IN THIS CHAPTER, you will learn what electromagnetic induction is and how it is used.

**What is an induced current?**
A magnetic field can create a current in a loop of wire, but only if the amount of field through the loop is changing.

- This is called an **induced current**.
- The process is called **electromagnetic induction**.

**LOOKING BACK:** Chapter 29 Magnetic fields
Chapter 30 Preview

What is magnetic flux?
A key idea will be the amount of magnetic field passing through a loop or coil. This is called magnetic flux. Magnetic flux depends on the strength of the magnetic field, the area of the loop, and the angle between them.

LOOKING BACK: Section 24.3 Electric flux

Chapter 30 Preview

What is Lenz’s law?
Lenz’s law says that a current is induced in a closed loop if and only if the magnetic flux through the loop is changing. Simply having a flux does nothing; the flux has to change. You’ll learn how to use Lenz’s law to determine the direction of an induced current around a loop.

Chapter 30 Preview

What is Faraday’s law?
Faraday’s law is the most important law connecting electric and magnetic fields, laying the groundwork for electromagnetic waves. Just as a battery has an emf that drives current, a loop of wire has an induced emf determined by the rate of change of magnetic flux through the loop.

LOOKING BACK: Section 26.4 Sources of potential
Chapter 30 Preview

What is an induced field?
At its most fundamental level, Faraday’s law tells us that a changing magnetic field creates an induced electric field. This is an entirely new way to create an electric field, independent of charges. It is the induced electric field that drives the induced current around a conducting loop.

Chapter 30 Preview

How is electromagnetic induction used?
Electromagnetic induction is one of the most important applications of electricity and magnetism. Generators use electromagnetic induction to turn the mechanical energy of a spinning turbine into electric energy. Inductors are important circuit elements that rely on electromagnetic induction. All forms of telecommunication are based on electromagnetic induction. And, not least, electromagnetic induction is the basis for light and other electromagnetic waves.

Chapter 30 Reading Questions
Currents circulate in a piece of metal that is pulled through a magnetic field. What are these currents called?

A. Induced currents
B. Displacement currents
C. Faraday’s currents
D. Eddy currents
E. This topic is not covered in Chapter 30.

Electromagnetic induction was discovered by

A. Faraday.
B. Henry.
C. Maxwell.
D. Both Faraday and Henry.
E. All three.
Electromagnetic induction was discovered by

A. Faraday.
B. Henry.
C. Maxwell.
D. Both Faraday and Henry.
E. All three.

The direction that an induced current flows in a circuit is given by

A. Faraday’s law.
B. Lenz’s law.
C. Henry’s law.
D. Hertz’s law.
E. Maxwell’s law.
After thinking about electromagnetic induction, James Clerk Maxwell was lead to propose that

A. An electric current can be induced by a changing magnetic flux.
B. A magnetic field can be produced by an electric current.
C. Light is an electromagnetic wave.
D. Moving charges accelerate in a magnetic field.
E. Nothing can travel faster than the speed of light.

Reading Question 30.5

A transformer

A. Boosts the maximum current provided by a battery.
B. Changes mechanical energy to electrical energy.
C. Changes the voltage of an alternating current.
D. Resists changes in current.
E. Converts alternating current to direct current.
A transformer

A. Boosts the maximum current provided by a battery.
B. Changes mechanical energy to electrical energy.
**C. Changes the voltage of an alternating current.**
D. Resists changes in current.
E. Converts alternating current to direct current.

---

Chapter 30 Content, Examples, and QuickCheck Questions

---

Faraday’s Discovery of 1831

- When one coil is placed directly above another, there is no current in the lower circuit while the switch is in the closed position.
- A momentary current appears whenever the switch is opened or closed.

![Diagram of coils and switch](image)
Faraday’s Discovery of 1831

- When a bar magnet is pushed into a coil of wire, it causes a momentary deflection of the current-meter needle.
- Holding the magnet inside the coil has no effect.
- A quick withdrawal of the magnet deflects the needle in the other direction.

Push or pull magnet.

Faraday’s Discovery of 1831

- A momentary current is produced by rapidly pulling a coil of wire out of a magnetic field.
- Pushing the coil into the magnet causes the needle to deflect in the opposite direction.

Push or pull coil.

Motional emf

Charge carriers in the conductor experience a force of magnitude \( F_B = qvB \). Positive charges are free to move and drift upward.
The magnetic force on the charge carriers in a moving conductor creates an electric field of strength \( E = vB \) inside the conductor.

For a conductor of length \( L \), the motional emf perpendicular to the magnetic field is:

\[ \mathcal{E} = vLB \]
QuickCheck 30.1

A metal bar moves through a magnetic field. The induced charges on the bar are

A.  
B.  
C.  
D.  
E.  

Correct Answer: E

QuickCheck 30.2

A metal bar moves through a magnetic field. The induced charges on the bar are

A.  
B.  
C.  
D.  
E.  

Correct Answer: E
QuickCheck 30.2

A metal bar moves through a magnetic field. The induced charges on the bar are

A. 
B. 
C. 
D. 
E. 

Example 30.1 Measuring the Earth’s Magnetic Field

EXAMPLE 30.1 Measuring the earth’s magnetic field

It is known that the earth’s magnetic field over northern Canada points straight down. The crew of a Boeing 747 aircraft flying at 260 m/s over northern Canada finds a 893 V potential difference between the wing tips. The wing span of a Boeing 747 is 65 m. What is the magnetic field strength there?

MODEL: The wing is a conductor moving through a magnetic field, so there is a motional emf.

Example 30.1 Measuring the Earth’s Magnetic Field

EXAMPLE 30.1 Measuring the earth’s magnetic field

SOLVE: The magnetic field is perpendicular to the velocity, so we can use Equation 30.5 to find

\[ B = \frac{E}{lv} \]

\[ B = \frac{893 \text{ V}}{260 \text{ m/s} \times 65 \text{ m}} = 0.05 \times 10^{-7} \text{T} \]

ASSESS: Chapter 29 noted that the earth’s magnetic field is roughly 5 \times 10^{-8} T. The field is somewhat stronger than this near the magnetic poles, somewhat weaker near the equator.
Induced Current

1. The charge carriers in the wire are pushed upward by the magnetic force.
2. The charge carriers flow around the conducting loop as an induced current.

- If we slide a conducting wire along a U-shaped conducting rail, we can complete a circuit and drive an electric current.
- If the total resistance of the circuit is \( R \), the induced current is given by Ohm's law as:
  \[ I = \frac{\mathcal{E}}{R} = \frac{vIB}{R} \]

To keep the wire moving at a constant speed \( v \), we must apply a pulling force \( F_{\text{pull}} = \frac{vLB^2}{R} \).

This pulling force does work at a rate:
\[ P_{\text{work}} = F_{\text{pull}}v = \frac{v^2B^2}{R} \]

All of this power is dissipated by the resistance of the circuit.

A device that converts mechanical energy to electric energy is called a generator.

The figure shows a conducting wire sliding to the left.

In this case, a pushing force is needed to keep the wire moving at constant speed.

Once again, this input power is dissipated in the electric circuit.

A device that converts mechanical energy to electric energy is called a generator.
QuickCheck 30.3

An induced current flows clockwise as the metal bar is pushed to the right. The magnetic field points

A. Up.
B. Down.
C. Into the screen.
D. Out of the screen.
E. To the right.

QuickCheck 30.3

An induced current flows clockwise as the metal bar is pushed to the right. The magnetic field points

A. Up.
B. Down.
C. Into the screen. [Corrected]
D. Out of the screen.
E. To the right.

Eddy Currents

- Consider pulling a sheet of metal through a magnetic field.
- Two “whirlpools” of current begin to circulate in the solid metal, called eddy currents.
- The magnetic force on the eddy currents is a retarding force.
- This is a form of magnetic braking.
The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area \( A = ab \) in front of a fan.
- The volume of air flowing through the loop each second depends on the angle between the loop and the direction of flow.
- No air goes through the same loop if it lies parallel to the flow.
- The flow is maximum through a loop that is perpendicular to the airflow.
- This occurs because the effective area is greatest at this angle.
- The effective area (as seen facing the fan) is \( A_{\text{eff}} = ab \cos \theta = A \cos \theta \)

Magnetic Flux Through a Loop

- Loop seen from the side:
  - \( \theta = 0^\circ \)
  - \( \theta = 90^\circ \)
- Seen in the direction of the magnetic field:
  - Loop perpendicular to field: \( \theta = 0^\circ \)
  - Loop rotated through angle \( \theta \): Fewer arrows pass through.
  - Loop rotated \( 90^\circ \): No arrows pass through.
The Area Vector

- Let's define an area vector \( \vec{A} = A \hat{n} \) to be a vector in the direction of, perpendicular to the surface, with a magnitude \( A \) equal to the area of the surface.
- Vector \( \vec{A} \) has units of \( \text{m}^2 \).

Magnetic Flux

- The magnetic flux measures the amount of magnetic field passing through a loop of area \( A \) if the loop is tilted at an angle \( \theta \) from the field.
- The SI unit of magnetic flux is the weber: 
  \[ 1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T m}^2 \]
**Example 30.4 A Circular Loop in a Magnetic Field**

**EXAMPLE 30.4**  
A circular loop in a magnetic field  
*Solution*  
Angle $\theta$ is the angle between the loop’s area vector $\mathbf{A}$, which is perpendicular to the plane of the loop, and the magnetic field $\mathbf{B}$. In this case, $\theta = 60^\circ$, not the $30^\circ$ angle shown in the figure. Vector $\mathbf{A}$ has magnitude $A = r^2 = 7.85 \times 10^{-4}$ m$^2$. Thus the magnetic flux is  
\[ \Phi_m = \mathbf{A} \cdot \mathbf{B} = (AL\sin\theta) = 2.6 \times 10^{-4} \text{ Wb} \]

**QuickCheck 30.4**

Which loop has the larger magnetic flux through it?

A. Loop A  
B. Loop B  
C. The fluxes are the same.  
D. Not enough information to tell.

---

**QuickCheck 30.4**

Which loop has the larger magnetic flux through it?

A. Loop A  
B. **Loop B** $\Phi_m = L^2B$  
C. The fluxes are the same.  
D. Not enough information to tell.
The metal loop is being pulled through a uniform magnetic field. Is the magnetic flux through the loop changing?

A. Yes
B. No
QuickCheck 30.6

The metal loop is rotating in a uniform magnetic field. Is the magnetic flux through the loop changing?

A. Yes
B. No

Magnetic Flux in a Nonuniform Field

- The figure shows a loop in a nonuniform magnetic field.
- The total magnetic flux through the loop is found with an area integral:

\[ \Phi_{\text{tot}} = \int \vec{B} \cdot d\vec{A} \]

area of loop

Lenz’s Law

Lenz’s law: There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the change in the flux.
Lenz’s Law

- Pushing the bar magnet into the loop causes the magnetic flux to increase in the downward direction.
- To oppose the change in flux, which is what Lenz’s law requires, the loop itself needs to generate an upward-pointing magnetic field.
- The induced current ceases as soon as the magnet stops moving.

Lenz’s Law

- Pushing the bar magnet away from the loop causes the magnetic flux to decrease in the downward direction.
- To oppose this decrease, a clockwise current is induced.

QuickCheck 30.7

The bar magnet is pushed toward the center of a wire loop. Which is true?

A. There is a clockwise induced current in the loop.
B. There is a counterclockwise induced current in the loop.
C. There is no induced current in the loop.
QuickCheck 30.7

The bar magnet is pushed toward the center of a wire loop. Which is true?

A. There is a clockwise induced current in the loop.
B. There is a counterclockwise induced current in the loop.
C. There is no induced current in the loop.

1. Upward flux from magnet is increasing.
2. To oppose the increase, the field of the induced current points down.
3. From the right-hand rule, a downward field needs a cw current.

QuickCheck 30.8

The bar magnet is pushed toward the center of a wire loop. Which is true?

A. There is a clockwise induced current in the loop.
B. There is a counterclockwise induced current in the loop.
C. There is no induced current in the loop.

Magnetic flux is zero, so there's no change of flux.
The Induced Current for Six Different Situations:

**Slide 1**

- \( \vec{B} \)
  - No induced current

  - \( \vec{B} \) up and steady
    - No change in flux
    - No induced field
    - No induced current

**Slide 2**

- \( \vec{B} \) up and increasing
  - Change in flux
  - Induced field
  - Induced current (cw)

**Slide 3**

- \( \vec{B} \) up and decreasing
  - Change in flux
  - Induced field
  - Induced current (cw)
**QuickCheck 30.9**

The current in the straight wire is decreasing. Which is true?

A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

C. There is no induced current in the loop.

1. The flux from wire’s field is into the screen and decreasing.
2. To oppose the decrease, the field of the induced current must point into the screen.
3. From the right-hand rule, an inward field needs a clockwise current.
QuickCheck 30.10

The magnetic field is confined to the region inside the dashed lines; it is zero outside. The metal loop is being pulled out of the magnetic field. Which is true?

A. There is a clockwise induced current in the loop.
B. There is a counterclockwise induced current in the loop.
C. There is no induced current in the loop.

QuickCheck 30.10

The magnetic field is confined to the region inside the dashed lines; it is zero outside. The metal loop is being pulled out of the magnetic field. Which is true?

A. There is a clockwise induced current in the loop.

1. The flux through the loop is into the screen and decreasing.
2. To oppose the decrease, the field of the induced current must point into the screen.
3. From the right-hand rule, an inward field needs a cw current.

QuickCheck 30.11

Immediately after the switch is closed, the lower loop exerts ____ on the upper loop.

A. a torque
B. an upward force
C. a downward force
D. no force or torque
QuickCheck 30.11

Immediately after the switch is closed, the lower loop exerts ____ on the upper loop.

A. a torque
B. an upward force
C. a downward force
D. no force or torque

1. The battery drives a ccw current that briefly increases rapidly.
2. The flux through the top loop is upward and increasing.
3. To oppose the increase, the field of the induced current must point downward.
4. From the right-hand rule, a downward field needs a cw current.
5. The ccw current in the lower loop makes the upper face a north pole. The cw induced current in the upper loop makes the lower face a north pole.
6. Facing north poles exert repulsive forces on each other.

Faraday’s Law

- An emf is induced in a conducting loop if the magnetic flux through the loop changes.
- The magnitude of the emf is
  \[ \varepsilon = \frac{d\Phi_m}{dt} \]
- The direction of the emf is such as to drive an induced current in the direction given by Lenz’s law.

Using Faraday’s Law

- If we slide a conducting wire along a U-shaped conducting rail, we can complete a circuit and drive an electric current.
- We can find the induced emf and current by using Faraday’s law and Ohm’s law:
  \[ \varepsilon = \frac{d\Phi_m}{dt} = \frac{d}{dt}(xIB) = \frac{dx}{dt}IB = vIB \]
  \[ I = \frac{\varepsilon}{R} = \frac{vIB}{R} \]
Problem-Solving Strategy: Electromagnetic Induction

**PROBLEM-SOLVING STRATEGY 30.1**

**Electromagnetic induction**

**MODEL** Make simplifying assumptions about wires and magnetic fields.

**VISUALIZE** Draw a picture or a circuit diagram. Use Lenz’s law to determine the direction of the induced current.

**SOLVE** The mathematical representation is based on Faraday’s law

\[ E = \frac{\partial \Phi_B}{\partial t} \]

For an \( N \)-turn coil, multiply by \( N \). The size of the induced current is \( I = E/R \).

**ASSESS** Check that your result has correct units and significant figures, is reasonable, and answers the question.

---

QuickCheck 30.12

The induced emf around this loop is

A. 200 V  
B. 50 V  
C. 2 V  
D. 0.5 V  
E. 0.02 V

---

QuickCheck 30.12

The induced emf around this loop is

A. 200 V  
B. 50 V  
C. 2 V  
D. 0.5 V  
E. **0.02 V**

\[ E = \frac{\partial \Phi_B}{\partial t} = \frac{A \cdot dB}{dt} = A \times \text{slope of graph} \]

Slope = 50 T/s
Induced Fields

- The figure shows a conducting loop in an increasing magnetic field.
- According to Lenz’s law, there is an induced current in the counterclockwise direction.
- Something has to act on the charge carriers to make them move, so we infer that there must be an induced electric field tangent to the loop at all points.

The Induced Electric Field

- When the magnetic field is increasing in a region of space, we may define a closed loop which is perpendicular to the magnetic field.
- Faraday’s Law specifies the loop integral of the induced electric field around this loop:

\[ \oint E \cdot d\vec{r} = A \frac{dB}{dt} \]

Induced Electric Field in a Solenoid Slide 1 of 3

- The current through the solenoid creates an upward pointing magnetic field.
- As the current is increasing, \( B \) is increasing, so it must induce an electric field.
We could use Lenz’s law to determine that if there were a conducting loop in the solenoid, the induced current would be clockwise.

The induced electric field must therefore be clockwise around the magnetic field lines.

To use Faraday’s law, integrate around a clockwise circle of radius $r$:

$$\oint E \cdot d\mathbf{s} = 2\pi r E$$

$$= A \left[ \frac{dB}{dt} \right] = \pi r^2 \left| \frac{dB}{dt} \right|$$

Thus the strength of the induced electric field inside the solenoid is

$$E_{\text{inside}} = \frac{r}{2} \left| \frac{dB}{dt} \right|$$

**Example 30.10 An Induced Electric Field**

**EXAMPLE 30.10** An induced electric field

A 4.0-cm diameter solenoid is wound with 2000 turns per meter. The current through the solenoid oscillates at 60 Hz with an amplitude of 2.0 A. What is the maximum strength of the induced electric field inside the solenoid?

**MODEL.** Assume that the magnetic field inside the solenoid is uniform.
Example 30.10 An Induced Electric Field

**EXAMPLE 30.10** An induced electric field

**VISUALIZE** The electric field lines are concentric circles around the magnetic field lines, as was shown in Figure 30.3b. They reverse direction twice every period as the current oscillates.

The induced electric field circulates around the magnetic field lines.

\[ \vec{E} \text{ increasing} \]

Induced \( \vec{E} \)

Induced \( \vec{E} \)

\[ E = \frac{\mu_0}{2\pi} \frac{d}{dr} (\mu_0 n I \sin \theta) = \mu_0 n I \cos \theta \]

The field strength is maximum at maximum radius \( r = R \) and at the instant when \( \cos \theta = 1 \). That is,

\[ E_{\text{max}} = \mu_0 n I R \cos \theta = 0.079 \text{ V/m} \]

**EXAMPLE 30.10** An induced electric field

**SOLVE** You learned in Chapter 29 that the magnetic field strength inside a solenoid with \( n \) turns per meter is \( B = \mu_0 n I \). In this case, the current through the solenoid is \( I = I_{\text{max}} \sin \omega t \), where \( I_{\text{max}} = 2.0 \text{ A} \) is the peak current and \( \omega = 2\pi(60 \text{ Hz}) = 377 \text{ rad/s} \). Thus, the induced electric field strength at radius \( r \) is

\[ E = \frac{\mu_0}{2\pi} \frac{d}{dr} (\mu_0 n I \sin \theta) = \mu_0 n I \cos \theta \]

**EXAMPLE 30.10** An induced electric field

**ASSESS** This field strength, although not large, is similar to the field strength that the emf of a battery creates in a wire. Hence, this induced electric field can drive a substantial induced current through a conducting loop if a loop is present. But the induced electric field exists inside the solenoid whether or not there is a conducting loop.
QuickCheck 30.13

The magnetic field is decreasing. Which is the induced electric field?

A. 
B. 
C. 
D. 
E. There's no induced field in this case.

QuickCheck 30.13

The magnetic field is decreasing. Which is the induced electric field?

The field is the same direction as induced current would flow if there were a loop in the field.

A. 
B. 
C. 
D. 
E. There's no induced field in this case.

The Induced Electric Field

A changing magnetic field creates an induced electric field.

- Faraday's law and Lenz's law may be combined by noting that the emf must oppose the change in $\Phi$.
- Mathematically, emf must have the opposite sign of $dB/dt$.
- Faraday's law may be written as $\mathcal{E} = \oint E \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$. 

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The Induced Magnetic Field

- As we know, changing the magnetic field induces a circular electric field.
- Symmetrically, changing the electric field induces a circular magnetic field.
- The induced magnetic field was first suggested as a possibility by James Clerk Maxwell in 1855.

Maxwell's Theory of Electromagnetic Waves

- A changing electric field creates a magnetic field, which then changes in just the right way to re-create the electric field, which then changes in just the right way to again re-create the magnetic field, and so on.
- This is an electromagnetic wave.

Generators

- A generator is a device that transforms mechanical energy into electric energy.

A generator inside a hydroelectric dam uses electromagnetic induction to convert the mechanical energy of a spinning turbine into electric energy.
Example 30.11 An AC Generator

**EXAMPLE 30.11** An AC generator

A coil with area 2.0 m$^2$ rotates in a 0.010 T magnetic field at a frequency of 60 Hz. How many turns are needed to generate a peak voltage of 160 V?

**SOLVE** The coil’s maximum voltage is found from Equation 30.29:

$$E_{\text{max}} = NABV = 2\pi fABV$$

The number of turns needed to generate $E_{\text{max}} = 160$ V is

$$N = \frac{E_{\text{max}}}{2\pi fAB} = \frac{160\text{ V}}{2\pi (60\text{ Hz})(2.0\text{ m}^2)(0.010\text{ T})} = 21\text{ turns}$$

**Example 30.11 An AC Generator**

**ASSESS** A 0.001 T field is modest, so you can see that generating large voltages is not difficult with large (2 m$^2$) coils. Commercial generators use water flowing through a dam, rotating windmill blades, or turbines spun by expanding steam to rotate the generator coils. Work is required to rotate the coil, just as work was required to pull the slide wire in Section 30.2, because the magnetic field exerts retarding forces on the currents in the coil. Thus a generator is a device that turns motion (mechanical energy) into a current (electric energy). A generator is the opposite of a motor, which turns a current into motion.
A transformer sends an alternating emf $V_1$ through the primary coil.

This causes an oscillating magnetic flux through the secondary coil and, hence, an induced emf $V_2$.

The induced emf of the secondary coil is delivered to the load:

$$V_2 = \frac{N_2}{N_1} V_1$$

A step-up transformer, with $N_2 >> N_1$, can boost the voltage of a generator up to several hundred thousand volts.

Delivering power with smaller currents at higher voltages reduces losses due to the resistance of the wires.

High-voltage transmission lines carry electric power to urban areas, where step-down transformers ($N_2 << N_1$) lower the voltage to 120 V.

Metal detectors consist of two coils: a transmitter coil and a receiver coil.

A high-frequency AC current in the transmitter coil causes a field which induces current in the receiver coil.

The net field at the receiver decreases when a piece of metal is inserted between the coils.

Electronic circuits detect the current decrease in the receiver coil and set off an alarm.
Inductors

- A coil of wire, or solenoid, can be used in a circuit to store energy in the magnetic field.
- We define the inductance of a solenoid having \( N \) turns, length \( l \) and cross-section area \( A \) as
  \[
  L_{\text{solenoid}} = \frac{\Phi_m}{I} = \frac{\mu_0 N^2 A}{l}
  \]
- The SI unit of inductance is the henry, defined as
  \[
  1 \text{ henry} = 1 \text{ H} = 1 \text{ Wb}/\text{A} = 1 \text{ T m}^2/\text{A}
  \]
- A coil of wire used in a circuit for the purpose of inductance is called an inductor.
- The circuit symbol for an ideal inductor is \( \text{─○─} \).

Example 30.12 The Length of an Inductor

**EXAMPLE 30.12** The length of an inductor

An inductor is made by tightly wrapping 0.36-mm-diameter wire around a 4.0-mm-diameter cylinder. What length cylinder has an inductance of 10 \( \mu \text{H} \)?

**Solve** The cross-section area of the solenoid is \( A = \pi r^2 \). If the wire diameter is \( d \), the number of turns of wire on a cylinder of length \( l \) is \( N = \pi rl \). Thus the inductance is

\[
L = \frac{\Phi_m}{I} = \frac{\mu_0 (\pi rl)^2 r^2}{l} = \frac{\mu_0 \pi^2 r^2 l}{d^2}
\]

The length needed to give inductance \( L = 1.0 \times 10^{-7} \text{ H} \) is

\[
I = \frac{d^2 L}{\mu_0 \pi r^2} = \frac{(0.00070 \text{ m})^2 (1.0 \times 10^{-7} \text{ H})}{(4\pi \times 10^{-7} \text{ T m/A})(0.0020 \text{ m})^2} = 0.057 \text{ m} = 5.7 \text{ cm}
\]
Potential Difference Across an Inductor

- The figure above shows a steady current into the left side of an inductor.
- The solenoid’s magnetic field passes through the coils, establishing a flux.
- The next slide shows what happens if the current increases.

Potential Difference Across an Inductor

- In the figure, the current into the solenoid is increasing.
- This creates an increasing flux to the left.
- Therefore the induced magnetic field must point to the right.
- The induced emf $\Delta V_L$ must be opposite to the current into the solenoid:

$$\Delta V_L = -\frac{d\Phi}{dt}$$

- The induced current is opposite the solenoid current.
- The induced magnetic field opposes the change in flux.
- The induced current carries positive charge carriers to the left and establishes a potential difference across the inductor.

Potential Difference Across an Inductor

- In the figure, the current into the solenoid is decreasing.
- To oppose the decrease in flux, the induced emf $\Delta V_L$ is in the same direction as the input current.
- The potential difference across an inductor, measured along the direction of the current, is

$$\Delta V_L = -\frac{d\Phi}{dt}$$

- The induced current carries positive charge carriers to the right. The potential difference is opposite that of Figure 36.38b.
Which current is changing more rapidly?

A. Current $I_1$
B. Current $I_2$
C. They are changing at the same rate.
D. Not enough information to tell.

$\Delta V_{res} = -IR$

$\Delta V_L = -L \frac{dI}{dt}$
Example 30.13 Large Voltage Across an Inductor

**EXAMPLE 30.13** Large voltage across an inductor

A 1.0 A current passes through a 10 mH inductor coil. What potential difference is induced across the coil if the current drops to zero in 5.0 μs?

**MODEL** Assume this is an ideal inductor, with $R = 0 \, \Omega$, and that the current decrease is linear with time.

**SOLVE** The rate of current decrease is

$$\frac{dl}{dt} = \frac{\Delta l}{\Delta t} = \frac{-1.0 \, \Lambda}{5.0 \times 10^6 \, \text{s}} = -2.0 \times 10^5 \, \text{A/s}$$

The induced voltage is

$$\Delta V_L = -l \frac{dl}{dt} = -(0.010 \, \text{H})(-2.0 \times 10^5 \, \text{A/s}) = 200 \, \text{V}$$

**ASSESS** Inductors may be physically small, but they can pack a punch if you try to change the current through them too quickly.

---

Energy in Inductors and Magnetic Fields

- As current passes through an inductor, the electric power is

$$P_{elec} = l \Delta V_L = -LI \frac{dl}{dt}$$

- $P_{elec}$ is negative because the current is losing energy.
- That energy is being transferred to the inductor, which is storing energy $U_l$, at the rate

$$\frac{dU_l}{dt} = LI \frac{dl}{dt}$$

- We can find the total energy stored in an inductor by integrating:

$$U_l = LI \int_0^t l \, dt = \frac{1}{2} LI^2$$
Energy in Inductors and Magnetic Fields

- Inside a solenoid, the magnetic field strength is \( B = \mu_0 NI/l \).
- The inductor’s energy can be related to \( B \):
  \[
  U_L = \frac{1}{2} LI^2 = \frac{\mu_0 N^2 I^2}{2l} = \frac{1}{2\mu_0} \left( \frac{\mu_0 NI}{l} \right)^2
  \]
  \[
  U_L = \frac{1}{2\mu_0} AlB^2
  \]
- But \( Al \) is the volume inside the solenoid.
- Dividing by \( Al \), the magnetic field energy density (energy per m\(^3\)) is
  \[
  u_B = \frac{1}{2\mu_0} B^2
  \]
Example 30.14 Energy Stored in an Inductor

**EXAMPLE 30.14 Energy stored in an inductor**

**SOLVE** The stored energy is

$$U_L = \frac{1}{2}L^2 = \frac{1}{2}(1.0 \times 10^{-3} \text{ H})(0.10 \text{ A})^2 = 5.0 \times 10^{-6} \text{ J}$$

The solenoid volume is $\left(\pi r^2\right)L = 7.16 \times 10^{-7} \text{ m}^3$. Using this gives the energy density of the magnetic field:

$$a_0 = \frac{5.0 \times 10^{-6} \text{ J}}{7.16 \times 10^{-7} \text{ m}^2} = 0.070 \text{ J/m}^3$$

From Equation 30.42, the magnetic field with this energy density is

$$B = \sqrt{\mu_0 a_0} = 4.2 \times 10^{-4} \text{ T}$$

---

**LC Circuits**

- The figure shows a capacitor with initial charge $Q_0$, an inductor, and a switch.
- The switch has been open for a long time, so there is no current in the circuit.
- At $t = 0$, the switch is closed.
- How does the circuit respond?

- The charge and current oscillate in a way that is analogous to a mass on a spring.

---

**LC Circuits: Step A**

Analogy:

Maximum capacitor charge is like a fully stretched spring.

The capacitor discharges until the current is a maximum.

- $Q = -Q_0$
- $t = 0$
**LC Circuits: Step B**

- $Q = 0$
- Maximum current is like the block having maximum speed.

The current continues until the capacitor is fully recharged with opposite polarization.

**LC Circuits: Step C**

- $Q = -Q_0$
- $v = 0$

Now the discharge goes in the opposite direction.

**LC Circuits: Step D**

- $Q = 0$
- Maximum current.

The current continues until the initial capacitor charge is reversed.
LC Circuits

- An LC circuit is an electric oscillator.
- The letters on the graph correspond to the four steps in the previous slides.
- The charge on the upper plate is \( Q = Q_0 \cos \omega t \) and the current through the inductor is \( I = I_{\text{max}} \sin \omega t \), where

\[
\omega = \sqrt{\frac{1}{LC}}
\]

QuickCheck 30.15

If the top circuit has an oscillation frequency of 1000 Hz, the frequency of the bottom circuit is

A. 500 Hz  
B. 707 Hz  
C. 1000 Hz  
D. 1410 Hz  
E. 2000 Hz

QuickCheck 30.15

If the top circuit has an oscillation frequency of 1000 Hz, the frequency of the bottom circuit is

A. 500 Hz  
B. 707 Hz  
C. 1000 Hz  
D. 1410 Hz  
E. 2000 Hz

Series capacitors have equivalent \( C/2 \).
A cell phone is actually a very sophisticated two-way radio that communicates with the nearest base station via high-frequency radio waves—roughly 1000 MHz. As in any radio or communications device, the transmission frequency is established by the oscillating current in an LC circuit.

**Example 30.15 An AM Radio Oscillator**

Once the current is driven through the LR circuit, how does the circuit respond?

The current through the circuit decays exponentially, with a time constant $r = L/R$. The figure shows an inductor and resistor in series.

Initially there is a steady current $I$, being driven through the LR circuit by an external battery.

At $t = 0$, the switch is closed.

How does the circuit respond?
QuickCheck 30.16

What is the battery current immediately after the switch has closed?

A. 0 A  
B. 1 A  
C. 2 A  
D. Undefined

QuickCheck 30.16

What is the battery current immediately after the switch has closed?

✓A. 0 A  
B. 1 A  
C. 2 A  
D. Undefined
QuickCheck 30.17

What is the battery current immediately after the switch has been closed for a very long time?

A. 0 A
B. 1 A
C. 2 A
D. Undefined

Correct Answer: C.

Chapter 30 Summary Slides
**General Principles**

**Lenz's Law**

There is an induced current in a closed conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the change in the flux.

**Faraday's Law**

An emf is induced around a closed loop if the magnetic flux through the loop changes.

- **Magnitude:** \( \mathcal{E} = \frac{d\Phi}{dt} \)
- **Direction:** As given by Lenz's law

**Using Electromagnetic Induction**

**MODEL:** Make simplifying assumptions.

**VISUALIZE:** Use Lenz's law to determine the direction of the induced current.

**SOLVE:** The induced emf is

\[ \mathcal{E} = \frac{d\Phi}{dt} \]

Multiply by \( N \) for an \( N \)-turn coil.

The size of the induced current is \( I = \mathcal{E}R \).

**ASSESS:** Is the result reasonable?
Important Concepts

**Magnetic flux**
Magnetic flux measures the amount of magnetic field passing through a surface.
\[ \Phi_B = \mathbf{A} \cdot \mathbf{B} = \mathcal{A} \mathbf{B} \cdot \mathbf{n} \]

![Diagram of magnetic flux](image)

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**Important Concepts**

**Three ways to change the flux**
1. A loop moves into or out of a magnetic field.
2. The loop changes area or rotates.
3. The magnetic field through the loop increases or decreases.

![Diagram of flux changes](image)

---

**Important Concepts**

**Two ways to create an induced current**
1. A motional emf is due to magnetic forces on moving charge carriers.
\[ \mathcal{E} = \mathcal{E}_m = \mathcal{E}_B \]
2. An induced electric field is due to a changing magnetic field.
\[ \oint \mathbf{E} \cdot d\mathbf{S} = -\frac{d\Phi_B}{dt} \]

![Diagram of induced current](image)
Applications

**Inductors**

- Self-inductance: $L = \frac{N^2 A}{l}$
- Potential difference: $\Delta V = -\frac{df}{dt}$
- Energy stored: $W_L = \frac{1}{2} L I^2$
- Magnetic energy density: $u_m = \frac{1}{2} B^2 / \mu_0$

Applications

**LC circuit**

- Oscillation: $\omega = \frac{1}{\sqrt{LC}}$

**LR circuit**

- Exponential change: $v = e^{-\frac{t}{\tau}}$