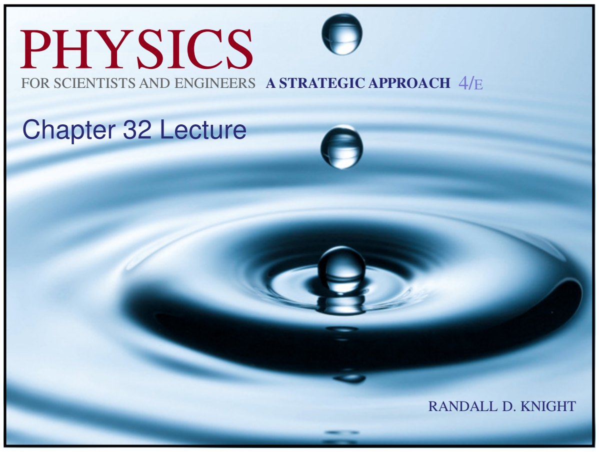


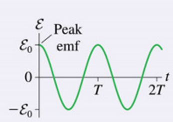
Chapter 32 AC Circuits

IN THIS CHAPTER, you will learn about and analyze AC circuits.

Chapter 32 Preview

What is an AC circuit?

The circuits of Chapter 28, with a steady current in one direction, are called DC circuits—*direct current*. A circuit with an oscillating emf is called an AC circuit, for *alternating current*. The wires that transport electricity across the country—the grid—use alternating current.



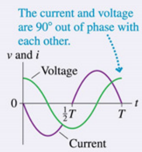
« LOOKING BACK Chapter 28 Circuits

Chapter 32 Preview

How do circuit elements act in an AC circuit?

Resistors in an AC circuit act as they do in a DC circuit. But you'll learn that capacitors and inductors are more useful in AC circuits than in DC circuits.

- The voltage across and the current through a capacitor or inductor are 90° out of phase. One is peaking when the other is zero, and vice versa.
- The peak voltage V and peak current I have an Ohm's-law-like relationship $V = IX$, where X, which depends on frequency, is called the reactance.
- Unlike resistors, capacitors and inductors do not dissipate energy.



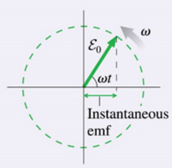
« LOOKING BACK Section 26.5 Capacitors

« LOOKING BACK Section 30.8 Inductors

Chapter 32 Preview

What is a phasor?

AC voltages oscillate sinusoidally, so the mathematics of AC circuits is that of SHM. You'll learn a new way to represent oscillating quantities—as a rotating vector called a phasor. The instantaneous value of a phasor quantity is its horizontal projection.

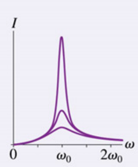


« LOOKING BACK Chapter 15 Simple harmonic motion and resonance

Chapter 32 Preview

What is an *RLC* circuit?

A circuit with a resistor, inductor, and capacitor in series is called an *RLC* circuit. An *RLC* circuit has a resonance—a large current over a narrow range of frequencies—that allows it to be tuned to a specific frequency. As a result, *RLC* circuits are very important in communications.



Chapter 32 Preview

Why are AC circuits important?
AC circuits are the backbone of our technological society. Generators automatically produce an oscillating emf, AC power is easily transported over large distances, and transformers allow engineers to shift the AC voltage up or down. The circuits of radio, television, and cell phones are also AC circuits because they work with oscillating voltages and currents—at much higher frequencies than the grid, but the physical principles are the same.

Chapter 32 Reading Questions

Reading Question 32.1

In Chapter 32, "AC" stands for

A. Air cooling.
B. Air conditioning.
C. All current.
D. Alternating current.
E. Analog current.

Reading Question 32.1

In Chapter 32, "AC" stands for

A. Air cooling.
B. Air conditioning.
C. All current.
✔**D. Alternating current.**
E. Analog current.

Slide 32-10

Reading Question 32.2

The analysis of AC circuits uses a rotating vector called a

A. Rotor.
B. Wiggler.
C. Phasor.
D. Motor.
E. Variator.

Slide 32-11

Reading Question 32.2

The analysis of AC circuits uses a rotating vector called a

A. Rotor.
B. Wiggler.
✔**C. Phasor.**
D. Motor.
E. Variator.

Slide 32-12

Reading Question 32.3

In a capacitor, the peak current and peak voltage are related by the

A. Capacitive resistance.
B. Capacitive reactance.
C. Capacitive impedance.
D. Capacitive inductance.

 Slide 32-13

Reading Question 32.3

In a capacitor, the peak current and peak voltage are related by the

A. Capacitive resistance.
✔**B. Capacitive reactance.**
C. Capacitive impedance.
D. Capacitive inductance.

 Slide 32-14

Reading Question 32.4

In a series *RLC* circuit, what quantity is maximum at resonance?

A. The voltage
B. The current
C. The impedance
D. The phase

 Slide 32-15

Reading Question 32.4

In a series *RLC* circuit, what quantity is maximum at resonance?

A. The voltage
✔**B. The current**
C. The impedance
D. The phase

Slide 32-16

Reading Question 32.5

In the United States a typical electrical outlet has a “line voltage” of 120 V. This is actually the

A. Average voltage.
B. Maximum voltage.
C. Maximum voltage minus the minimum voltage.
D. Minimum voltage.
E. rms voltage.

Slide 32-17

Reading Question 32.5

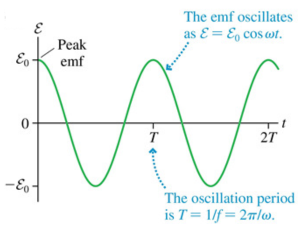
In the United States a typical electrical outlet has a “line voltage” of 120 V. This is actually the

A. Average voltage.
B. Maximum voltage.
C. Maximum voltage minus the minimum voltage.
D. Minimum voltage.
✔**E. rms voltage.**

Slide 32-18

Chapter 32 Content, Examples, and QuickCheck Questions

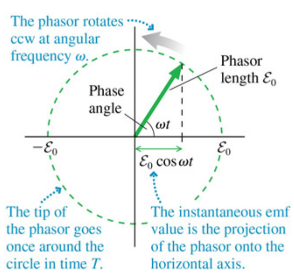
AC Sources and Phasors



- Circuits powered by a sinusoidal emf are called **AC circuits**, where AC stands for *alternating current*.
- Steady-current circuits studied in Chapter 28 are called DC circuits, for *direct current*.
- The instantaneous emf of an AC generator or oscillator can be written

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$

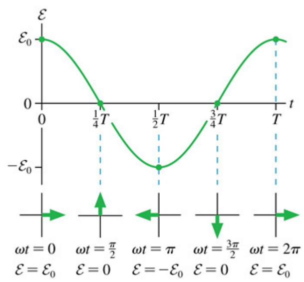
AC Sources and Phasors



- An alternative way to represent the emf and other oscillatory quantities is with a *phasor diagram*, as shown.
- A **phasor** is a vector that rotates *counterclockwise* (ccw) around the origin at angular frequency ω.
- The quantity's value at time t is the projection of the phasor onto the horizontal axis.

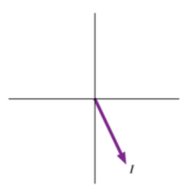
AC Sources and Phasors

- The figure below helps you visualize the phasor rotation by showing how the phasor corresponds to the more familiar graph at several specific points in the cycle.



QuickCheck 32.1

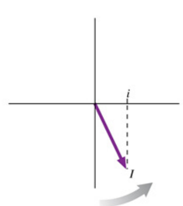
This is a current phasor. The magnitude of the instantaneous value of the current is



A. Increasing.
B. Decreasing.
C. Constant.
D. Can't tell without knowing which way it is rotating.

QuickCheck 32.1

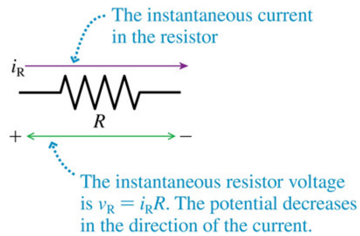
This is a current phasor. The magnitude of the instantaneous value of the current is



✔ **A. Increasing.**
B. Decreasing.
C. Constant.
D. Can't tell without knowing which way it is rotating.

Resistor Circuits

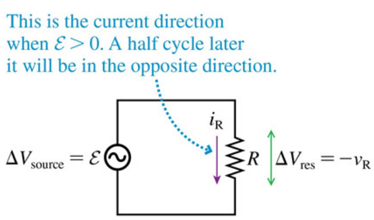
- In Chapter 28 we used the symbols I and V for DC current and voltage.
- Now, because the current and voltage are oscillating, we will use lowercase i to represent the instantaneous current and v for the instantaneous voltage.



Slide 32-25

Resistor Circuits

- The figure shows a resistor R connected across an AC generator of peak emf equal to V_R.



- The current through the resistor is

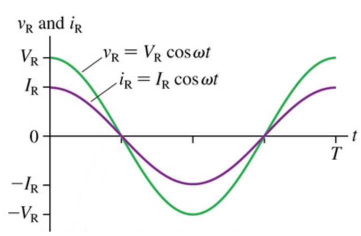
$$i_R = \frac{v_R}{R} = \frac{V_R \cos\omega t}{R} = I_R \cos\omega t$$

where $I_R = V_R/R$ is the peak current.

Slide 32-26

Resistor Circuits

- The resistor's instantaneous current and voltage are *in phase*, both oscillating as $\cos\omega t$.

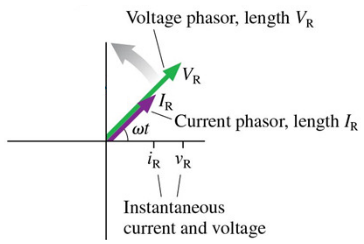


$$i_R = \frac{v_R}{R} = \frac{V_R \cos\omega t}{R} = I_R \cos\omega t$$

Slide 32-27

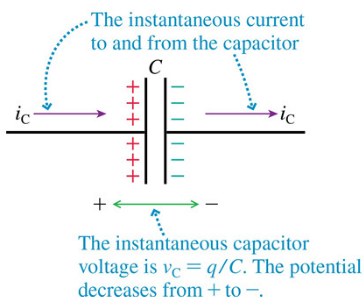
Resistor Circuits

- Below is the phasor diagram for the resistor circuit.
- V_R and I_R point in the same direction, indicating that resistor voltage and current oscillate in phase.



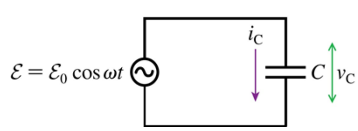
Capacitor Circuits

- The figure shows current i_C charging a capacitor with capacitance C.



Capacitor Circuits

- The figure shows a capacitor C connected across an AC generator of peak emf equal to V_C.



- The charge sitting on the positive plate of the capacitor at a particular instant is

$$q = Cv_C = CV_C \cos \omega t$$

Capacitor Circuits

- The current is the rate at which charge flows through the wires, $i_C = dq/dt$, thus

$$i_C = \frac{dq}{dt} = \frac{d}{dt}(CV_C \cos \omega t) = -\omega CV_C \sin \omega t$$

- We can most easily see the relationship between the capacitor voltage and current if we use the trigonometric identity

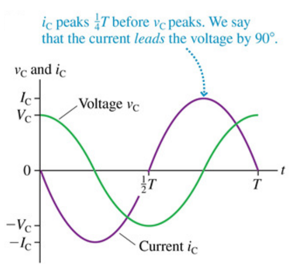
$$-\sin(x) = \cos(x + \pi/2)$$

to write

$$i_C = \omega CV_C \cos\left(\omega t + \frac{\pi}{2}\right)$$

Capacitor Circuits

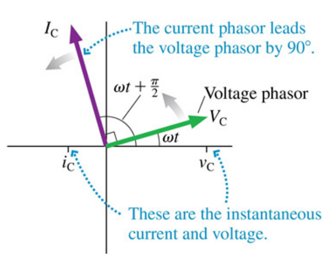
- A capacitor's current and voltage are *not* in phase.
- The current peaks one-quarter of a period *before* the voltage peaks.



$$i_C = \omega CV_C \cos\left(\omega t + \frac{\pi}{2}\right)$$

Capacitor Circuits

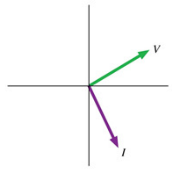
- Below is the phasor diagram for the capacitor circuit.
- The AC current of a capacitor *leads* the capacitor voltage by $\pi/2$ rad, or 90º.



QuickCheck 32.2

In the circuit represented by these phasors, the current ____ the voltage

A. leads
B. lags
C. is perpendicular to
D. is out of phase with

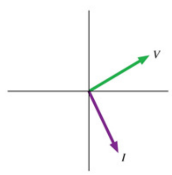


Slide 32-34

QuickCheck 32.2

In the circuit represented by these phasors, the current ____ the voltage

A. leads
B. lags
C. is perpendicular to
D. is out of phase with

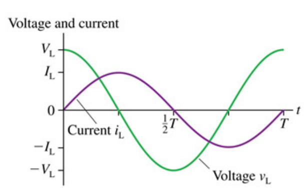


Slide 32-35

QuickCheck 32.3

In the circuit represented by these graphs, the current ____ the voltage

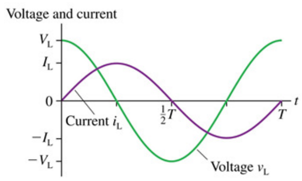
A. leads
B. lags
C. is less than
D. is out of phase with



Slide 32-36

QuickCheck 32.3

In the circuit represented by these graphs, the current _____ the voltage

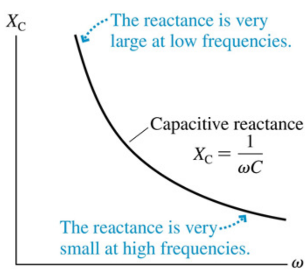


A. leads
✔ **B. lags**
C. is less than
D. is out of phase with

Capacitive Reactance

- The peak current to and from a capacitor is $I_C = \omega C V_C$.
- We can find a relationship that looks similar to Ohm's Law if we define the **capacitave reactance** to be

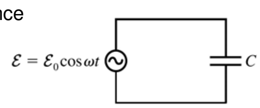
$$X_C \equiv \frac{1}{\omega C}$$



$$I_C = \frac{V_C}{X_C} \quad \text{or} \quad V_C = I_C X_C$$

QuickCheck 32.4

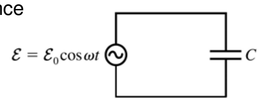
If the value of the capacitance is doubled, the capacitive reactance



A. Is quartered.
B. Is halved.
C. Is doubled.
D. Is quadrupled.
E. Can't tell without knowing ω.

QuickCheck 32.4

If the value of the capacitance is doubled, the capacitive reactance



A. Is quartered.

✔**B. Is halved.** $X_C = \dfrac{1}{\omega C}$

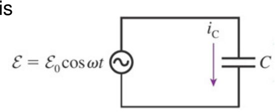
C. Is doubled.

D. Is quadrupled.

E. Can't tell without knowing ω.

QuickCheck 32.5

If the value of the capacitance is doubled, the peak current



A. Is quartered.

B. Is halved.

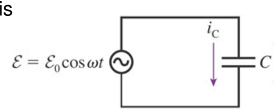
C. Is doubled.

D. Is quadrupled.

E. Can't tell without knowing C.

QuickCheck 32.5

If the value of the capacitance is doubled, the peak current



A. Is quartered.

B. Is halved.

✔**C. Is doubled.** $I_C = \dfrac{V_C}{X_C} = \dfrac{\mathcal{E}}{X_C}$

D. Is quadrupled.

E. Can't tell without knowing C.

Example 32.2 Capacitive Reactance

EXAMPLE 32.2 **Capacitive reactance**

What is the capacitive reactance of a 0.10 μF capacitor at 100 Hz (an audio frequency) and at 100 MHz (an FM-radio frequency)?

SOLVE At 100 Hz,

$$X_C(\text{at } 100\text{ Hz}) = \frac{1}{\omega C} = \frac{1}{2\pi(100\text{ Hz})(1.0 \times 10^{-7}\text{ F})} = 16{,}000\ \Omega$$

Increasing the frequency by a factor of 10^6 decreases X_C by a factor of 10^6, giving

$$X_C(\text{at } 100\text{ MHz}) = 0.016\ \Omega$$

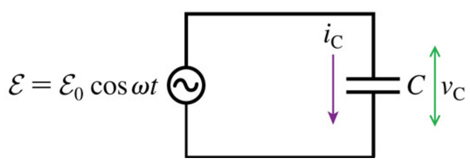
ASSESS A capacitor with a substantial reactance at audio frequencies has virtually no reactance at FM-radio frequencies.

Example 32.3 Capacitor Current

EXAMPLE 32.3 **Capacitor current**

A 10 μF capacitor is connected to a 1000 Hz oscillator with a peak emf of 5.0 V. What is the peak current to the capacitor?

VISUALIZE Figure 32.7b showed the circuit diagram. It is a simple one-capacitor circuit.



Example 32.3 Capacitor Current

EXAMPLE 32.3 **Capacitor current**

SOLVE The capacitive reactance at $\omega = 2\pi f = 6280$ rad/s is

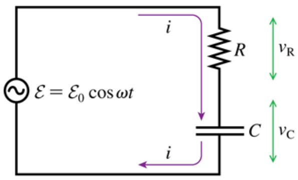
$$X_C = \frac{1}{\omega C} = \frac{1}{(6280\text{ rad/s})(10 \times 10^{-6}\text{ F})} = 16\ \Omega$$

The peak voltage across the capacitor is $V_C = \mathcal{E}_0 = 5.0$ V; hence the peak current is

$$I_C = \frac{V_C}{X_C} = \frac{5.0\text{ V}}{16\ \Omega} = 0.31\text{ A}$$

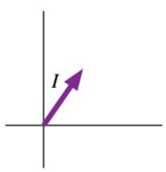
ASSESS Using reactance is just like using Ohm's law, but don't forget it applies to only the *peak* current and voltage, not the instantaneous values.

RC Filter Circuits



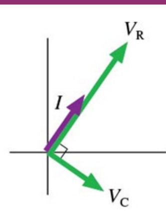
- The figure shows a circuit in which a resistor R and capacitor C are in series with an emf oscillating at angular frequency ω.
- If the frequency is very low, the capacitive reactance will be very large, and thus the peak current I_C will be very small.
- If the frequency is very high, the capacitive reactance approaches zero and the peak current, determined by the resistance alone, will be $I_R = \mathcal{E}_0/R$.

Using Phasors to Analyze an *RC* Circuit Step 1 of 4



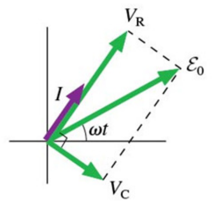
- Begin by drawing a current phasor of length I.
- This is the starting point because the series circuit elements have the same current i.
- The angle at which the phasor is drawn is not relevant.

Using Phasors to Analyze an *RC* Circuit Step 2 of 4



- The current and voltage of a resistor are in phase, so draw a resistor voltage phasor of length V_R parallel to the current phasor I.
- The capacitor current leads the capacitor voltage by $90°$, so draw a capacitor voltage phasor of length V_C that is $90°$ behind the current phasor.

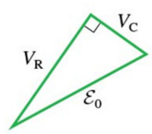
Using Phasors to Analyze an *RC* Circuit Step 3 of 4



- The series resistor and capacitor are in parallel with the emf, so their *instantaneous* voltages satisfy $v_R + v_C = \mathcal{E}$.
- This is a *vector* addition of phasors.
- The emf is $\mathcal{E} = \mathcal{E}_0 \cos \omega t$, hence the emf phasor is at angle ωt.

 Slide 32-49

Using Phasors to Analyze an *RC* Circuit Step 4 of 4



- The length of the emf phasor, $\mathcal{E}_0$, is the hypotenuse of a right triangle formed by the resistor and capacitor phasors.
- Thus $\mathcal{E}_0^2 = V_R^2 + V_C^2$

 Slide 32-50

RC Filter Circuits

- The relationship $\mathcal{E}_0^2 = V_R^2 + V_C^2$ is based on the peak values.
- The peak voltages are related to the peak current by $V_R = IR$ and $V_C = IX_C$, so

$$\mathcal{E}_0^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 = (R^2 + X_C^2)I^2$$
$$= (R^2 + 1/\omega^2 C^2)I^2$$

- This can be solved for the peak current, which in turn gives us the two peak voltages:

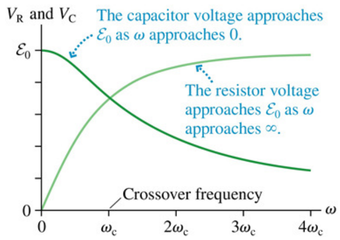
$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$
$$V_C = IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0/\omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

 Slide 32-51

RC Filter Circuits

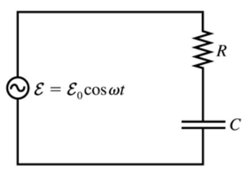
- The figure shows a graph of the resistor and capacitor peak voltages as functions of the emf angular frequency ω.
- The frequency at which $V_R = V_C$ is called the **crossover frequency**:

$$\omega_c = \frac{1}{RC}$$



QuickCheck 32.6

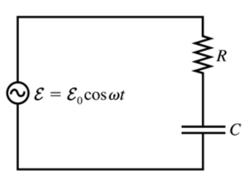
Does $V_R + V_C = \mathcal{E}_0$?



A. Yes
B. No.
C. Can't tell without knowing ω.

QuickCheck 32.6

Does $V_R + V_C = \mathcal{E}_0$?

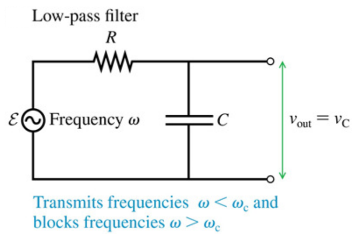


A. Yes
✔**B. No**
C. Can't tell without knowing ω.

Instantaneous voltages add.
Peak voltages don't because the voltages are not in phase.

RC Filter Circuits

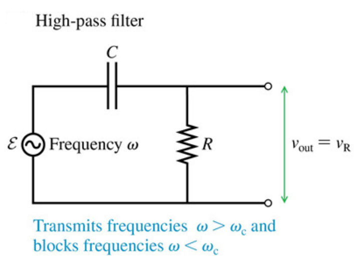
- The figure below shows an *RC* circuit in which v_C is the *output voltage*.
- This circuit is called a **low-pass filter**.



Slide 32-55

RC Filter Circuits

- The figure below shows an *RC* circuit in which v_R is the *output voltage*.
- This circuit is called a **high-pass filter**.



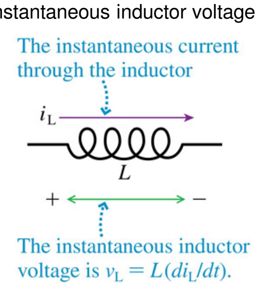
Slide 32-56

Inductor Circuits

- The figure shows the instantaneous current i_L through an inductor.
- If the current is *changing*, the instantaneous inductor voltage is

$$v_L = L\frac{di_L}{dt}$$

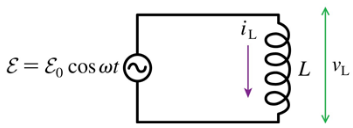
- The potential decreases in the direction of the current if the current is increasing, and increases if the current is decreasing.



Slide 32-57

Inductor Circuits

- The figure shows an inductor L connected across an AC generator of peak emf equal to V_L.



- The instantaneous inductor voltage is equal to the emf:

$$v_L = V_L \cos \omega t$$

Inductor Circuits

- Combining the two previous equations for v_L gives

$$di_L = \frac{v_L}{L}dt = \frac{V_L}{L}\cos \omega t\, dt$$

- Integrating gives

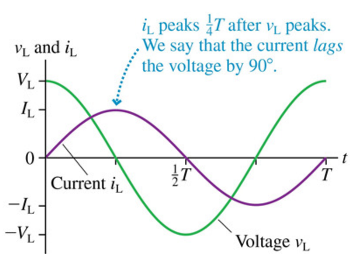
$$i_L = \frac{V_L}{L}\int \cos \omega t\, dt = \frac{V_L}{\omega L}\sin \omega t = \frac{V_L}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_L \cos\left(\omega t - \frac{\pi}{2}\right)$$

where $I_L = V_L/\omega L$ is the peak or maximum inductor current.

Inductor Circuits

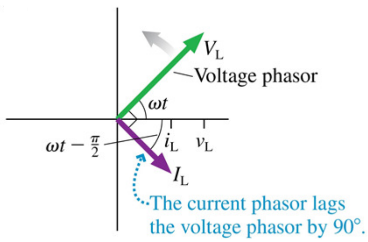
- An inductor's current and voltage are *not* in phase.
- The current peaks one-quarter of a period after the voltage peaks.



$$i_L = I_L \cos\left(\omega t - \frac{\pi}{2}\right)$$

Inductor Circuits

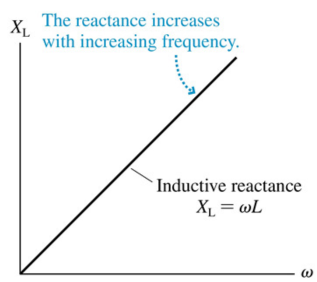
- Below is the phasor diagram for the inductor circuit.
- The AC current through an inductor *lags* the inductor voltage by $\pi/2$ rad, or 90º.



Inductive Reactance

- We define the **inductive reactance**, analogous to the capacitive reactance, to be

$$X_L \equiv \omega L$$



$$I_L = \frac{V_L}{X_L} \quad \text{or} \quad V_L = I_L X_L$$

Example 32.5 Current and Voltage of an Inductor

EXAMPLE 32.5 | **Current and voltage of an inductor**

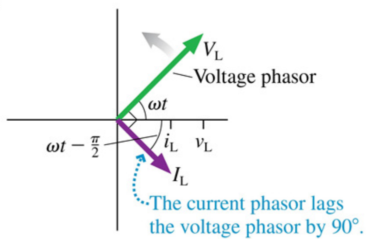
A 25 μH inductor is used in a circuit that oscillates at 100 kHz. The current through the inductor reaches a peak value of 20 mA at $t = 5.0\ \mu$s. What is the peak inductor voltage, and when, closest to $t = 5.0\ \mu$s, does it occur?

MODEL The inductor current lags the voltage by 90°, or, equivalently, the voltage reaches its peak value one-quarter period *before* the current.

Example 32.5 Current and Voltage of an Inductor

EXAMPLE 32.5 Current and voltage of an inductor

VISUALIZE The circuit looks like Figure 32.15b.



Example 32.5 Current and Voltage of an Inductor

EXAMPLE 32.5 Current and voltage of an inductor

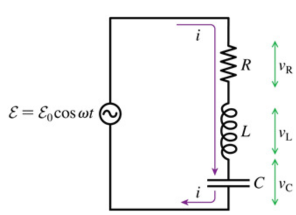
SOLVE The inductive reactance at $f = 100$ kHz is

$$X_L = \omega L = 2\pi(1.0 \times 10^5 \text{ Hz})(25 \times 10^{-6} \text{ H}) = 16\ \Omega$$

Thus the peak voltage is $V_L = I_L X_L = (20 \text{ mA})(16\ \Omega) = 320$ mV. The voltage peak occurs one-quarter period before the current peaks, and we know that the current peaks at $t = 5.0\ \mu$s. The period of a 100 kHz oscillation is 10.0 μs, so the voltage peaks at

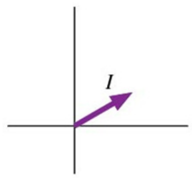
$$t = 5.0\ \mu\text{s} - \frac{10.0\ \mu\text{s}}{4} = 2.5\ \mu\text{s}$$

The Series *RLC* Circuit



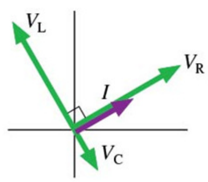
- The circuit shown, where a resistor, inductor, and capacitor are in series, is called a series *RLC* circuit.
- The instantaneous current of all three elements is the same: $i = i_R = i_L = i_C$
- The sum of the instantaneous voltages matches the emf: $\mathcal{E} = v_R + v_L + v_C$

Using Phasors to Analyze an *RLC* Circuit Step 1 of 4



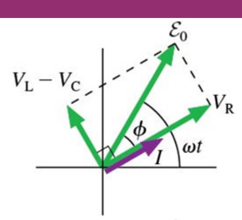
- Begin by drawing a current phasor of length I.
- This is the starting point because the series circuit elements have the same current I.
- The angle at which the phasor is drawn is not relevant.

Using Phasors to Analyze an *RLC* Circuit Step 2 of 4



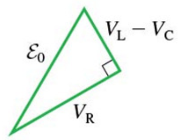
- The current and voltage of a resistor are in phase, so draw a resistor voltage phasor parallel to the current phasor I.
- The capacitor current *leads* the capacitor voltage, so draw a capacitor voltage phasor that is $90°$ behind the current phasor.
- The inductor current *lags* the voltage, so draw an inductor voltage phasor that is $90°$ ahead of the current phasor.

Using Phasors to Analyze an *RLC* Circuit Step 3 of 4



- The *instantaneous* voltages satisfy $\mathcal{E} = v_R + v_L + v_C$.
- This is a *vector* addition of phasors.
- Because the capacitor and inductor phasors are in opposite directions, their vector sum has length $V_L - V_C$.
- Adding the resistor phasor, at right angles, then gives the emf phasor $\mathcal{E}$ at angle ωt.

Using Phasors to Analyze an *RLC* Circuit
Step 4 of 4



- The length of the emf phasor, $\mathcal{E}_0$, is the hypotenuse of a right triangle.
- Thus $\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2$

Slide 32-70

The Series *RLC* Circuit

- If $V_L < V_C$, which we've assumed, then the instantaneous current i lags the emf by a phase angle ϕ:

$$i = I\cos(\omega t - \phi)$$

- Based on the right-triangle, the square of the peak voltage is

$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = [R^2 + (X_L - X_C)^2]I^2$$

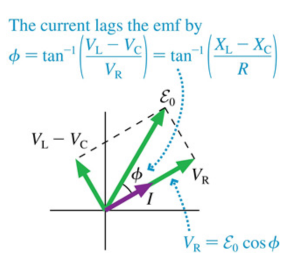
where we wrote each of the peak voltages in terms of the peak current I and a resistance or a reactance.

- Consequently, the peak current in the *RLC* circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Slide 32-71

Phase Angle in a Series *RLC* Circuit

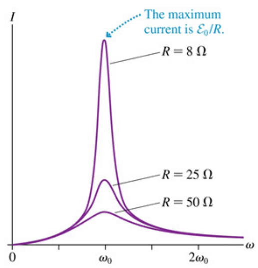


- It is often useful to know the phase angle ϕ between the emf and the current in an *RLC* circuit:

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Slide 32-72

Resonance in a Series *RLC* Circuit



- Suppose we vary the emf frequency ω while keeping everything else constant.
- There is very little current at very low or very high frequencies.
- I is maximum when $X_L = X_C$, which occurs at the **resonance frequency**:

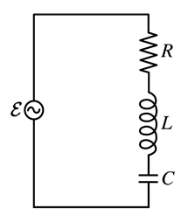
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Slide 32-73

QuickCheck 32.7

If the value of R is increased, the resonance frequency of this circuit

A. Increases.
B. Decreases.
C. Stays the same.



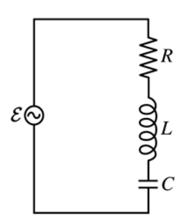
Slide 32-74

QuickCheck 32.7

If the value of R is increased, the resonance frequency of this circuit

A. Increases.
B. Decreases.
✔**C. Stays the same.**

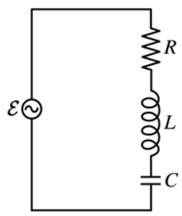
The resonance frequency depends on C and L but not on R.



Slide 32-75

QuickCheck 32.8

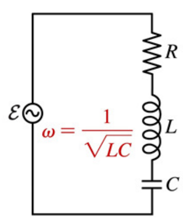
The resonance frequency of this circuit is 1000 Hz. To change the resonance frequency to 2000 Hz, replace the capacitor with one having capacitance



A. C/4
B. C/2
C. 2C
D. 4C
E. It's impossible to change the resonance frequency by changing only the capacitor.

QuickCheck 32.8

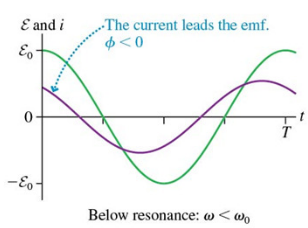
The resonance frequency of this circuit is 1000 Hz. To change the resonance frequency to 2000 Hz, replace the capacitor with one having capacitance



✔ **A. C/4**
B. C/2
C. 2C
D. 4C
E. It's impossible to change the resonance frequency by changing only the capacitor.

Resonance in a Series *RLC* Circuit

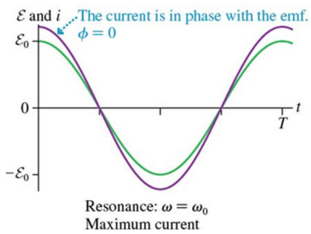
- Below is a graph of the instantaneous emf and current in a series *RLC* circuit driven *below* the resonance frequency: $\omega < \omega_0$
- In this case, $X_L < X_C$, and ϕ is negative.



Below resonance: $\omega < \omega_0$

Resonance in a Series *RLC* Circuit

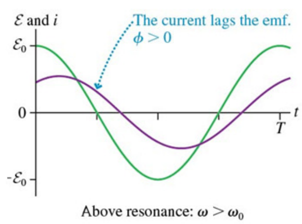
- Below is a graph of the instantaneous emf and current in a series *RLC* circuit driven *at* the resonance frequency: $\omega = \omega_0$
- In this case, $X_L = X_C$, and $\phi = 0$



Slide 32-79

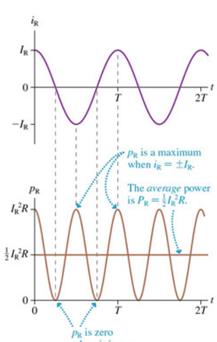
Resonance in a Series *RLC* Circuit

- Below is a graph of the instantaneous emf and current in a series *RLC* circuit driven *above* the resonance frequency: $\omega > \omega_0$
- In this case, $X_L > X_C$, and ϕ is positive.



Slide 32-80

Power in AC Circuits



- The graphs show the instantaneous power loss in a resistor R carrying a current i_R:

$$p_R = i_R^2R = I_R^2R\cos^2\omega t$$

- The average power P_R is the total energy dissipated per second:

$$P_R = \tfrac{1}{2}I_R^2R \qquad \text{(average power loss in a resistor)}$$

Slide 32-81

Power in AC Circuits

- We can define the **root-mean-square** current and voltage as

$$I_{rms} = \frac{I_R}{\sqrt{2}} \qquad V_{rms} = \frac{V_R}{\sqrt{2}}$$

- The resistor's average power loss in terms of the rms quantities is

$$P_R = (I_{rms})^2 R = \frac{(V_{rms})^2}{R} = I_{rms} V_{rms}$$

- The average power supplied by the emf is

$$P_{source} = I_{rms} \mathcal{E}_{rms}$$

The power rating on a lightbulb is its average power at $V_{rms} = 120$ V.

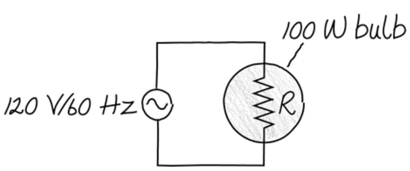
Example 32.7 Lighting a Bulb

EXAMPLE 32.7 Lighting a bulb

A 100 W incandescent lightbulb is plugged into a 120 V/60 Hz outlet. What is the resistance of the bulb's filament? What is the peak current through the bulb?

MODEL The filament in a lightbulb acts as a resistor.

VISUALIZE FIGURE 32.21 is a simple one-resistor circuit.



Example 32.7 Lighting a Bulb

EXAMPLE 32.7 Lighting a bulb

SOLVE A bulb labeled 100 W is designed to dissipate an average 100 W at $V_{rms} = 120$ V. We can use Equation 32.39 to find

$$R = \frac{(V_{rms})^2}{P_R} = \frac{(120\text{ V})^2}{100\text{ W}} = 144\ \Omega$$

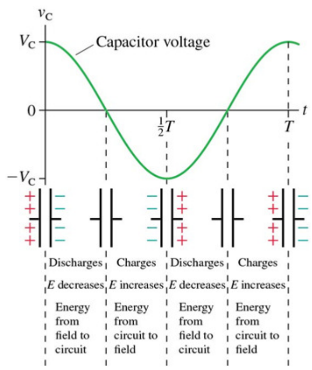
The rms current is then found from

$$I_{rms} = \frac{P_R}{V_{rms}} = \frac{100\text{ W}}{120\text{ V}} = 0.833\text{ A}$$

The peak current is $I_R = \sqrt{2} I_{rms} = 1.18$ A.

ASSESS Calculations with rms values are just like the calculations for DC circuits.

Capacitors in AC Circuits



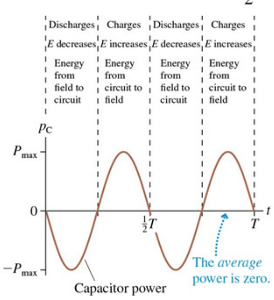
- Energy flows into and out of a capacitor as it is charged and discharged.
- The energy is not dissipated, as it would be by a resistor.
- The energy is stored as potential energy in the capacitor's electric field.

 Slide 32-85

Capacitors in AC Circuits

- The instantaneous power flowing into a capacitor is

$$p_C = v_C i_C = (V_C \cos \omega t)(-\omega C V_C \sin \omega t) = -\frac{1}{2}\omega C V_C^2 \sin 2\omega t$$



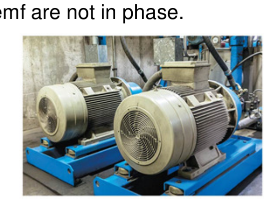
 Slide 32-86

The Power Factor in *RLC* Circuits

- In an *RLC* circuit, energy is supplied by the emf and dissipated by the resistor.
- The average power supplied by the emf is:

$$P_{source} = \frac{1}{2} I \mathcal{E}_0 \cos\phi = I_{rms}\mathcal{E}_{rms}\cos\phi$$

- The term cos ϕ, called the **power factor,** arises because the current and the emf are not in phase.
- Large industrial motors, such as the one shown, operate most efficiently, doing the maximum work per second, when the power factor is as close to 1 as possible.



 Slide 32-87

Chapter 32 Summary Slides

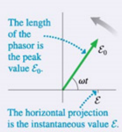
Important Concepts

AC circuits are driven by an emf

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$

that oscillates with angular frequency $\omega = 2\pi f$.

Phasors can be used to represent the oscillating emf, current, and voltage.



Important Concepts

Basic circuit elements

Element	i and v	Resistance/ reactance	I and V	Power
Resistor	In phase	R is fixed	$V = IR$	$I_{rms}V_{rms}$
Capacitor	i leads v by 90°	$X_C = 1/\omega C$	$V = IX_C$	0
Inductor	i lags v by 90°	$X_L = \omega L$	$V = IX_L$	0

For many purposes, especially calculating power, the **root-mean-square** (rms) quantities

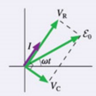
$$V_{rms} = V/\sqrt{2} \qquad I_{rms} = I/\sqrt{2} \qquad \mathcal{E}_{rms} = \mathcal{E}_0/\sqrt{2}$$

are equivalent to the corresponding DC quantities.

Key Skills

Using phasor diagrams

- Start with a phasor (v or i) common to two or more circuit elements.
- The sum of instantaneous quantities is vector addition.
- Use the Pythagorean theorem to relate peak quantities.



For an RC circuit, shown here,

$$v_R + v_C = \mathcal{E}$$
$$V_R^2 + V_C^2 = \mathcal{E}_0^2$$

Key Skills

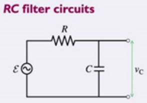
Instantaneous and peak quantities The instantaneous quantities v and i vary sinusoidally. The peak quantities V and I are the maximum values of v and i. For capacitors and inductors, the peak quantities are related by $V = IX$, where X is the reactance, but this relationship does *not* apply to v and i.

Kirchhoff's loop law says that the sum of the potential differences around a loop is zero.

Charge conservation says that circuit elements in series all have the same instantaneous current i and the same peak current I.

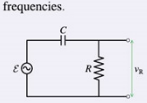
Applications

RC filter circuits



$$v_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}}$$
$$v_C \to \mathcal{E}_0 \text{ as } \omega \to 0$$

A **low-pass filter** transmits low frequencies and blocks high frequencies.

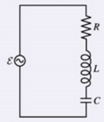


$$v_R = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}}$$
$$v_R \to \mathcal{E}_0 \text{ as } \omega \to \infty$$

A **high-pass filter** transmits high frequencies and blocks low frequencies.

Applications

Series *RLC* circuits



$I = \mathcal{E}_0/Z$ where Z is the **impedance**

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$V_R = IR \qquad V_L = IX_L \qquad V_C = IX_C$

When $\omega = \omega_0 = 1/\sqrt{LC}$ (the **resonance frequency**), the current in the circuit is a maximum $I_{max} = \mathcal{E}_0/R$.

In general, the current i lags behind $\mathcal{E}$ by the **phase angle** $\phi = \tan^{-1}\left((X_L - X_C)/R\right)$.

The power supplied by the emf is $P_{source} = I_{rms}\mathcal{E}_{rms}\cos\phi$, where $\cos\phi$ is called the **power factor.**

The power lost in the resistor is $P_R = I_{rms}V_{rms} = (I_{rms})^2R$.