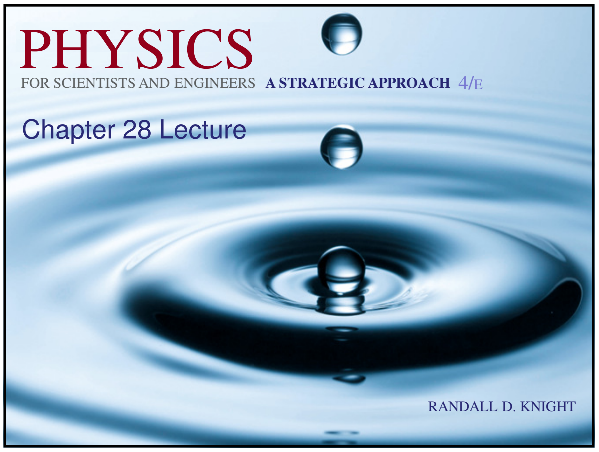
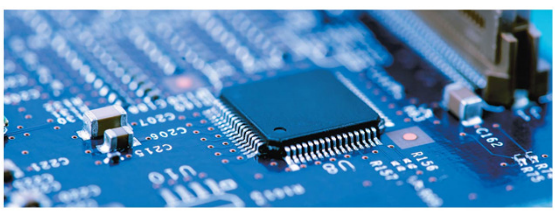



## Chapter 28 Fundamentals of Circuits



IN THIS CHAPTER, you will learn the fundamental physical principles that govern electric circuits.


Slide 28-2

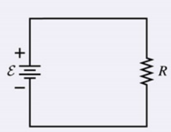
## Chapter 28 Preview

### What is a circuit?

Circuits—from flashlights to computers—are the controlled motion of charges through conductors and resistors.

- This chapter focuses on DC circuits, meaning *direct current*, in which potentials and currents are constant.
- You'll learn to draw circuit diagrams.

« LOOKING BACK Section 26.4 Sources of potential





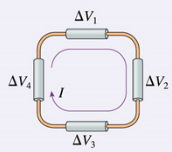
Slide 28-3

## Chapter 28 Preview

### How are circuits analyzed?

Any circuit, no matter how complex, can be analyzed with Kirchhoff's two laws:

- The junction law (charge conservation) relates the currents at a junction.
- The loop law (energy conservation) relates the voltages around a closed loop.
- We'll also use Ohm's law for resistors.



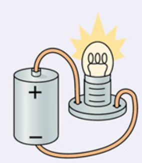




## Chapter 28 Preview

### How do circuits use energy?

Circuits use energy to do things, such as lighting a bulb or turning a motor. You'll learn to calculate power, the rate at which the battery supplies energy to a circuit and the rate at which a resistor dissipates energy. Many circuit elements are *rated* by their power consumption in watts.



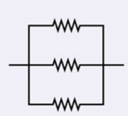




## Chapter 28 Preview

### How are resistors combined?

Resistors, like capacitors, often occur in series or in parallel. These combinations of resistors can be simplified by replacing them with a single resistor with equivalent resistance.



« LOOKING BACK Sections 27.3–27.5 Current, resistance, and Ohm's law

## Chapter 28 Preview

**What is an *RC* circuit?**

Capacitors are charged and discharged by current flowing through a resistor. These important circuits are called *RC* circuits. Their uses range from heart defibrillators to digital electronics. You'll learn that capacitors charge and discharge exponentially with time constant $\tau = RC$.


« LOOKING BACK Section 26.5 Capacitors




## Chapter 28 Preview

**Why are circuits important?**

We live in an electronic era, and electric circuits surround you: your household wiring, the ignition system in your car, your music and communication devices, and your tablets and computers. Electric circuits are one of the most important applications of physics, and in this chapter you will see how the seemingly abstract ideas of electric charge, field, and potential are the foundation for many of the things we take for granted in the 21st century.




## Chapter 28 Reading Questions

## Reading Question 28.1

How many laws are named after Kirchhoff?

A. 0
B. 1
C. 2
D. 3
E. 4


Slide 28-10

## Reading Question 28.1

How many laws are named after Kirchhoff?

A. 0
B. 1
✔ **C. 2**
D. 3
E. 4


Slide 28-11

## Reading Question 28.2

What property of a real battery makes its potential difference slightly different than that of an ideal battery?

A. Short circuit
B. Chemical potential
C. Internal resistance
D. Effective capacitance
E. Inductive constant


Slide 28-12

## Reading Question 28.2

What property of a real battery makes its potential difference slightly different than that of an ideal battery?

A. Short circuit
B. Chemical potential
✔ **C. Internal resistance**
D. Effective capacitance
E. Inductive constant




## Reading Question 28.3

In an *RC* circuit, what is the name of the quantity represented by the symbol $\tau$?

A. Period
B. Torque
C. Terminal voltage
D. Time constant
E. Coefficient of thermal expansion




## Reading Question 28.3

In an *RC* circuit, what is the name of the quantity represented by the symbol $\tau$?

A. Period
B. Torque
C. Terminal voltage
✔ **D. Time constant**
E. Coefficient of thermal expansion

## Reading Question 28.4

Which of the following are *ohmic* materials?

A. Batteries
B. Wires.
C. Resistors
D. Materials A and B
E. Materials B and C




## Reading Question 28.4

Which of the following are *ohmic* materials?

A. Batteries
B. Wires
C. Resistors
D. Materials A and B
✓ **E. Materials B and C**




## Reading Question 28.5

The equivalent resistance for a group of parallel resistors is

A. Less than any resistor in the group.
B. Equal to the smallest resistance in the group.
C. Equal to the average resistance of the group.
D. Equal to the largest resistance in the group.
E. Larger than any resistor in the group.

## Reading Question 28.5

The equivalent resistance for a group of parallel resistors is

✔ **A. Less than any resistor in the group.**
B. Equal to the smallest resistance in the group.
C. Equal to the average resistance of the group.
D. Equal to the largest resistance in the group.
E. Larger than any resistor in the group.

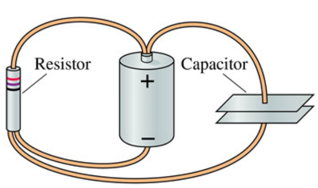


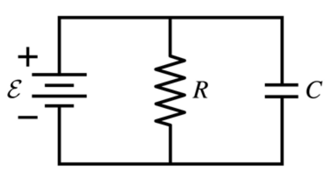

Chapter 28 Content, Examples, and QuickCheck Questions



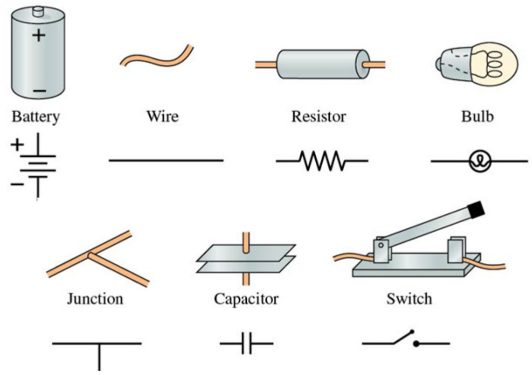

## Circuit Diagrams

- The top figure shows a literal picture of a resistor and a capacitor connected by wires to a battery.
- The bottom figure is a **circuit diagram** of the same circuit.
- A circuit diagram is a *logical* picture of what is connected to what.



## Circuit Elements

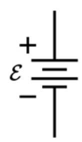




Slide 28-22

## Circuit Diagrams

- A circuit diagram replaces pictures of the circuit elements with symbols.
- The longer line at one end of the battery symbol represents the positive terminal of the battery.
- The battery's emf is shown beside the battery.
- + and – symbols, even though somewhat redundant, are shown beside the terminals.



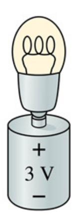


Slide 28-23

## QuickCheck 28.1

Does the bulb light?

A. Yes
B. No
C. I'm not sure.





Slide 28-24

## QuickCheck 28.1

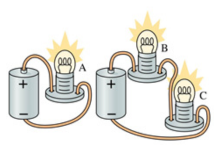
Does the bulb light?


A. Yes
✔**B. No** Not a complete circuit
C. I'm not sure.


Slide 28-25

## QuickCheck 28.2

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

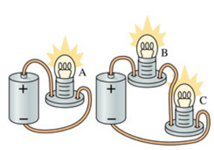



A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C


Slide 28-26

## QuickCheck 28.2

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.



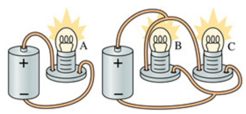

A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C

This question is checking your initial intuition. We'll return to it later.


Slide 28-27

## QuickCheck 28.3

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

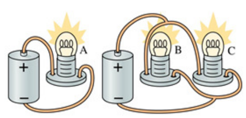



A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C


Slide 28-28

## QuickCheck 28.3

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.




A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C

This question is checking your initial intuition. We'll return to it later.

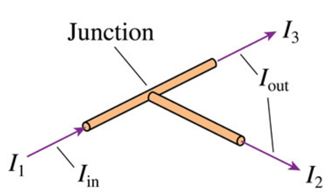

Slide 28-29

## Kirchhoff's Junction Law

- For a *junction*, the law of conservation of current requires that:

$$\sum I_{in} = \sum I_{out}$$

where the Σ symbol means summation.
- This basic conservation statement is called **Kirchhoff's junction law**.




Junction law: $I_1 = I_2 + I_3$

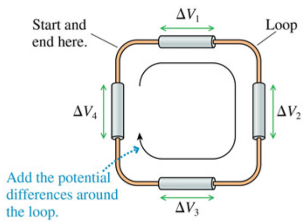

Slide 28-30

## Kirchhoff's Loop Law

- For any path that starts and ends at the same point,

$$\Delta V_{\text{loop}} = \sum (\Delta V)_i = 0$$

- The sum of all the potential differences encountered while moving around a loop or closed path is zero.
- This statement is known as **Kirchhoff's loop law**.




Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

 Slide 28-31

## Tactics: Using Kirchhoff's Loop Law

**TACTICS BOX 28.1**

**Using Kirchhoff's loop law**

1. **Draw a circuit diagram.** Label all known and unknown quantities.
2. **Assign a direction to the current.** Draw and label a current arrow $I$ to show your choice.
   - If you know the actual current direction, choose that direction.
   - If you don't know the actual current direction, make an arbitrary choice. All that will happen if you choose wrong is that your value for $I$ will end up negative.

 Slide 28-32

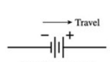
## Tactics: Using Kirchhoff's Loop Law

**TACTICS BOX 28.1**

**Using Kirchhoff's loop law**

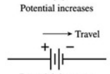
3. **"Travel" around the loop.** Start at any point in the circuit, then go all the way around the loop in the direction you assigned to the current in step 2. As you go through each circuit element, $\Delta V$ is interpreted to mean $\Delta V = V_{\text{downstream}} - V_{\text{upstream}}$.
   - For an ideal battery in the negative-to-positive direction:

$$\Delta V_{\text{bat}} = +\mathcal{E}$$

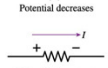



   - For an ideal battery in the positive-to-negative direction:

$$\Delta V_{\text{bat}} = -\mathcal{E}$$




   - For a resistor: $\Delta V_{\text{res}} = -\Delta V_R = -IR$



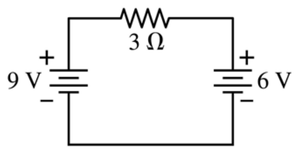

4. **Apply the loop law:** $\sum (\Delta V)_i = 0$

Exercises 4–7

 Slide 28-33

## QuickCheck 28.4

The current through the 3 Ω resistor is



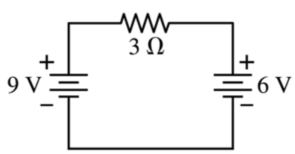

A. 9 A
B. 6 A
C. 5 A
D. 3 A
E. 1 A



Slide 28-34

## QuickCheck 28.4

The current through the 3 Ω resistor is

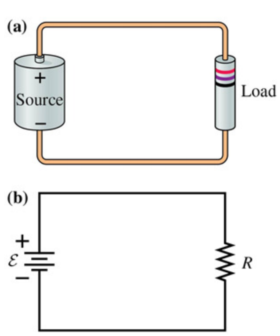



A. 9 A
B. 6 A
C. 5 A
D. 3 A
✔ **E. 1 A**



Slide 28-35

## The Basic Circuit

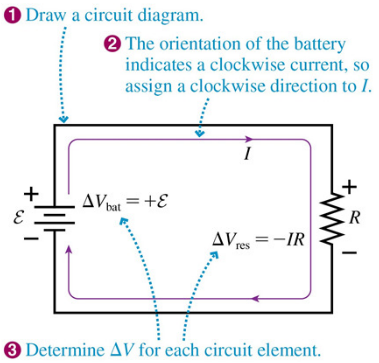



- The most basic electric circuit is a single resistor connected to the two terminals of a battery.
- Figure (a) shows a literal picture of the circuit elements and the connecting wires.
- Figure (b) is the circuit diagram.
- This is a **complete circuit**, forming a continuous path between the battery terminals.



Slide 28-36

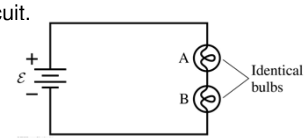
## Analyzing the Basic Circuit




 Slide 28-37

## Lightbulb Puzzle #1

- The figure shows two identical lightbulbs in a circuit.
- The current through both bulbs is *exactly the same*!
- It's not the current that the bulbs use up, it's *energy*.
- The battery creates a potential difference, which supplies potential energy to the charges.
- As the charges move through the lightbulbs, they lose some of their potential energy, transferring the energy to the bulbs.

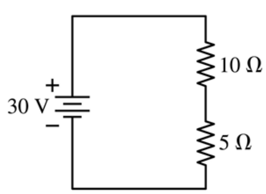



 Slide 28-38

## QuickCheck 28.5

The potential difference across the 10 resistor is

A. 30 V
B. 20 V
C. 15 V
D. 10 V
E. 5 V

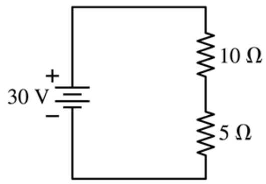



 Slide 28-39

## QuickCheck 28.5

The potential difference across the 10 resistor is

A. 30 V
✔**B. 20 V**
C. 15 V
D. 10 V
E. 5 V



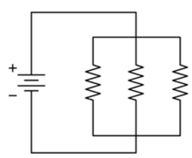


Slide 28-40

## QuickCheck 28.6

What things about the resistors in this circuit are the same for all three?

A. Current $I$
B. Potential difference $\Delta V$
C. Resistance $R$
D. A and B
E. B and C

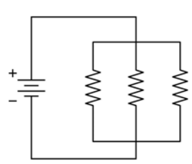



Slide 28-41

## QuickCheck 28.6

What things about the resistors in this circuit are the same for all three?

A. Current $I$
✔**B. Potential difference $\Delta V$**
C. Resistance $R$
D. A and B
E. B and C




Slide 28-42

## Energy and Power

- The power supplied by a battery is

$$P_{bat} = I\mathcal{E} \quad \text{(power delivered by an emf)}$$

- The units of power are J/s or W.
- The power dissipated by a resistor is

$$P_R = \frac{dE_{th}}{dt} = \frac{dq}{dt}\Delta V_R = I\Delta V_R$$

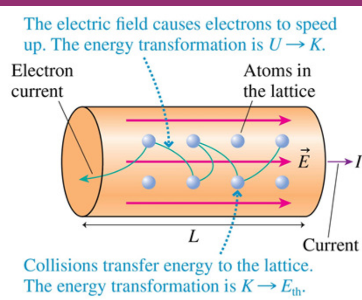
- Or, in terms of the potential drop across the resistor,

$$P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R} \quad \text{(power dissipated by a resistor)}$$



Slide 28-43

## Power Dissipation in a Resistor




- A current-carrying resistor dissipates power because the electric force does work on the charges.

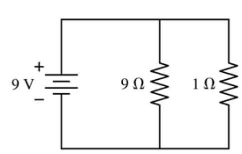


Slide 28-44

## QuickCheck 28.7

Which resistor dissipates more power?

A. The $9\ \Omega$ resistor
B. The $1\ \Omega$ resistor
C. They dissipate the same power.

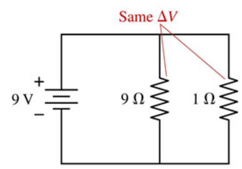





Slide 28-45

## QuickCheck 28.7

Which resistor dissipates more power?




A. The 9 Ω resistor
✔**B. The 1 Ω resistor**
C. They dissipate the same power.

$$P = \frac{(\Delta V)^2}{R}$$


Slide 28-46

## QuickCheck 28.8

Which has a larger resistance, a 60 W lightbulb or a 100 W lightbulb?

A. The 60 W bulb
B. The 100 W bulb
C. Their resistances are the same.
D. There's not enough information to tell.


Slide 28-47

## QuickCheck 28.8

Which has a larger resistance, a 60 W lightbulb or a 100 W lightbulb?

A. The 60 W bulb
✔**B. The 100 W bulb** $P = \frac{(\Delta V)^2}{R}$ with both used at $\Delta V = 120$ V
C. Their resistances are the same.
D. There's not enough information to tell.


Slide 28-48

## Example 28.2 The Power of Light

**EXAMPLE 28.2** | **The power of light**

How much current is "drawn" by a 100 W lightbulb connected to a 120 V outlet?

**MODEL** Most household appliances, such as a 100 W lightbulb or a 1500 W hair dryer, have a power rating. The rating does *not* mean that these appliances *always* dissipate that much power. These appliances are intended for use at a standard household voltage of 120 V, and their rating is the power they will dissipate *if* operated with a potential difference of 120 V. Their power consumption will differ from the rating if they are operated at any other potential difference.

**SOLVE** Because the lightbulb is operating as intended, it will dissipate 100 W of power. Thus

$$I = \frac{P_R}{\Delta V_R} = \frac{100\ \text{W}}{120\ \text{V}} = 0.833\ \text{A}$$

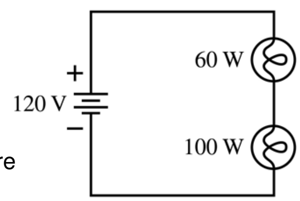
**ASSESS** A current of 0.833 A in this lightbulb transfers 100 J/s to the thermal energy of the filament, which, in turn, dissipates 100 J/s as heat and light to its surroundings.





## QuickCheck 28.9

Which bulb is brighter?



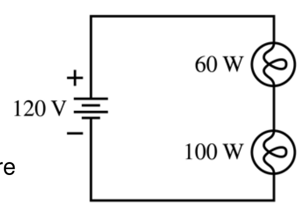

A. The 60 W bulb
B. The 100 W bulb
C. Their brightnesses are the same.
D. There's not enough information to tell.





## QuickCheck 28.9

Which bulb is brighter?




✔ **A. The 60 W bulb**
B. The 100 W bulb
C. Their brightnesses are the same.
D. There's not enough information to tell.

$P = I^2R$ and both have the same current.

## Example 28.3 The Power of Sound

**EXAMPLE 28.3** The power of sound

Most loudspeakers are designed to have a resistance of 8 Ω. If an 8 Ω loudspeaker is connected to a stereo amplifier with a rating of 100 W, what is the maximum possible current to the loudspeaker?

**MODEL** The rating of an amplifier is the *maximum* power it can deliver. Most of the time it delivers far less, but the maximum might be reached for brief, intense sounds like cymbal crashes.

**SOLVE** The loudspeaker is a resistive load. The maximum current to the loudspeaker occurs when the amplifier delivers maximum power $P_{max} = (I_{max})^2R$. Thus

$$I_{max} = \sqrt{\frac{P_{max}}{R}} = \sqrt{\frac{100\text{ W}}{8\ \Omega}} = 3.5\text{ A}$$



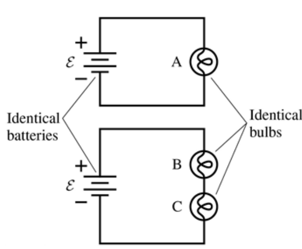

## Kilowatt Hours

- The product of watts and seconds is joules, the SI unit of energy.
- However, most electric companies prefers to use the **kilowatt hour**, to measure the energy you use each month.
- Examples:
  - A 4000 W electric water heater uses 40 kWh of energy in 10 hours.
  - A 1500 W hair dryer uses 0.25 kWh of energy in 10 minutes.
- The average cost of electricity in the United States is ≈ 10¢ per kWh ($0.10/kWh).




## Lightbulb Puzzle #2

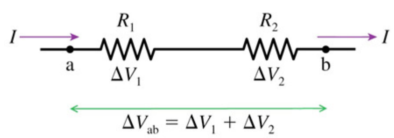
- The figure shows three identical lightbulbs in two different circuits.
- The voltage drop across A is the *same* as the total voltage drop across both B and C.
- More current will pass through Bulb A, and it will be *brighter* than either B or C.



## Series Resistors

- The figure below shows two resisters connected *in series* between points a and b.
- The total potential difference between points a and b is the sum of the individual potential differences across $R_1$ and $R_2$:

$$\Delta V_{\text{ab}} = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I(R_1 + R_2)$$

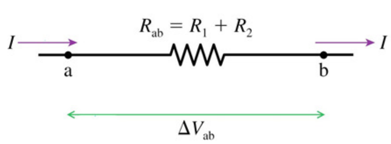



 Slide 28-55

## Series Resistors

- Suppose we replace $R_1$ and $R_2$ with a single resistor with the same current $I$ and the same potential difference $\Delta V_{\text{ab}}$.
- Ohm's law gives resistance between points a and b:

$$R_{\text{ab}} = \frac{\Delta V_{\text{ab}}}{I} = \frac{I(R_1 + R_2)}{I} = R_1 + R_2$$




 Slide 28-56

## Series Resistors

- Resistors that are aligned end to end, *with no junctions between them,* are called **series resistors** or, sometimes, resistors "in series."
- The current $I$ is the same through all resistors placed in series.
- If we have $N$ resistors in series, their **equivalent resistance** is

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_N \qquad \text{(series resistors)}$$

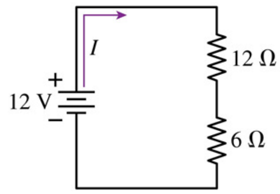
- The behavior of the circuit will be unchanged if the $N$ series resistors are replaced by the single resistor $R_{\text{eq}}$.

 Slide 28-57

## QuickCheck 28.10

The battery current $I$ is

A. 3 A
B. 2 A
C. 1 A
D. 2/3 A
E. 1/2 A



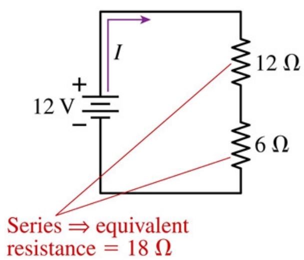




## QuickCheck 28.10

The battery current $I$ is

A. 3 A
B. 2 A
C. 1 A
✔ **D. 2/3 A**
E. 1/2 A





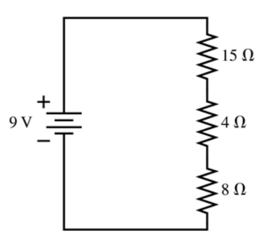


## Example 28.4 A Series Resistor Circuit

**EXAMPLE 28.4** | **A series resistor circuit**

a. What is the current in the circuit of FIGURE 28.15a?

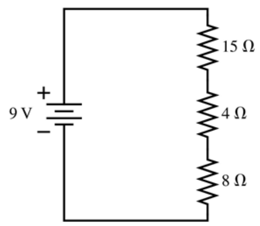
b. Draw a graph of potential versus position in the circuit, going cw from $V = 0$ V at the battery's negative terminal.



## Example 28.4 A Series Resistor Circuit

**EXAMPLE 28.4** | A series resistor circuit

**MODEL** The three resistors are end to end, with no junctions between them, and thus are in series. Assume ideal connecting wires and an ideal battery.







## Example 28.4 A Series Resistor Circuit

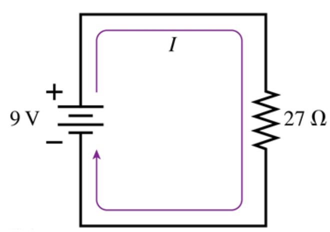
**EXAMPLE 28.4** | A series resistor circuit

**SOLVE** a. The battery "acts" the same—it provides the same current at the same potential difference—if we replace the three series resistors by their equivalent resistance

$$R_{eq} = 15\ \Omega + 4\ \Omega + 8\ \Omega = 27\ \Omega$$

This is shown as an equivalent circuit in FIGURE 28.15b. Now we have a circuit with a single battery and a single resistor, for which we know the current to be

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{9\ \text{V}}{27\ \Omega} = 0.333\ \text{A}$$



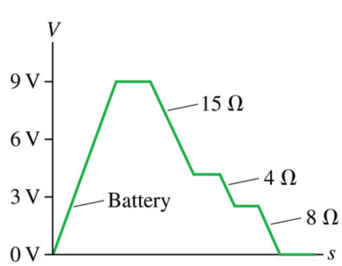




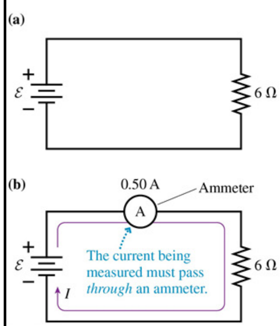
## Example 28.4 A Series Resistor Circuit

**EXAMPLE 28.4** | A series resistor circuit

**SOLVE** b. $I = 0.333$ A is the current in each of the three resistors in the original circuit. Thus the potential differences across the resistors are $\Delta V_{res\,1} = -IR_1 = -5.0$ V, $\Delta V_{res\,2} = -IR_2 = -1.3$ V, and $\Delta V_{res\,3} = -IR_3 = -2.7$ V for the 15 Ω, the 4 Ω, and the 8 Ω resistors. FIGURE 28.15c shows that the potential increases by 9 V due to the battery, then decreases by 9 V in three steps.



## Ammeters



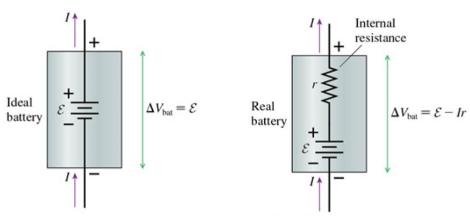

- Figure (a) shows a simple one-resistor circuit.
- We can measure the current by breaking the connection and inserting an ammeter *in series*.
- The resistance of the ammeter is negligible.
- The potential difference across the resistor must be $\Delta V_R = IR = 3.0$ V.
- So the battery's emf must be 3.0 V.



Slide 28-64

## Real Batteries

- Real batteries have what is called an internal resistance, which is symbolized by $r$.

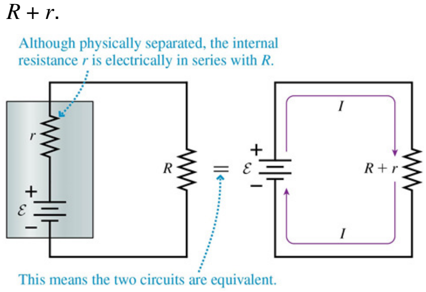





Slide 28-65

## Real Batteries

- A single resistor connected to a real battery is in series with the battery's internal resistance, giving $R_{eq} = R + r$.






Slide 28-66

## Example 28.5 Lighting Up a Flashlight

**EXAMPLE 28.5** **Lighting up a flashlight**

A 6 Ω flashlight bulb is powered by a 3 V battery with an internal resistance of 1 Ω. What are the power dissipation of the bulb and the terminal voltage of the battery?

**MODEL** Assume ideal connecting wires but not an ideal battery.



Slide 28-67

## Example 28.5 Lighting Up a Flashlight

**EXAMPLE 28.5** **Lighting up a flashlight**

**SOLVE** Equation 28.17 gives us the current:

$$I = \frac{\mathcal{E}}{R + r} = \frac{3\text{ V}}{6\ \Omega + 1\ \Omega} = 0.43\text{ A}$$

This is 15% less than the 0.5 A an ideal battery would supply. The potential difference across the resistor is $\Delta V_R = IR = 2.6$ V, thus the power dissipation is

$$P_R = I\Delta V_R = 1.1\text{ W}$$

The battery's terminal voltage is

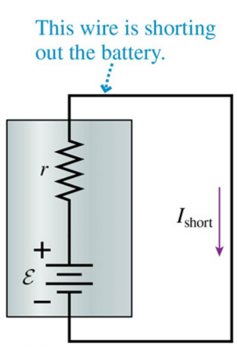
$$\Delta V_{bat} = \frac{R}{R + r}\mathcal{E} = \frac{6\ \Omega}{6\ \Omega + 1\ \Omega}3\text{ V} = 2.6\text{ V}$$

**ASSESS** 1 Ω is a typical internal resistance for a flashlight battery. The internal resistance causes the battery's terminal voltage to be 0.4 V less than its emf in this circuit.



Slide 28-68

## A Short Circuit




- The figure shows an ideal wire *shorting out* a battery.
- If the battery were ideal, shorting it with an ideal wire ($R = 0\ \Omega$) would cause the current to be infinite!
- In reality, the battery's internal resistance $r$ becomes the only resistance in the circuit.
- The *short-circuit current* is

$$I_{short} = \frac{\mathcal{E}}{r}$$



Slide 28-69

## Example 28.6 A Short-Circuited Battery

**EXAMPLE 28.6** | **A short-circuited battery**

What is the short-circuit current of a 12 V car battery with an internal resistance of 0.020 Ω? What happens to the power supplied by the battery?

**SOLVE** The short-circuit current is

$$I_{\text{short}} = \frac{\mathcal{E}}{r} = \frac{12\ \text{V}}{0.02\ \Omega} = 600\ \text{A}$$

Power is generated by chemical reactions in the battery and dissipated by the load resistance. But with a short-circuited battery, the load resistance is *inside* the battery! The "shorted" battery has to dissipate power $P = I^2 r = 7200$ W *internally*.

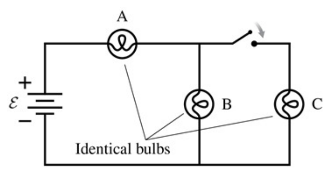
**ASSESS** This value is realistic. Car batteries are designed to drive the starter motor, which has a very small resistance and can draw a current of a few hundred amps. That is why the battery cables are so thick. A shorted car battery can produce an *enormous* amount of current. The normal response of a shorted car battery is to explode; it simply cannot dissipate this much power. Shorting a flashlight battery can make it rather hot, but your life is not in danger. Although the voltage of a car battery is relatively small, a car battery can be dangerous and should be treated with great respect.





## Lightbulb Puzzle #3

- The figure shows three identical lightbulbs in a circuit.
- When the switch is closed, an alternate pathway for the current to get from bulb A back to the battery is created.
- This *decreases* the overall resistance of the circuit, and the brightness of bulb A *increases*.





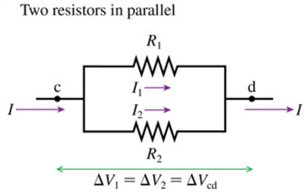



## Parallel Resistors

- The figure below shows two resisters connected *in parallel* between points c and d.
- By Kirchhoff's junction law, the input current is the sum of the current through each resistor: $I = I_1 + I_2$

$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\Delta V_{\text{cd}}}{R_1} + \frac{\Delta V_{\text{cd}}}{R_2} = \Delta V_{\text{cd}}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

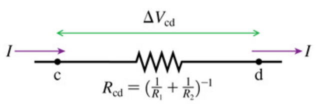
Two resistors in parallel



## Series Resistors

- Suppose we replace $R_1$ and $R_2$ with a single resistor with the same current $I$ and the same potential difference $\Delta V_{cd}$.
- Ohm's law gives resistance between points c and d:

$$R_{cd} = \frac{\Delta V_{cd}}{I} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$




An equivalent resistor


Slide 28-73

## Parallel Resistors

- Resistors connected *at both ends* are called **parallel resistors** or, sometimes, resistors "in parallel."
- The left ends of all the resistors connected in parallel are held at the same potential $V_1$, and the right ends are all held at the same potential $V_2$.
- The potential differences $\Delta V$ are the *same* across all resistors placed in parallel.
- If we have $N$ resistors in parallel, their **equivalent resistance** is

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1} \quad \text{(parallel resistors)}$$

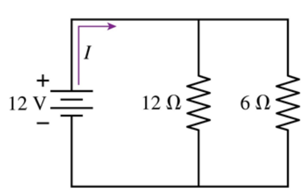
- The behavior of the circuit will be unchanged if the $N$ parallel resistors are replaced by the single resistor $R_{eq}$.


Slide 28-74

## QuickCheck 28.11

The battery current $I$ is

A. 3 A
B. 2 A
C. 1 A
D. 2/3 A
E. 1/2 A



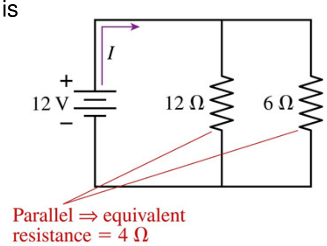


Slide 28-75

## QuickCheck 28.11

The battery current $I$ is

✔ **A. 3 A**
B. 2 A
C. 1 A
D. 2/3 A
E. 1/2 A



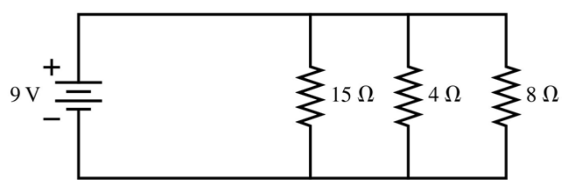

 Slide 28-76

## Example 28.7 A Parallel Resistor Circuit

**EXAMPLE 28.7** | **A parallel resistor circuit**

The three resistors of FIGURE 28.22 are connected to a 9 V battery. Find the potential difference across and the current through each resistor.

**MODEL** The resistors are in parallel. Assume an ideal battery and ideal connecting wires.




 Slide 28-77

## Example 28.7 A Parallel Resistor Circuit

**EXAMPLE 28.7** | **A parallel resistor circuit**

**SOLVE** The three parallel resistors can be replaced by a single equivalent resistor

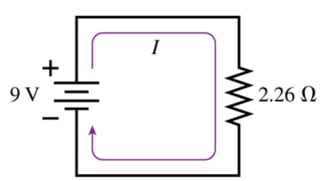
$$R_{eq} = \left(\frac{1}{15\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{8\ \Omega}\right)^{-1} = (0.4417\ \Omega^{-1})^{-1} = 2.26\ \Omega$$

The equivalent circuit is shown in FIGURE 28.23a, from which we find the current to be

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{9\ \text{V}}{2.26\ \Omega} = 3.98\ \text{A}$$

The potential difference across $R_{eq}$ is $\Delta V_{eq} = \mathcal{E} = 9.0$ V. Now we have to be careful. Current $I$ divides at the junction into the smaller currents $I_1$, $I_2$, and $I_3$ shown in FIGURE 28.23b. However, the division is *not* into three equal currents. According to Ohm's law, resistor $i$ has current $I_i = \Delta V_i / R_i$. Because the three resistors are each connected to the battery by ideal wires, as is the equivalent resistor, their potential differences are equal:

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V_{eq} = 9.0\ \text{V}$$




 Slide 28-78

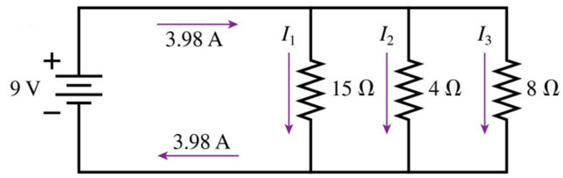
## Example 28.7 A Parallel Resistor Circuit

**EXAMPLE 28.7** A parallel resistor circuit

**SOLVE** Thus the currents are

$$I_1 = \frac{9\text{ V}}{15\ \Omega} = 0.60\text{ A} \qquad I_2 = \frac{9\text{ V}}{4\ \Omega} = 2.25\text{ A} \qquad I_3 = \frac{9\text{ V}}{8\ \Omega} = 1.13\text{ A}$$

**ASSESS** The *sum* of the three currents is 3.98 A, as required by Kirchhoff's junction law.



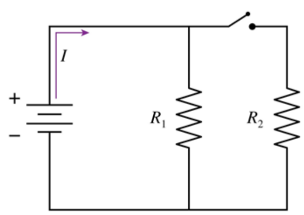


Slide 28-79

## QuickCheck 28.12

When the switch closes, the battery current

A. Increases.
B. Stays the same.
C. Decreases.



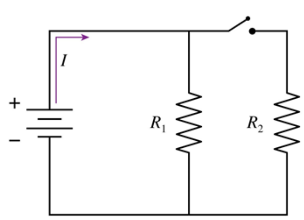


Slide 28-80

## QuickCheck 28.12

When the switch closes, the battery current

✔**A. Increases.**
B. Stays the same.
C. Decreases.



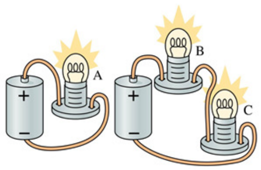

Equivalent resistance decreases.
Potential difference is unchanged.


Slide 28-81

## QuickCheck 28.13

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C



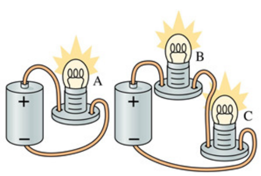



Slide 28-82

## QuickCheck 28.13

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

A. A > B > C
B. A > C > B
✔**C. A > B = C**
D. A < B = C
E. A = B = C



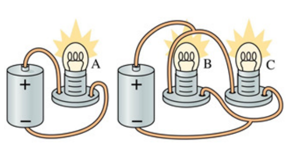



Slide 28-83

## QuickCheck 28.14

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C

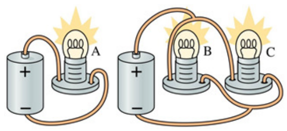





Slide 28-84

## QuickCheck 28.14

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

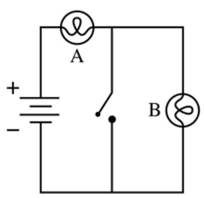



A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
✔ **E. A = B = C**




## QuickCheck 28.15

The lightbulbs are identical. Initially both bulbs are glowing. What happens when the switch is closed?

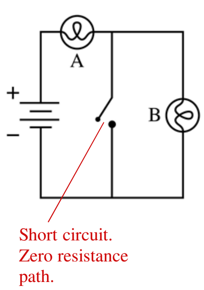



A. Nothing
B. A stays the same; B gets dimmer.
C. A gets brighter; B stays the same.
D. Both get dimmer.
E. A gets brighter; B goes out.



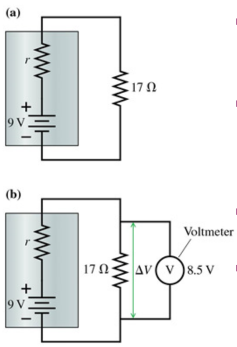

## QuickCheck 28.15

The lightbulbs are identical. Initially both bulbs are glowing. What happens when the switch is closed?




A. Nothing
B. A stays the same; B gets dimmer.
C. A gets brighter; B stays the same.
D. Both get dimmer.
✔ **E. A gets brighter; B goes out.**

## Voltmeters



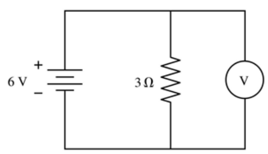

- Figure (a) shows a simple circuit with a resistor and a real battery.
- We can measure the potential difference across the resistor by connecting a voltmeter *in parallel* across the resistor.
- The resistance of the voltmeter must be very high.
- The internal resistance is

$$r = \frac{\mathcal{E} - \Delta V_R}{\Delta V_R} R = \frac{0.5\ \text{V}}{8.5\ \text{V}} 17\ \Omega = 1.0\ \Omega$$


Slide 28-88

## QuickCheck 28.16

What does the voltmeter read?

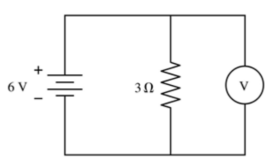



A. 6 V
B. 3 V
C. 2 V
D. Some other value
E. Nothing because this will fry the meter.


Slide 28-89

## QuickCheck 28.16

What does the voltmeter read?



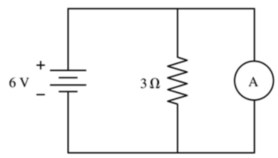

✔ **A. 6 V**
B. 3 V
C. 2 V
D. Some other value
E. Nothing because this will fry the meter.


Slide 28-90

## QuickCheck 28.17

What does the ammeter read?

A. 6 A
B. 3 A
C. 2 A
D. Some other value
E. Nothing because this will fry the meter.



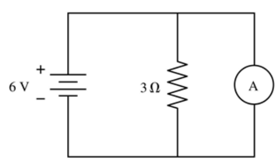


Slide 28-91

## QuickCheck 28.17

What does the ammeter read?

A. 6 A
B. 3 A
C. 2 A
D. Some other value
**E. Nothing because this will fry the meter.** ✔





Slide 28-92

## Problem-Solving Strategy: Resistor Circuits

**PROBLEM-SOLVING STRATEGY 28.1** MP

**Resistor circuits**

**MODEL** Model wires as ideal and, where appropriate, batteries as ideal.
**VISUALIZE** Draw a circuit diagram. Label all known and unknown quantities.


Slide 28-93

## Problem-Solving Strategy: Resistor Circuits

**PROBLEM-SOLVING STRATEGY 28.1**

**Resistor circuits**

**SOLVE** Base your mathematical analysis on Kirchhoff's laws and on the rules for series and parallel resistors.

- Step by step, reduce the circuit to the smallest possible number of equivalent resistors.
- Write Kirchhoff's loop law for each independent loop in the circuit.
- Determine the current through and the potential difference across the equivalent resistors.
- Rebuild the circuit, using the facts that the current is the same through all resistors in series and the potential difference is the same for all parallel resistors.

**ASSESS** Use two important checks as you rebuild the circuit.

- Verify that the sum of the potential differences across series resistors matches $\Delta V$ for the equivalent resistor.
- Verify that the sum of the currents through parallel resistors matches $I$ for the equivalent resistor.

Exercise 26

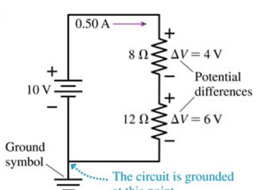



## Getting Grounded

- The earth itself is a conductor.
- If we connect one point of a circuit to the earth by an ideal wire, we can agree to call potential of this point to be that of the earth: $V_{earth} = 0\text{ V}$
- The wire connecting the circuit to the earth is not part of a complete circuit, so there is no current in this wire!
- A circuit connected to the earth in this way is said to be **grounded**, and the wire is called the *ground wire*.
- The circular prong of a three-prong plug is a connection to ground.




## A Circuit That Is Grounded

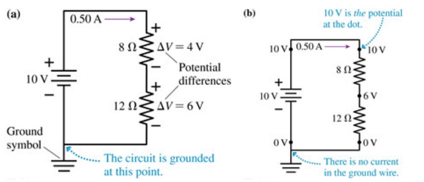



- The figure shows a circuit with a $10\text{ V}$ battery and two resistors in series.
- The symbol beneath the circuit is the ground symbol.
- The potential at the ground is $V = 0$.
- Grounding the circuit allows us to have *specific values* for potential at each point in the circuit, rather than just potential differences.

## Example 28.11 A Grounded Circuit

**EXAMPLE 28.11** | **A grounded circuit**

Suppose the circuit of Figure 28.32 were grounded at the junction between the two resistors instead of at the bottom. Find the potential at each corner of the circuit.



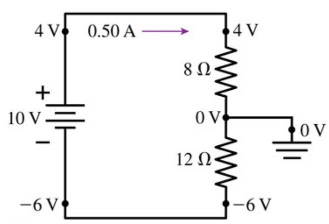




## Example 28.11 A Grounded Circuit

**EXAMPLE 28.11** | **A grounded circuit**

**VISUALIZE** **FIGURE 28.33** shows the new circuit. (It is customary to draw the ground symbol so that its "point" is always down.)



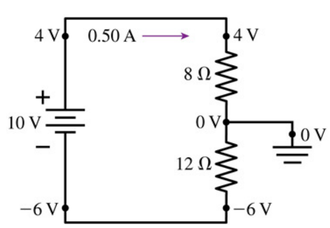




## Example 28.11 A Grounded Circuit

**EXAMPLE 28.11** | **A grounded circuit**

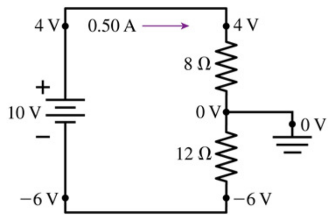
**SOLVE** Changing the ground point does not affect the circuit's behavior. The current is still 0.50 A, and the potential differences across the two resistors are still 4 V and 6 V. All that has happened is that we have moved the $V = 0$ V reference point. Because the earth has $V_{earth} = 0$ V, the junction itself now has a potential of 0 V. The potential decreases by 4 V as charge flows through the 8 Ω resistor. Because it *ends* at 0 V, the potential at the top of the 8 Ω resistor must be +4 V.



## Example 28.11 A Grounded Circuit

**EXAMPLE 28.11** A grounded circuit

**SOLVE** Similarly, the potential decreases by 6 V through the 12 Ω resistor. Because it *starts* at 0 V, the bottom of the 12 Ω resistor must be at −6 V. The negative battery terminal is at the same potential as the bottom of the 12 Ω resistor, because they are connected by a wire, so $V_{neg} = -6$ V. Finally, the potential increases by 10 V as the charge flows through the battery, so $V_{pos} = +4$ V, in agreement, as it should be, with the potential at the top of the 8 Ω resistor.



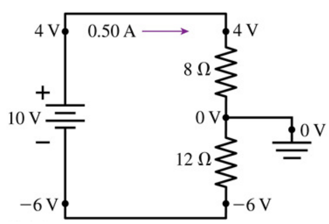




## Example 28.11 A Grounded Circuit

**EXAMPLE 28.11** A grounded circuit

**ASSESS** A negative voltage means only that the potential at that point is less than the potential at some other point that we chose to call $V = 0$ V. Only potential *differences* are physically meaningful, and only potential differences enter into Ohm's law: $I = \Delta V/R$. The potential difference across the 12 Ω resistor in this example is 6 V, decreasing from top to bottom, regardless of which point we choose to call $V = 0$ V.

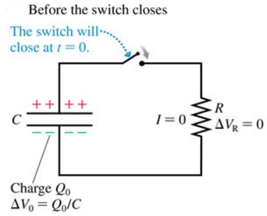






## *RC* Circuits

- The figure shows a charged capacitor, a switch, and a resistor.
- At $t = 0$, the switch closes and the capacitor begins to discharge through the resistor.

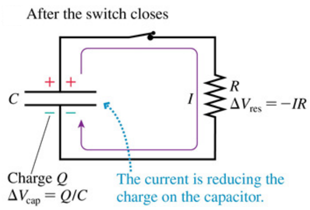



- A circuit such as this, with resistors and capacitors, is called an ***RC* circuit**.
- We wish to determine how the current through the resistor will vary as a function of time after the switch is closed.

## *RC* Circuits

- The figure shows an $RC$ circuit, some time after the switch was closed.
- Kirchhoff's loop law applied to this circuit clockwise is

$$\Delta V_{\text{cap}} + \Delta V_{\text{res}} = \frac{Q}{C} - IR = 0$$




- $Q$ and $I$ in this equation are the instantaneous values of the capacitor charge and the resistor current.
- The resistor current is the rate at which charge is *removed* from the capacitor:

$$I = -\frac{dQ}{dt}$$




## *RC* Circuits

- Knowing that $I = -dQ/dt$, the loop law for a simple closed $RC$ circuit is

$$\frac{dQ}{dt} + \frac{Q}{RC} = 0$$

- Rearranging and integrating:

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt$$

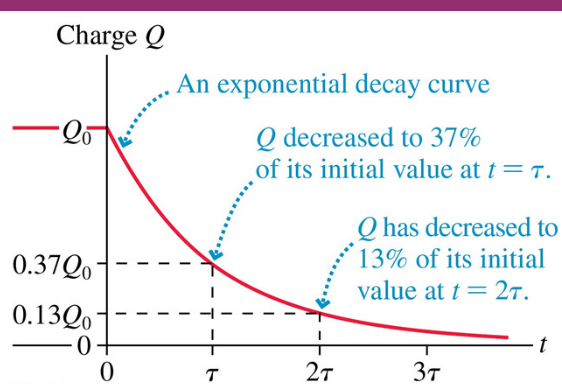
$$Q = Q_0 e^{-t/\tau}$$

where the **time constant** $\tau$ is

$$\tau = RC$$



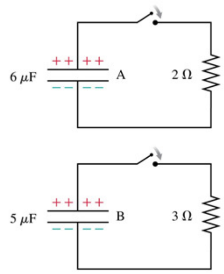

## *RC* Circuits



## QuickCheck 28.18

Which capacitor discharges more quickly after the switch is closed?

A. Capacitor A
B. Capacitor B
C. They discharge at the same rate.
D. Can't say without knowing the initial amount of charge.





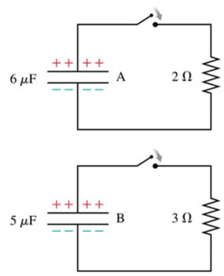
Slide 28-106

## QuickCheck 28.18

Which capacitor discharges more quickly after the switch is closed?

Smaller time constant $\tau = RC$

✔ **A. Capacitor A**
B. Capacitor B
C. They discharge at the same rate.
D. Can't say without knowing the initial amount of charge.





Slide 28-107

## *RC* Circuits

- The charge on the capacitor of an $RC$ circuit is

$$Q = Q_0 e^{-t/\tau}$$

where $Q_0$ is the charge at $t = 0$, and $\tau = RC$ is the time constant.

- The capacitor voltage is directly proportional to the charge, so

$$\Delta V_C = \Delta V_0 e^{-t/\tau}$$

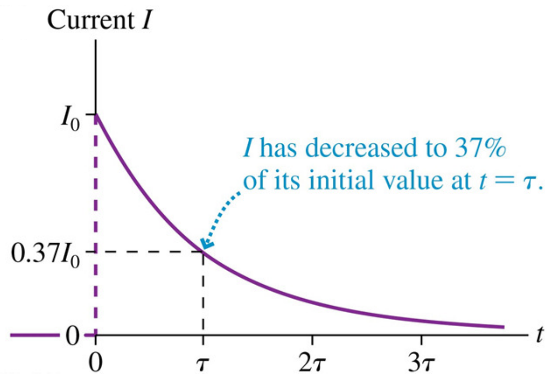
where $\Delta V_0$ is the voltage at $t = 0$.

- The current also can be found to decay exponentially:

$$I = -\frac{dQ}{dt} = \frac{Q_0}{\tau} e^{-t/\tau} = \frac{Q_0}{RC} e^{-t/\tau} = \frac{\Delta V_0}{R} e^{-t/\tau} = I_0 e^{-t/\tau}$$


Slide 28-108

## *RC* Circuits

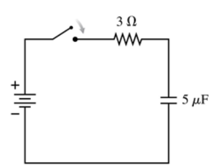





Slide 28-109

## QuickCheck 28.19

The capacitor is initially unchanged. Immediately after the switch closes, the capacitor voltage is



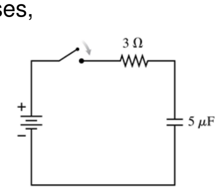

A. 0 V
B. Somewhere between 0 V and 6 V
C. 6 V
D. Undefined.



Slide 28-110

## QuickCheck 28.19

The capacitor is initially unchanged. Immediately after the switch closes, the capacitor voltage is

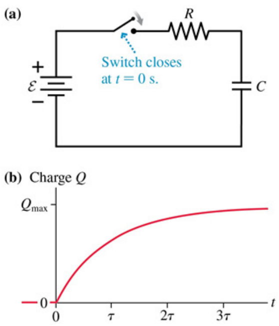



✔ **A. 0 V**
B. Somewhere between 0 V and 6 V
C. 6 V
D. Undefined.



Slide 28-111

## Charging a Capacitor




- Figure (a) shows a circuit that charges a capacitor.
- The capacitor charge and the circuit current at time $t$ are

$$Q = Q_0(1 - e^{-t/\tau})$$
$$I = I_0 e^{-t/\tau}$$

where $I_0 = \mathcal{E}/R$ and $\tau = RC$.

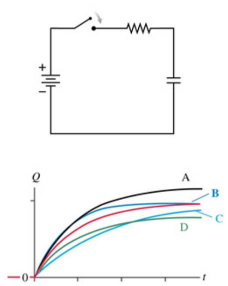
- This "upside-down decay" is shown in figure (b).





## QuickCheck 28.20

The red curve shows how the capacitor charges after the switch is closed at $t = 0$. Which curve shows the capacitor charging if the value of the resistor is reduced?

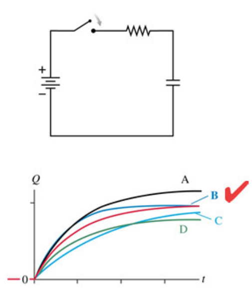







## QuickCheck 28.20

The red curve shows how the capacitor charges after the switch is closed at $t = 0$. Which curve shows the capacitor charging if the value of the resistor is reduced?




Smaller time constant. Same ultimate amount of charge.

Chapter 28 Summary Slides




## General Strategy

**Solving Circuit Problems**

**MODEL** Assume that wires and, where appropriate, batteries are ideal.

**VISUALIZE** Draw a circuit diagram. Label all quantities.

**SOLVE** Base the solution on Kirchhoff's laws.

- Reduce the circuit to the smallest possible number of equivalent resistors.
- Write one loop equation for each independent loop.
- Find the current and the potential difference.
- Rebuild the circuit to find $I$ and $\Delta V$ for each resistor.

**ASSESS** Verify that

- The sum of potential differences across series resistors matches $\Delta V$ for the equivalent resistor.
- The sum of the currents through parallel resistors matches $I$ for the equivalent resistor.

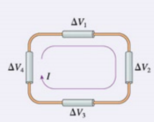



## General Strategy

**Kirchhoff's loop law**

For a closed loop:

- Assign a direction to the current $I$.
- $\sum(\Delta V)_i = 0$

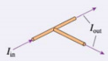


## General Strategy

**Kirchhoff's junction law**

For a junction:

- $\Sigma I_{in} = \Sigma I_{out}$






Slide 28-118

## Important Concepts

**Ohm's Law**

A potential difference $\Delta V$ between the ends of a conductor with resistance $R$ creates a current

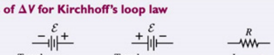
$$I = \frac{\Delta V}{R}$$



Slide 28-119

## Important Concepts

**Signs of $\Delta V$ for Kirchhoff's loop law**




$\Delta V_{bat} = +\mathcal{E}$    $\Delta V_{bat} = -\mathcal{E}$    $\Delta V_{res} = -IR$



Slide 28-120

## Important Concepts

The **energy used by a circuit** is supplied by the emf $\mathcal{E}$ of the battery through the energy transformations

$$E_{chem} \rightarrow U \rightarrow K \rightarrow E_{th}$$

The battery *supplies* energy at the rate

$$P_{bat} = I\mathcal{E}$$

The resistors *dissipate* energy at the rate

$$P_R = I\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R}$$



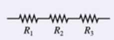

## Applications

**Equivalent resistance**

Groups of resistors can often be reduced to one equivalent resistor.

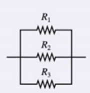
**Series resistors**

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$




**Parallel resistors**

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1}$$







## Applications

***RC* circuits**

The charge on and current through a discharging capacitor are

$$Q = Q_0 e^{-t/\tau}$$

$$I = -\frac{dQ}{dt} = \frac{Q_0}{\tau} e^{-t/\tau} = I_0 e^{-t/\tau}$$

where $\tau = RC$ is the **time constant.**