the edge of the bowl.

\[ x' = -8.3 \text{ cm} \]

Lenses

- In an actual lens, rays refract twice, at spherical surfaces having radii of curvature \( R_1 \) and \( R_2 \).

<table>
<thead>
<tr>
<th>Objective</th>
<th>Image</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual</td>
<td>Virtual</td>
<td>( f &lt; 0 )</td>
</tr>
<tr>
<td>Real</td>
<td>Virtual</td>
<td>( f &gt; 0 )</td>
</tr>
</tbody>
</table>

The Thin Lens Equation

- The object distance \( s \) is related to the image distance \( s' \) by

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \] (thin-lens equation)

where \( f \) is the focal length of the lens, which can be found from

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \] (lens maker’s equation)

where \( R_1 \) is the radius of curvature of the first surface, and \( R_2 \) is the radius of curvature of the second surface, and the material surrounding the lens has \( n = 1 \).
A lens creates an image as shown. In this situation,

A. \( s < f \)
B. \( f < s < 2f \)
C. \( s > 2f \)
D. There’s not enough information to compare \( s \) to \( f \).


---

A lens creates an image as shown. In this situation,

A. \( s < f \)
B. \( f < s < 2f \)
C. \( s > 2f \)
D. There’s not enough information to compare \( s \) to \( f \).

The image is real, which requires \( s > f \).
The image is taller than the object, and \( s' > s \) requires \( s < 2f \).

---

**Example 34.12 Focal Length of a Meniscus Lens**

**EXAMPLE 34.12** Focal length of a meniscus lens

What is the focal length of the glass meniscus lens shown in FIGURE 34.42? Is this a converging or diverging lens?

\[
\begin{align*}
R_1 & = 40 \text{ cm} \\
n & = 1.50 \\
R_2 & = 20 \text{ cm}
\end{align*}
\]
Example 34.12 Focal Length of a Meniscus Lens

**EXAMPLE 34.12** Focal length of a meniscus lens

**SOLVE** If the object is on the left, then the first surface has $R_1 = -40 \text{ cm}$ (concave toward the object) and the second surface has $R_2 = -20 \text{ cm}$ (also concave toward the object). The index of refraction of glass is $n = 1.50$, so the lens maker’s equation is

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = (1.50 - 1) \left( \frac{1}{-40 \text{ cm}} - \frac{1}{-20 \text{ cm}} \right)$$

$$= 0.0125 \text{ cm}^{-1}$$

Inverting this expression gives $f = 80 \text{ cm}$. This is a converging lens, as seen both from the positive value of $f$ and from the fact that the lens is thicker in the center.

---

Example 34.14 A Magnifying Lens

**EXAMPLE 34.14** A magnifying lens

A stamp collector sees a magnifying lens that sits 2.0 cm above the stamp. The magnification is 4.0. What is the focal length of the lens?

**MODEL** A magnifying lens is a converging lens with the object distance less than the focal length ($s < f$). Assume it is a thin lens.

---

Example 34.14 A Magnifying Lens

**EXAMPLE 34.14** A magnifying lens

**VISUALIZE** Figure 34.44 shows the lens and a ray-tracing diagram. We do not need to know the actual shape of the lens, so the figure shows a generic converging lens.

[Diagram of a magnifying lens with focal points, lens plane, and virtual image]
Example 34.14 A Magnifying Lens

**EXAMPLE 34.14** A magnifying lens

**Solve** A virtual image is upright, so \( m = +4.0 \). The magnification is \( m = -s'/s \), thus

\[
s' = -4.0s = -(4.0)(2.0 \text{ cm}) = -8.0 \text{ cm}
\]

We can use \( s \) and \( s' \) in the thin-lens equation to find the focal length:

\[
f = \frac{1}{s} + \frac{1}{s'} = \frac{1}{2.0 \text{ cm}} + \frac{1}{-8.0 \text{ cm}} = 0.375 \text{ cm}^{-1}
\]

**Assess** \( f > 2 \text{ cm} \), as expected.

---

Image Formation with Concave Spherical Mirrors

- The figure shows a **concave mirror**, a mirror in which the edges curve **toward** the light source.
- Rays parallel to the optical axis reflect and pass through the focal point of the mirror.

---

A Real Image Formed by a Concave Mirror

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

(mirror equation)
The figure shows parallel light rays approaching a mirror in which the edges curve away from the light source. This is called a **convex mirror**.

The reflected rays appear to come from a point behind the mirror.

---

### A Real Image Formed by a Convex Mirror

This ray entered parallel to the optical axis, and then appeared to have come from the focal point. This ray was heading for the focal point, and thus emerges parallel to the optical axis.

\[
\frac{1}{x} + \frac{1}{x'} = \frac{1}{f} \quad \text{(mirror equation)}
\]

---

### Image Formation with Spherical Mirrors

A city skyline is reflected in this polished sphere.
For a spherical mirror with negligible thickness, the object and image distances are related by:

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(mirror equation)} \]

where the focal length \( f \) is related to the mirror’s radius of curvature by:

\[ f = \frac{R}{2} \]

### Table 34.4: Sign convention for spherical mirrors

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R, f )</td>
<td>Convex toward the object</td>
</tr>
<tr>
<td>( s' )</td>
<td>Real image, same side as object</td>
</tr>
</tbody>
</table>
QuickCheck 34.15

You see an upright, magnified image of your face when you look into magnifying "cosmetic mirror." The image is located

A. In front of the mirror’s surface.
B. On the mirror’s surface.
C. Behind the mirror’s surface.
D. Only in your mind because it’s a virtual image.

QuickCheck 34.15

You see an upright, magnified image of your face when you look into magnifying "cosmetic mirror." The image is located

A. In front of the mirror’s surface.
B. On the mirror’s surface.
C. **Behind the mirror’s surface.**
D. Only in your mind because it’s a virtual image.

Example 34.16 Analyzing a Concave Mirror

**EXAMPLE 34.16** Analyzing a concave mirror

A 3.0-cm-high object is located 20 cm from a concave mirror. The mirror’s radius of curvature is 80 cm. Determine the position, orientation, and height of the image.

**METHODOLOGY:** Treat the mirror as a thin mirror.
Example 34.16 Analyzing a Concave Mirror

**EXAMPLE 34.16**  Analyzing a concave mirror

**VISUALIZE** The mirror's focal length is \( f = \frac{r}{2} = \frac{+40}{2} = +20 \text{ cm} \), where we used the sign convention from Table 34.4. With the focal length known, the three special rays in Figure 34.51 show that the image is a magnified, virtual image behind the mirror.

\[
\begin{align*}
\text{Object} & \quad \text{Mirror plane} \\
\quad & \quad \text{Virtual image}
\end{align*}
\]

**EXHIBIT**

\[
\begin{align*}
f &= 40 \text{ cm} \\
 s &= 20 \text{ cm} \\
 s' &= \text{virtual image}
\end{align*}
\]

Example 34.16 Analyzing a Concave Mirror

**EXAMPLE 34.16**  Analyzing a concave mirror

**VISUALIZE** The mirror's focal length is \( f = \frac{r}{2} = +40 \text{ cm} \), where we used the sign convention from Table 34.4. With the focal length known, the three special rays in Figure 34.51 show that the image is a magnified, virtual image behind the mirror.

\[
\begin{align*}
\text{Object} & \quad \text{Mirror plane} \\
\quad & \quad \text{Virtual image}
\end{align*}
\]

**EXHIBIT**

\[
\begin{align*}
f &= 40 \text{ cm} \\
 s &= 20 \text{ cm} \\
 s' &= \text{virtual image}
\end{align*}
\]

Example 34.16 Analyzing a Concave Mirror

**EXAMPLE 34.16**  Analyzing a concave mirror

**ASSESS** This is a virtual image because light rays diverge from the image point. You could see this enlarged image by standing behind the object and looking into the mirror. In fact, this is how magnifying cosmetic mirrors work.

\[
\begin{align*}
\text{Object} & \quad \text{Mirror plane} \\
\quad & \quad \text{Virtual image}
\end{align*}
\]

**EXHIBIT**

\[
\begin{align*}
f &= 40 \text{ cm} \\
 s &= 20 \text{ cm} \\
 s' &= \text{virtual image}
\end{align*}
\]
Chapter 34 Summary Slides

General Principles

**Reflection**
Law of reflection: $\theta_i = \theta_r$
Reflection can be specular (mirror-like) or diffuse (fuzzy rough surfaces). Please mirror: A virtual image is formed at $P'$ with $\theta' = \theta$.

General Principles

**Refraction**
Snell's law of refraction:
$n_x \sin \theta = n_y \sin \theta_y$
Index of refraction $n = c/v$, The ray is closer to the normal on the side with the larger index of refraction.
If $n_x < n_y$, total internal reflection (TIR) occurs when the angle of incidence $\theta_i = \theta = \sin^{-1}(n_y/n_x)$.
Important Concepts

The ray model of light
Light travels along straight lines, called light rays, at speed $c = 3 \times 10^8 \text{ m/s}$. A light ray continues forever unless an interaction with matter causes it to reflect, refract, scatter, or be absorbed. Light rays come from objects. Each point on the object sends rays in all directions. The eye sees an object (or an image) when diverging rays are collected by the pupil and focused on the retina.

Ray optics is valid when lenses, mirrors, and apertures are larger than $\lambda = 1 \text{ mm}$. 

Important Concepts

Image formation
If rays diverge from P and intersect with a lens or mirror so that the reflected/refracted rays converge at P', then P' is a real image of P.

If rays diverge from P and intersect with a lens or mirror so that the reflected/refracted rays diverge from P', then P' is a virtual image of P.

Spherical surfaces: Object and image distances are related by
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f},$$
where $f$ is the focal length.

Plane surfaces: $R = \infty$, so $d_i = -\frac{n}{n_i} d_o$.

Applications

Ray tracing
Three special rays in 3 basic situations:

1. Converging lens: Real image
2. Convex lens: Virtual image
3. Diverging lens: Virtual image

Magnification $m = \frac{d_v}{d_o}$
$m = -$ for an upright image, $-$ for inverted.
The height ratio is $\frac{h_v}{h_o} = |m|$. 
Applications

Thin lenses
The image and object distances are related by
\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]
where the focal length is given by the lens maker’s equation:
\[
f = \frac{1}{R} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]
- \( R \) = for surface convex toward object — for concave
- \( f \) = for a converging lens — for diverging
- \( s' \) = for a real image — for virtual

Applications

Spherical mirrors
The image and object distances are related by
\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]
- \( R_f \) = for convex mirror
- \( s' \) = for a real image
- \( f' \) = for virtual
- \( f' = R_f \)

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