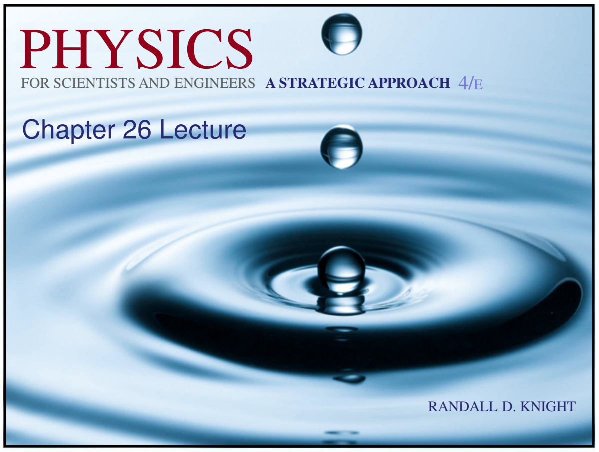


PHYSICS

FOR SCIENTISTS AND ENGINEERS **A STRATEGIC APPROACH** 4/E

Chapter 26 Lecture

RANDALL D. KNIGHT

Chapter 26 Potential and Field

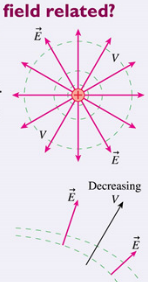
IN THIS CHAPTER, you will learn how the electric potential is related to the electric field.

Chapter 26 Preview

How are electric potential and field related?

The electric field and the electric potential are intimately connected. In fact, they are simply two different perspectives on how source charges alter the space around them.

- The electric potential can be found if you know the electric field.
- The electric field can be found if you know the electric potential.
- Electric field lines are always perpendicular to equipotential surfaces.
- The electric field points "downhill" in the direction of decreasing potential.
- The electric field is stronger where equipotentials are closer together.



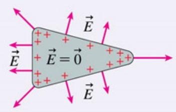
« LOOKING BACK Sections 25.4– 25.6 The electric potential and its graphical representations

Chapter 26 Preview

What are the properties of conductors?

You'll learn about the properties of conductors in electrostatic equilibrium, finding the same results as using Gauss's law:

- Any excess charge is on the surface.
- The interior electric field is zero.
- The exterior electric field is perpendicular to the surface.
- The entire conductor is an equipotential.



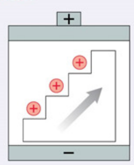
Slide 26-4

Chapter 26 Preview

What are sources of electric potential?

A potential difference—voltage—is created by separating positive and negative charges.

- Work must be done to separate charges. The work done per charge is called the emf of a device. Emf is measured in volts.
- We'll use a charge escalator model of a battery in which chemical reactions "lift" charges from one terminal to the other.



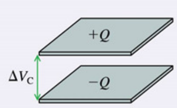
Slide 26-5

Chapter 26 Preview

What is a capacitor?

Any two electrodes with equal and opposite charges form a capacitor. Their capacitance indicates their capacity for storing charge. The energy stored in a capacitor will lead us to recognize that electric energy is stored in the electric field.

« LOOKING BACK Section 23.5 Parallel-plate capacitors



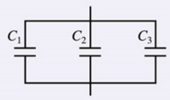
Slide 26-6

Chapter 26 Preview

How are capacitors used?

Capacitors are important circuit elements that store charge and energy.

- You'll learn to work with combinations of capacitors arranged in series and parallel.
- You'll learn that an insulator—called a dielectric—between the capacitor plates alters the capacitor in useful ways.



Chapter 26 Reading Questions

Reading Question 26.1

What quantity is represented by the symbol $\mathcal{E}$?

A. Electronic potential
B. Excitation potential
C. emf
D. Electric stopping power
E. Exosphericity

Reading Question 26.1

What quantity is represented by the symbol $\mathcal{E}$?

A. Electronic potential
B. Excitation potential
✔ **C. emf**
D. Electric stopping power
E. Exosphericity

Slide 26-10

Reading Question 26.2

What is the SI unit of capacitance?

A. Capaciton
B. Faraday
C. Hertz
D. Henry
E. Exciton

Slide 26-11

Reading Question 26.2

What is the SI unit of capacitance?

A. Capaciton
✔ **B. Faraday**
C. Hertz
D. Henry
E. Exciton

Slide 26-12

Reading Question 26.3

The electric field

A. Is always perpendicular to an equipotential surface.
B. Is always tangent to an equipotential surface.
C. Always bisects an equipotential surface.
D. Makes an angle to an equipotential surface that depends on the amount of charge.

 Slide 26-13

Reading Question 26.3

The electric field

✔**A. Is always perpendicular to an equipotential surface.**
B. Is always tangent to an equipotential surface.
C. Always bisects an equipotential surface.
D. Makes an angle to an equipotential surface that depends on the amount of charge.

 Slide 26-14

Reading Question 26.4

This chapter investigated

A. Parallel capacitors.
B. Perpendicular capacitors.
C. Series capacitors.
D. Both A and B.
E. Both A and C.

 Slide 26-15

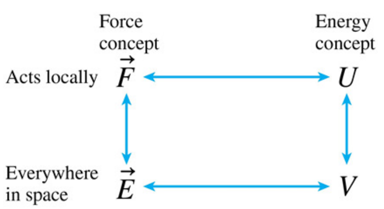
Reading Question 26.4

This chapter investigated

A. Parallel capacitors.
B. Perpendicular capacitors.
C. Series capacitors.
D. Both A and B.
✔**E. Both A and C.**

Chapter 26 Content, Examples, and QuickCheck Questions

Connecting Potential and Field



- The figure shows the four key ideas of force, field, potential energy, and potential.
- We know, from Chapters 9 and 10, that **force** and **potential energy** are closely related.
- The focus of this chapter is to establish a similar relationship between the **electric field** and the **electric potential**.

Finding the Potential from the Electric Field

- The potential difference between two points in space is

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s\, ds = -\int_i^f \vec{E} \cdot d\vec{s}$$

where s is the position along a line from point i to point f.

- We can find the potential difference between two points if we know the electric field.
- Thus a graphical interpretation of the equation above is

$V_f = V_i -$ (area under the E_s-versus-s curve between s_i and s_f)

Slide 26-19

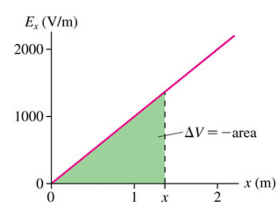
Example 26.1 Finding the Potential

EXAMPLE 26.1 **Finding the potential**

FIGURE 26.2 is a graph of E_x, the x-component of the electric field, versus position along the x-axis. Find and graph $V(x)$. Assume $V = 0$ V at $x = 0$ m.

MODEL The potential difference is the *negative* of the area under the curve.

VISUALIZE E_x is positive throughout this region of space, meaning that $\vec{E}$ points in the positive x-direction.



Slide 26-20

Example 26.1 Finding the Potential

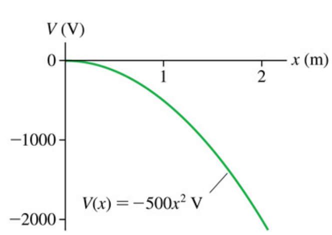
EXAMPLE 26.1 **Finding the potential**

SOLVE We can see that $E_x = 1000x$ V/m, where x is in m. Thus

$$V_f = V(x) = 0 - (\text{area under the } E_x \text{ curve})$$
$$= -\tfrac{1}{2} \times \text{base} \times \text{height} = -\tfrac{1}{2}(x)(1000x) = -500x^2 \text{ V}$$

FIGURE 26.3 shows that the electric potential in this region of space is parabolic, decreasing from 0 V at $x = 0$ m to -2000 V at $x = 2$ m.

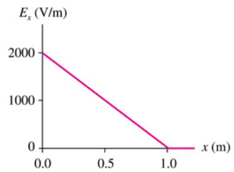
ASSESS The electric field points in the direction in which V is *decreasing*. We'll soon see that this is a general rule.



Slide 26-21

QuickCheck 26.1

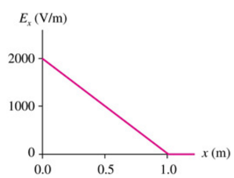
This is a graph of the x-component of the electric field along the x-axis. The potential is zero at the origin. What is the potential at $x = 1\text{m}$?



A. 2000 V
B. 1000 V
C. 0 V
D. –1000 V
E. –2000 V

QuickCheck 26.1

This is a graph of the x-component of the electric field along the x-axis. The potential is zero at the origin. What is the potential at $x = 1\text{m}$?



A. 2000 V
B. 1000 V
C. 0 V
✔D. **–1000 V** ΔV = –area under curve
E. –2000 V

Tactics: Finding the Potential From the Electric Field

TACTICS BOX 26.1

Finding the potential from the electric field

1. Draw a picture and identify the point at which you wish to find the potential. Call this position f.
2. Choose the zero point of the potential, often at infinity. Call this position i.
3. Establish a coordinate axis from i to f along which you already know or can easily determine the electric field component E_s.
4. Carry out the integration of Equation 26.3 to find the potential.

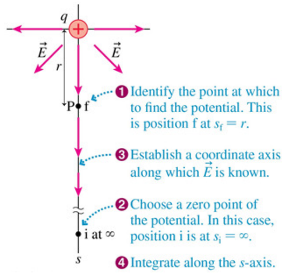
Exercise 1

Finding the Potential of a Point Charge

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{q}{s^2}$$

$$V(r) = V(\infty) + \frac{q}{4\pi\epsilon_0}\int_r^{\infty}\frac{ds}{s^2}$$

$$V_{\text{point charge}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



Slide 26-25

Example 26.2 The Potential of a Parallel-Plate Capacitor

EXAMPLE 26.2 **The potential of a parallel-plate capacitor**

In Chapter 23, the electric field inside a capacitor was found to be

$$\vec{E} = \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

Find the electric potential inside the capacitor. Let $V = 0$ V at the negative plate.

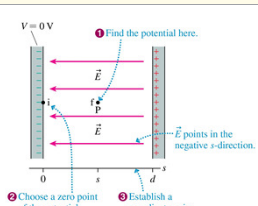
MODEL The electric field inside a capacitor is a uniform field.

Slide 26-26

Example 26.2 The Potential of a Parallel-Plate Capacitor

EXAMPLE 26.2 **The potential of a parallel-plate capacitor**

VISUALIZE **FIGURE 26.5** shows the capacitor and establishes a point P where we want to find the potential. We've chosen an s-axis measured from the negative plate, which is the zero point of the potential.



Slide 26-27

Example 26.2 The Potential of a Parallel-Plate Capacitor

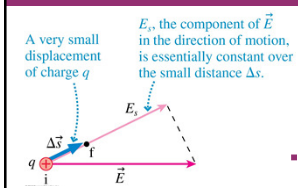
EXAMPLE 26.2 **The potential of a parallel-plate capacitor**

SOLVE We'll integrate along the s-axis from $s_i = 0$ (where $V_i = 0$ V) to $s_f = s$. Notice that $\vec{E}$ points in the negative s-direction, so $E_s = -Q/\epsilon_0 A$. $Q/\epsilon_0 A$ is a constant, so

$$V(s) = V_f = V_i - \int_0^s E_s\,ds = -\left(-\frac{Q}{\epsilon_0 A}\right)\int_0^s ds = \frac{Q}{\epsilon_0 A}s = Es$$

ASSESS $V = Es$ is the capacitor potential we deduced in Chapter 25 by working directly with the potential energy. The potential increases linearly from $V = 0$ at the negative plate to $V = Ed$ at the positive plate. Here we found the potential by explicitly recognizing the connection between the potential and the field.

Finding the Electric Field from the Potential



- The figure shows two points i and f separated by a small distance Δs.
- The work done by the electric field as a small charge q moves from i to f is $W = F_s\Delta s = qE_s\Delta s$.
- The potential difference between the points is $\Delta V = \dfrac{\Delta U_{q+\text{sources}}}{q} = \dfrac{-W}{q} = -E_s\,\Delta s$
- The electric field in the s-direction is $E_s = -\Delta V/\Delta s$. In the limit $\Delta s \rightarrow 0$: $$E_s = -\frac{dV}{ds}$$

Finding the Electric Field from the Potential: Quick Example

- Suppose we knew the potential of a point charge to be $V = q/4\pi\epsilon_0 r$ but didn't remember the electric field.
- Symmetry requires that the field point straight outward from the charge, with only a radial component E_r.
- If we choose the s-axis to be in the radial direction, parallel to $\vec{E}$, we find

$$E_r = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{q}{4\pi\epsilon_0 r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

- This is, indeed, the well-known electric field of a point charge!

Example 26.3 The Electric Field of a Ring of Charge

EXAMPLE 26.3 | **The electric field of a ring of charge**

In Chapter 25, we found the on-axis potential of a ring of radius R and charge Q to be

$$V_{\text{ring}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{z^2+R^2}}$$

Find the on-axis electric field of a ring of charge.

Slide 26-31

Example 26.3 The Electric Field of a Ring of Charge

EXAMPLE 26.3 | **The electric field of a ring of charge**

SOLVE Symmetry requires the electric field along the axis to point straight outward from the ring with only a z-component E_z. The electric field at position z is

$$E_z = -\frac{dV}{dz} = -\frac{d}{dz}\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{z^2+R^2}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

ASSESS This result is in perfect agreement with the electric field we found in Chapter 23, but this calculation was easier because we didn't have to deal with angles.

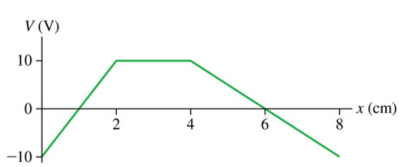
Slide 26-32

Example 26.4 Finding *E* From the Slope of *V*

EXAMPLE 26.4 | **Finding *E* from the slope of *V***

FIGURE 26.7 is a graph of the electric potential in a region of space where $\vec{E}$ is parallel to the x-axis. Draw a graph of E_x versus x.

MODEL The electric field is the *negative* of the slope of the potential graph.



Slide 26-33

Example 26.4 Finding E From the Slope of V

EXAMPLE 26.4 Finding E from the slope of V

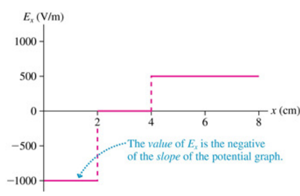
SOLVE There are three regions of different slope:

$$0 < x < 2 \text{ cm} \quad \begin{cases} \Delta V/\Delta x = (20 \text{ V})/(0.020 \text{ m}) = 1000 \text{ V/m} \\ E_x = -1000 \text{ V/m} \end{cases}$$

$$2 < x < 4 \text{ cm} \quad \begin{cases} \Delta V/\Delta x = 0 \text{ V/m} \\ E_x = 0 \text{ V/m} \end{cases}$$

$$4 < x < 8 \text{ cm} \quad \begin{cases} \Delta V/\Delta x = (-20 \text{ V})/(0.040 \text{ m}) = -500 \text{ V/m} \\ E_x = 500 \text{ V/m} \end{cases}$$

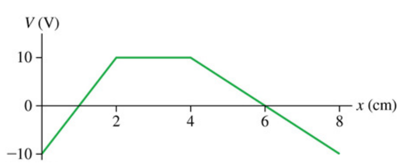
The results are shown in FIGURE 26.8.



Example 26.4 Finding E From the Slope of V

EXAMPLE 26.4 Finding E from the slope of V

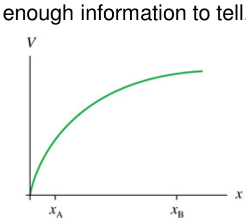
ASSESS The electric field $\vec{E}$ points to the left (E_x is negative) for $0 < x < 2$ cm and to the right (E_x is positive) for $4 < x < 8$ cm. Notice that **the electric field is zero in a region of space where the potential is not changing.**



QuickCheck 26.2

At which point is the electric field stronger?

A. At x_A
B. At x_B
C. The field is the same strength at both.
D. There's not enough information to tell.



QuickCheck 26.2

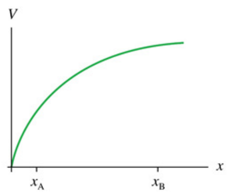
At which point is the electric field stronger?

✔**A. At x_A** $|E|$ = slope of potential graph

B. At x_B

C. The field is the same strength at both.

D. There's not enough information to tell.



QuickCheck 26.3

An electron is released from rest at $x = 2$ m in the potential shown. What does the electron do right after being released?

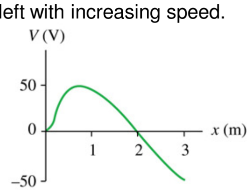
A. Stay at $x = 2$ m

B. Move to the right ($+x$) at steady speed.

C. Move to the right with increasing speed.

D. Move to the left ($-x$) at steady speed.

E. Move to the left with increasing speed.



QuickCheck 26.3

An electron is released from rest at $x = 2$ m in the potential shown. What does the electron do right after being released?

A. Stay at $x = 2$ m

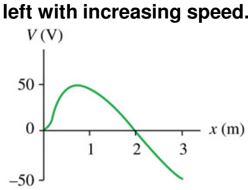
B. Move to the right ($+x$) at steady speed.

C. Move to the right with increasing speed.

D. Move to the left ($-x$) at steady speed.

✔**E. Move to the left with increasing speed.**

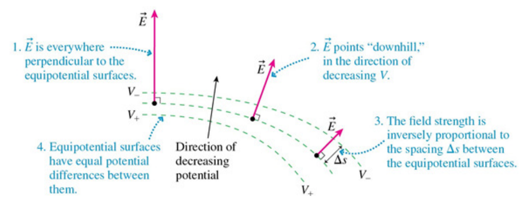
Slope of V negative => E_x is positive (field to the right). Electron is negative => force to the left. Force to the left => acceleration to the left.



The Geometry of Potential and Field

- In three dimensions, we can find the electric field from the electric potential as

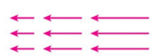
$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

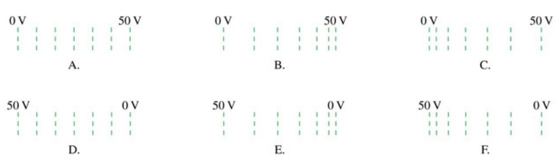


Slide 26-40

QuickCheck 26.4

Which set of equipotential surfaces matches this electric field?

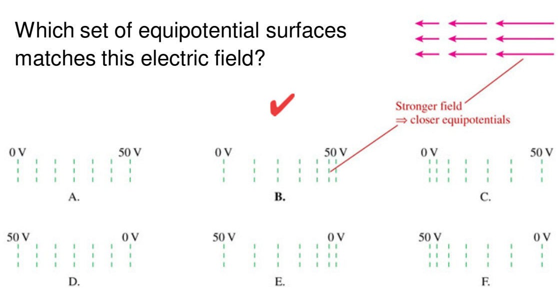




Slide 26-41

QuickCheck 26.4

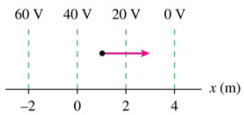
Which set of equipotential surfaces matches this electric field?



Slide 26-42

QuickCheck 26.5

The electric field at the dot is

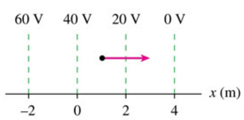


A. $10\hat{\imath}$ V/m
B. $-10\hat{\imath}$ V/m
C. $20\hat{\imath}$ V/m
D. $30\hat{\imath}$ V/m
E. $-30\hat{\imath}$ V/m

Slide 26-43

QuickCheck 26.5

The electric field at the dot is



✔**A. $10\hat{\imath}$ V/m**
B. $-10\hat{\imath}$ V/m
C. $20\hat{\imath}$ V/m
D. $30\hat{\imath}$ V/m
E. $-30\hat{\imath}$ V/m

20 V over 2 m, pointing toward lower potential

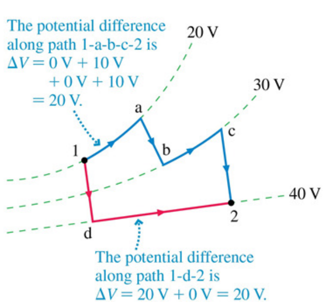
Slide 26-44

Kirchhoff's Loop Law

- For any path that starts and ends at the same point:

$$\Delta V_{loop} = \sum_i (\Delta V)_i = 0$$

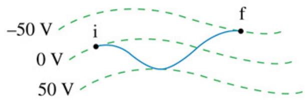
- The sum of all the potential differences encountered while moving around a loop or closed path is zero.
- This statement is known as **Kirchhoff's loop law**.



Slide 26-45

QuickCheck 26.6

A particle follows the trajectory shown from initial position i to final position f. The potential difference ΔV *is*

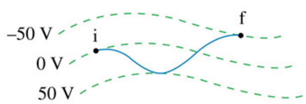


A. 100 V
B. 50 V
C. 0 V
D. −50 V
E. −100 V

Slide 26-46

QuickCheck 26.6

A particle follows the trajectory shown from initial position i to final position f. The potential difference ΔV *is*

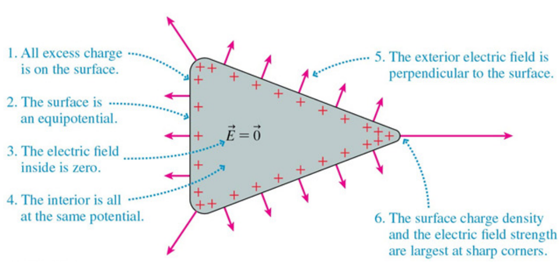


A. 100 V
B. 50 V
C. 0 V
✔D. −50 V
E. −100 V

$\Delta V = V_{\text{final}} - V_{\text{initial}}$, independent of the path

Slide 26-47

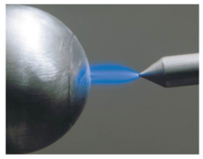
A Conductor in Electrostatic Equilibrium



Slide 26-48

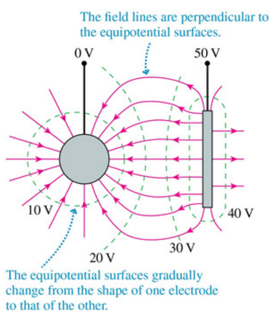
A Conductor in Electrostatic Equilibrium

- When a conductor is in equilibrium:
  - All excess charge sits on the surface.
  - The surface is an equipotential.
  - The electric field inside is zero.
  - The external electric field is perpendicular to the surface at the surface.
  - The electric field is strongest at sharp corners of the conductor's surface.



A corona discharge occurs at pointed metal tips where the electric field can be very strong.

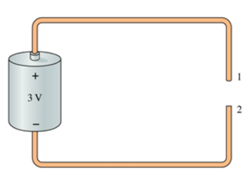
A Conductor in Electrostatic Equilibrium



- The figure shows a negatively charged metal sphere near a flat metal plate.
- Since a conductor surface must be an equipotential, the equipotential surfaces close to each electrode roughly match the shape of the electrode.

QuickCheck 26.7

Metal wires are attached to the terminals of a 3 V battery. What is the potential difference between points 1 and 2?



A. 6 V

B. 3 V

C. 0 V

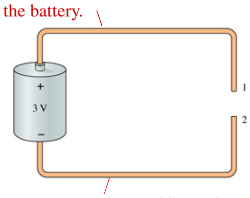
D. Undefined.

E. Not enough information to tell.

QuickCheck 26.7

Metal wires are attached to the terminals of a 3 V battery. What is the potential difference between points 1 and 2?

A. 6 V
✔**B. 3 V**
C. 0 V
D. Undefined.
E. Not enough information to tell.

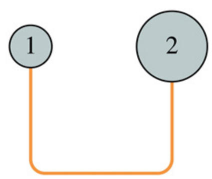


Slide 26-52

QuickCheck 26.8

Metal spheres 1 and 2 are connected by a metal wire. What quantities do spheres 1 and 2 have in common?

A. Same potential
B. Same electric field
C. Same charge
D. Both A and B
E. Both A and C

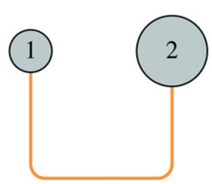


Slide 26-53

QuickCheck 26.8

Metal spheres 1 and 2 are connected by a metal wire. What quantities do spheres 1 and 2 have in common?

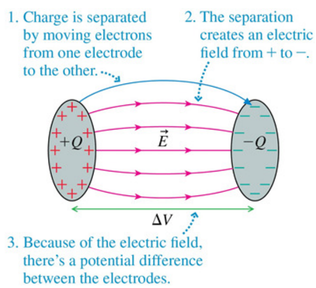
✔**A. Same potential**
B. Same electric field
C. Same charge
D. Both A and B
E. Both A and C



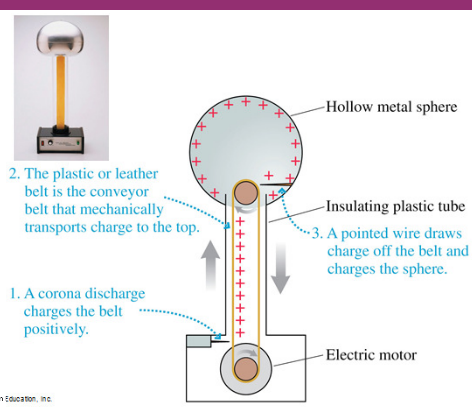
Slide 26-54

Sources of Electric Potential

- A *separation of charge* creates an electric potential difference.
- Shuffling your feet on the carpet transfers electrons from the carpet to you, creating a potential difference between you and other objects in the room.
- This potential difference can cause sparks.



Van de Graaff Generator



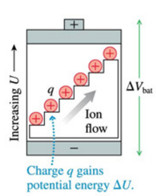
Charge escalator model of a battery

MODEL 26.1

Charge escalator model of a battery

A battery uses chemical reactions to separate charge.

- The charge escalator "lifts" positive charges from the negative terminal to the positive terminal. This requires *work*, with the energy being supplied by the chemical reactions.
- The work done *per charge* is called the **emf** of the battery: $\mathcal{E} = W_{chem}/q$.
- The charge separation creates a potential difference ΔV_{bat} between the terminals. An *ideal battery* has $\Delta V_{bat} = \mathcal{E}$.
- Limitations: $\Delta V_{bat} < \mathcal{E}$ if current flows through the battery. In most cases, the difference is small and a battery can be considered ideal.



Batteries and emf

- **emf** is the work done per charge to pull positive and negative charges apart.
- In an ideal battery, this work creates a potential difference $\Delta V_{bat} = \mathcal{E}$ between the positive and negative terminals.
- This is called the terminal voltage.

A battery constructed to have an emf of 1.5 V creates a 1.5 V potential difference between its positive and negative terminals.

 Slide 26-58

QuickCheck 26.9

The charge escalator in a battery does 4.8×10^{-19} J of work for each positive ion that it moves from the negative to the positive terminal. What is the battery's emf?

A. 9 V

B. 4.8 V

C. 3 V

D. 4.8×10^{-19} V

E. I have no idea.

 Slide 26-59

QuickCheck 26.9

The charge escalator in a battery does 4.8×10^{-19} J of work for each positive ion that it moves from the negative to the positive terminal. What is the battery's emf?

A. 9 V

B. 4.8 V

✔**C. 3 V** $\mathcal{E} = \frac{W}{q}$ and $q = e = 1.6 \times 10^{-19}$ C for an ion.

D. 4.8×10^{-19} V

E. I have no idea.

 Slide 26-60

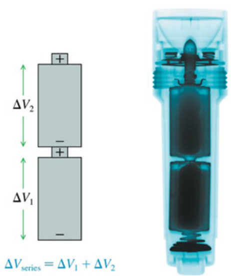
Batteries in Series

- The total potential difference of batteries in series is simply the sum of their individual terminal voltages:

$$\Delta V_{\text{series}} = \Delta V_1 + \Delta V_2 + \cdots$$

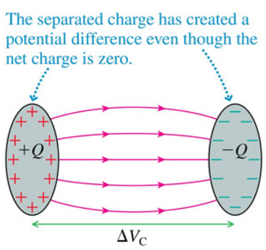
- Flashlight batteries are placed in series to create twice the potential difference of one battery.
- For this flashlight:

$$\Delta V_{\text{series}} = \Delta V_1 + \Delta V_2$$
$$= 1.5\text{ V} + 1.5\text{ V}$$
$$= 3.0\text{ V}$$



Slide 26-61

Capacitance and Capacitors



- The figure shows two arbitrary electrodes charged to $\pm Q$.
- There is a potential difference ΔV_C that is directly proportional to Q.
- The ratio of the charge Q to the potential difference ΔV_C is called the **capacitance** C:

$$C \equiv \frac{Q}{\Delta V_C} = \frac{\epsilon_0 A}{d} \qquad \text{(parallel-plate capacitor)}$$

Slide 26-62

Capacitance and Capacitors

- Capacitance is a purely *geometric* property of two electrodes because it depends only on their surface area and spacing.
- The SI unit of capacitance is the **farad**:

$$1\text{ farad} = 1\text{ F} = 1\text{ C/V}$$

- **The charge on the capacitor plates is directly proportional to the potential difference between the plates:**

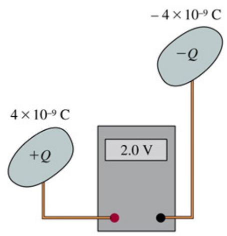
$$Q = C\,\Delta V_C \qquad \text{(charge on a capacitor)}$$

Slide 26-63

QuickCheck 26.10

What is the capacitance of these two electrodes?

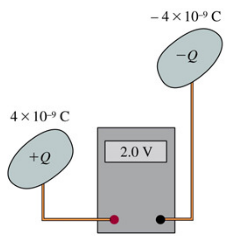
A. 8 nF
B. 4 nF
C. 2 nF
D. 1 nF
E. Some other value



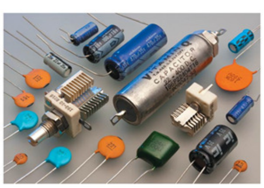
QuickCheck 26.10

What is the capacitance of these two electrodes?

A. 8 nF
B. 4 nF
✔**C. 2 nF** $C = \frac{Q}{\Delta V}$
D. 1 nF
E. Some other value



Capacitance and Capacitors



Capacitors are important elements in electric circuits. They come in a variety of sizes and shapes.

The keys on most computer keyboards are capacitor switches. Pressing the key pushes two capacitor plates closer together, increasing their capacitance.

Example 26.6 Charging a Capacitor

EXAMPLE 26.6 | Charging a capacitor

The spacing between the plates of a 1.0 μF capacitor is 0.050 mm.

a. What is the surface area of the plates?

b. How much charge is on the plates if this capacitor is charged to 1.5 V?

MODEL Assume the capacitor is a parallel-plate capacitor.

SOLVE a. From the definition of capacitance,

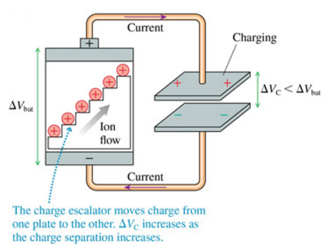
$$A = \frac{dC}{\epsilon_0} = 5.65\ \text{m}^2$$

b. The charge is $Q = C\,\Delta V_C = 1.5 \times 10^{-6}\ \text{C} = 1.5\ \mu\text{C}$.

ASSESS The surface area needed to construct a 1.0 μF capacitor (a fairly typical value) is enormous. We'll see in Section 26.7 how the area can be reduced by inserting an insulator between the capacitor plates.

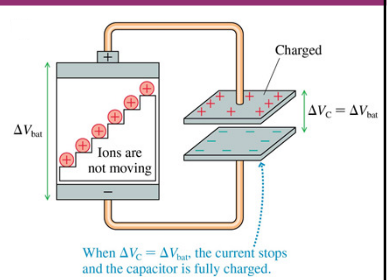
Charging a Capacitor

- The figure shows a capacitor *just after* it has been connected to a battery.
- Current will flow in this manner for a nanosecond or so until the capacitor is fully charged.



Charging a Capacitor

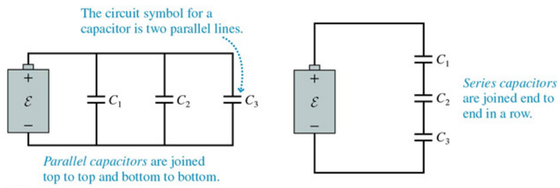
- The figure shows a *fully charged* capacitor.
- Now the system is in electrostatic equilibrium.



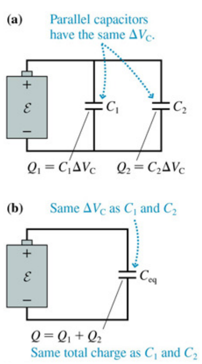
- Capacitance always refers to the charge per voltage on a *fully charged* capacitor.

Combinations of Capacitors

- In practice, two or more capacitors are sometimes joined together.
- The circuit diagrams below illustrate two basic combinations: **parallel capacitors** and **series capacitors**.



Capacitors Combined in Parallel



- Consider two capacitors C_1 and C_2 connected in parallel.
- The total charge drawn from the battery is $Q = Q_1 + Q_2$.
- In figure (b) we have replaced the capacitors with a single "equivalent" capacitor:

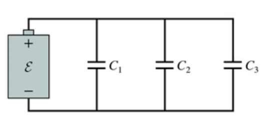
$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C}$$

$$C_{eq} = C_1 + C_2$$

Capacitors Combined in Parallel

- If capacitors C_1, C_2, C_3, … are in parallel, their equivalent capacitance is:

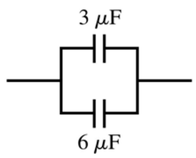
$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad \text{(parallel capacitors)}$$



QuickCheck 26.11

The equivalent capacitance is

A. 9 μF
B. 6 μF
C. 3 μF
D. 2 μF
E. 1 μF

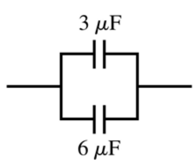


Slide 26-73

QuickCheck 26.11

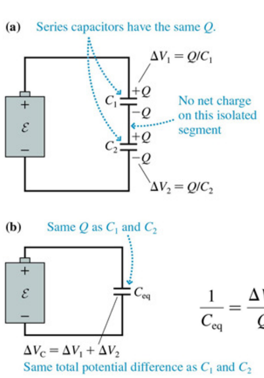
The equivalent capacitance is

✔ A. **9 μF** Parallel => add
B. 6 μF
C. 3 μF
D. 2 μF
E. 1 μF



Slide 26-74

Capacitors Combined in Series

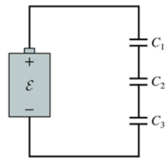


- Consider two capacitors C_1 and C_2 connected in series.
- The total potential difference across both capacitors is $\Delta V_C = \Delta V_1 + \Delta V_2$.
- The inverse of the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

Slide 26-75

Capacitors Combined in Series

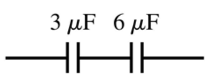


- If capacitors C_1, C_2, C_3, … are in series, their equivalent capacitance is

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1} \quad \text{(series capacitors)}$$

QuickCheck 26.12

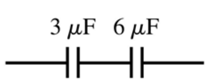
The equivalent capacitance is



A. 9 μF
B. 6 μF
C. 3 μF
D. 2 μF
E. 1 μF

QuickCheck 26.12

The equivalent capacitance is



A. 9 μF
B. 6 μF
C. 3 μF
✔**D. 2 μF** Series => inverse of sum of inverses
E. 1 μF

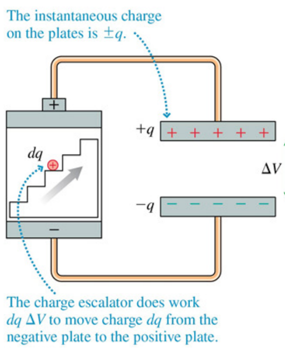
The Energy Stored in a Capacitor

- The figure shows a capacitor being charged.
- As a small charge *dq* is lifted to a higher potential, the potential energy of the capacitor increases by

$$dU = dq\,\Delta V = \frac{q\,dq}{C}$$

- The total energy transferred from the battery to the capacitor is

$$U_C = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$



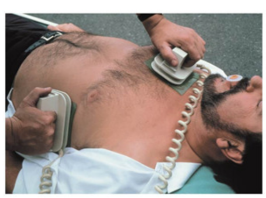
The Energy Stored in a Capacitor

- Capacitors are important elements in electric circuits because of their ability to store energy.
- The charge on the two plates is $\pm q$ and this charge separation establishes a potential difference $\Delta V = q/C$ between the two electrodes.
- In terms of the capacitor's potential difference, the potential energy stored in a capacitor is

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_C)^2$$

The Energy Stored in a Capacitor

- A capacitor can be charged slowly but then can release the energy very quickly.
- An important medical application of capacitors is the *defibrillator*.



- A heart attack or a serious injury can cause the heart to enter a state known as *fibrillation* in which the heart muscles twitch randomly and cannot pump blood.
- A strong electric shock through the chest completely stops the heart, giving the cells that control the heart's rhythm a chance to restore the proper heartbeat.

QuickCheck 26.13

A capacitor charged to 1.5 V stores 2.0 mJ of energy. If the capacitor is charged to 3.0 V, it will store

A. 1.0 mJ
B. 2.0 mJ
C. 4.0 mJ
D. 6.0 mJ
E. 8.0 mJ

Slide 26-82

QuickCheck 26.13

A capacitor charged to 1.5 V stores 2.0 mJ of energy. If the capacitor is charged to 3.0 V, it will store

A. 1.0 mJ
B. 2.0 mJ
C. 4.0 mJ
D. 6.0 mJ
✔E. **8.0 mJ** $U_C \propto (\Delta V)^2$

Slide 26-83

Example 26.8 Storing Energy in a Capacitor

EXAMPLE 26.8 Storing energy in a capacitor

How much energy is stored in a 220 μF camera-flash capacitor that has been charged to 330 V? What is the average power dissipation if this capacitor is discharged in 1.0 ms?

Slide 26-84

Example 26.8 Storing Energy in a Capacitor

EXAMPLE 26.8 | **Storing energy in a capacitor**

SOLVE The energy stored in the charged capacitor is

$$U_C = \tfrac{1}{2}C(\Delta V_C)^2 = \tfrac{1}{2}(220 \times 10^{-6}\ \text{F})(330\ \text{V})^2 = 12\ \text{J}$$

If this energy is released in 1.0 ms, the average power dissipation is

$$P = \frac{\Delta E}{\Delta t} = \frac{12\ \text{J}}{1.0 \times 10^{-3}\ \text{s}} = 12{,}000\ \text{W}$$

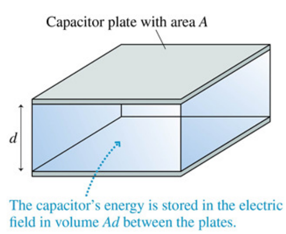
ASSESS The stored energy is equivalent to raising a 1 kg mass 1.2 m. This is a rather large amount of energy, which you can see by imagining the damage a 1 kg mass could do after falling 1.2 m. When this energy is released very quickly, which is possible in an electric circuit, it provides an *enormous* amount of power.

The Energy in the Electric Field

- The **energy density** of an electric field, such as the one inside a capacitor, is:

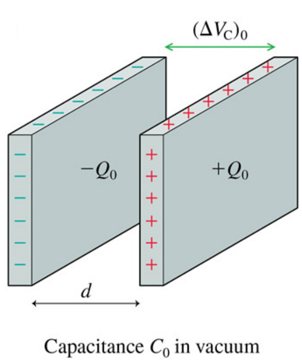
$$u_E = \frac{\text{energy stored}}{\text{volume in which it is stored}} = \frac{U_C}{Ad} = \frac{\epsilon_0}{2}E^2$$

- The energy density has units J/m^3.



Dielectrics

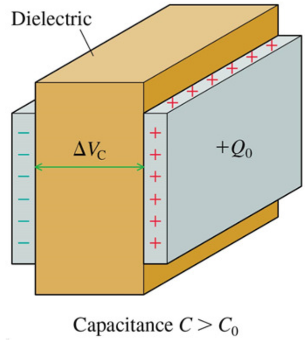
- The figure shows a parallel-plate capacitor with the plates separated by a vacuum.
- When the capacitor is fully charged to voltage $(\Delta V_C)_0$, the charge on the plates will be $\pm Q_0$, where $Q_0 = C_0(\Delta V_C)_0$.
- In this section the subscript 0 refers to a vacuum-filled capacitor.



Dielectrics

- Now an insulating material is slipped between the capacitor plates.
- An insulator in an electric field is called a dielectric.
- The charge on the capacitor plates does not change ($Q = Q_0$).
- However, the voltage has *decreased*:

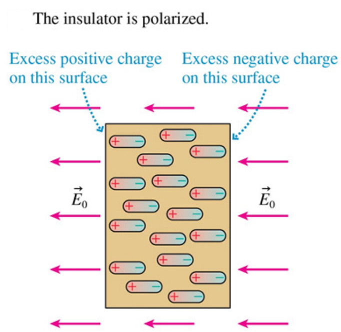
$$\Delta V_C < (\Delta V_C)_0$$



Slide 26-88

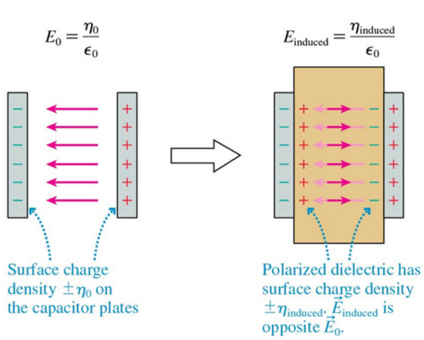
Dielectrics

- The figure shows how an insulating material becomes polarized in an external electric field.
- The insulator as a whole is still neutral, but the external electric field separates positive and negative charge.



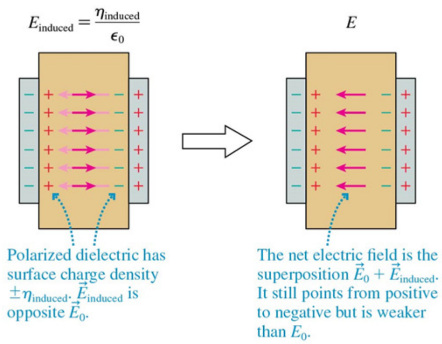
Slide 26-89

Dielectrics



Slide 26-90

Dielectrics



 Slide 26-91

Dielectrics

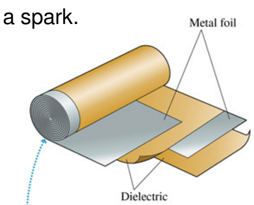
- We define the **dielectric constant**: $\kappa \equiv \frac{E_0}{E}$
- The dielectric constant, like density or specific heat, is a property of a material.
- Easily polarized materials have larger dielectric constants than materials not easily polarized.
- Vacuum has $\kappa = 1$ exactly.
- **Filling a capacitor with a dielectric increases the capacitance by a factor equal to the dielectric constant:**

$$C = \frac{Q}{\Delta V_C} = \frac{Q_0}{(\Delta V_C)_0/\kappa} = \kappa\frac{Q_0}{(\Delta V_C)_0} = \kappa C_0$$

 Slide 26-92

Dielectrics

- The production of a practical capacitor, as shown, almost always involves the use of a solid or liquid dielectric.
- All materials have a maximum electric field they can sustain without breakdown—the production of a spark.
- The breakdown electric field of air is about 3×10^6 V/m.
- A material's maximum sustainable electric field is called its **dielectric strength**.



Many real capacitors are a rolled-up sandwich of metal foils and thin, insulating dielectrics.

 Slide 26-93

Dielectrics

TABLE 26.1 Properties of dielectrics

Material	Dielectric constant κ	Dielectric strength E_{max} (10^6 V/m)
Vacuum	1	—
Air (1 atm)	1.0006	3
Teflon	2.1	60
Polystyrene plastic	2.6	24
Mylar	3.1	7
Paper	3.7	16
Pyrex glass	4.7	14
Pure water (20°C)	80	—
Titanium dioxide	110	6
Strontium titanate	300	8

Example 26.9 A Water-Filled Capacitor

EXAMPLE 26.9 A water-filled capacitor

A 5.0 nF parallel-plate capacitor is charged to 160 V. It is then disconnected from the battery and immersed in distilled water. What are (a) the capacitance and voltage of the water-filled capacitor and (b) the energy stored in the capacitor before and after its immersion?

MODEL Pure distilled water is a good insulator. (The conductivity of tap water is due to dissolved ions.) Thus the immersed capacitor has a dielectric between the electrodes.

Example 26.9 A Water-Filled Capacitor

EXAMPLE 26.9 A water-filled capacitor

SOLVE a. From Table 26.1, the dielectric constant of water is $\kappa = 80$. The presence of the dielectric increases the capacitance to

$$C = \kappa C_0 = 80 \times 5.0\ \text{nF} = 400\ \text{nF}$$

At the same time, the voltage decreases to

$$\Delta V_C = \frac{(\Delta V_C)_0}{\kappa} = \frac{160\ \text{V}}{80} = 2.0\ \text{V}$$

Example 26.9 A Water-Filled Capacitor

EXAMPLE 26.9 **A water-filled capacitor**

SOLVE b. The presence of a dielectric does not alter the derivation leading to Equation 26.25 for the energy stored in a capacitor. Right after being disconnected from the battery, the stored energy was

$$(U_C)_0 = \tfrac{1}{2}C_0(\Delta V_C)_0^2 = \tfrac{1}{2}(5.0 \times 10^{-9}\ \text{F})(160\ \text{V})^2 = 6.4 \times 10^{-5}\ \text{J}$$

After being immersed, the stored energy is

$$U_C = \tfrac{1}{2}C(\Delta V_C)^2 = \tfrac{1}{2}(400 \times 10^{-9}\ \text{F})(2.0\ \text{V})^2 = 8.0 \times 10^{-7}\ \text{J}$$

Slide 26-97

Example 26.9 A Water-Filled Capacitor

EXAMPLE 26.9 **A water-filled capacitor**

ASSESS Water, with its large dielectric constant, has a *big* effect on the capacitor. But where did the energy go? We learned in Chapter 23 that a dipole is drawn into a region of stronger electric field. The electric field inside the capacitor is much stronger than just outside the capacitor, so the polarized dielectric is actually *pulled* into the capacitor. The "lost" energy is the work the capacitor's electric field did pulling in the dielectric.

Slide 26-98

Example 26.10 Energy Density of a Defibrillator

EXAMPLE 26.10 **Energy density of a defibrillator**

A defibrillator unit contains a 150 μF capacitor that is charged to 2100 V. The capacitor plates are separated by a 0.050-mm-thick insulator with dielectric constant 120.

a. What is the area of the capacitor plates?

b. What are the stored energy and the energy density in the electric field when the capacitor is charged?

MODEL Model the defibrillator as a parallel-plate capacitor with a dielectric.

Slide 26-99

Example 26.10 Energy Density of a Defibrillator

EXAMPLE 26.10 Energy density of a defibrillator

SOLVE a. The capacitance of a parallel-plate capacitor in a vacuum is $C_0 = \epsilon_0 A/d$. A dielectric increases the capacitance by the factor κ, to $C = \kappa C_0$, so the area of the capacitor plates is

$$A = \frac{Cd}{\kappa\epsilon_0} = \frac{(150 \times 10^{-6}\ \text{F})(5.0 \times 10^{-5}\ \text{m})}{120(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)} = 7.1\ \text{m}^2$$

Although the surface area is very large, Figure 26.32 shows how very large sheets of very thin metal can be rolled up into capacitors that you hold in your hand.

 Slide 26-100

Example 26.10 Energy Density of a Defibrillator

EXAMPLE 26.10 Energy density of a defibrillator

SOLVE b. The energy stored in the capacitor is

$$U_C = \tfrac{1}{2}C(\Delta V_C)^2 = \tfrac{1}{2}(150 \times 10^{-6}\ \text{F})(2100\ \text{V})^2 = 330\ \text{J}$$

Because the dielectric has increased C by a factor of κ, the energy density of Equation 26.27 is increased by a factor of κ to $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$. The electric field strength in the capacitor is

$$E = \frac{\Delta V_C}{d} = \frac{2100\ \text{V}}{5.0 \times 10^{-5}\ \text{m}} = 4.2 \times 10^7\ \text{V/m}$$

Consequently, the energy density is

$$u_E = \tfrac{1}{2}(120)(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)(4.2 \times 10^7\ \text{V/m})^2 = 9.4 \times 10^5\ \text{J/m}^3$$

 Slide 26-101

Example 26.10 Energy Density of a Defibrillator

EXAMPLE 26.10 Energy density of a defibrillator

ASSESS 330 J is a substantial amount of energy—equivalent to that of a 1 kg mass traveling at 25 m/s. And it can be delivered very quickly as the capacitor is discharged through the patient's chest.

 Slide 26-102

Chapter 26 Summary Slides

General Principles

Connecting V and $\vec{E}$

The electric potential and the electric field are two different perspectives of how source charges alter the space around them. V and $\vec{E}$ are related by

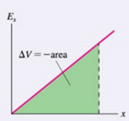
$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s\, ds$$

where s is measured from point i to point f and E_s is the component of $\vec{E}$ parallel to the line of integration.

Graphically

$\Delta V =$ the negative of the area under the E_s graph

$E_s = -\dfrac{dV}{ds}$ = the negative of the slope of the potential graph

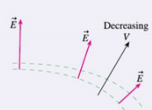


General Principles

The Geometry of Potential and Field

The electric field

- Is perpendicular to the equipotential surfaces.
- Points "downhill" in the direction of decreasing V.
- Is inversely proportional to the spacing Δs between the equipotential surfaces.

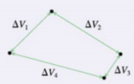


General Principles

Conservation of Energy

The sum of all potential differences around a closed path is zero.

$$\Sigma(\Delta V)_i = 0$$



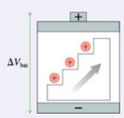
Slide 26-106

Important Concepts

A **battery** is a **source of potential**. The charge escalator in a battery uses chemical reactions to move charges from the negative terminal to the positive terminal:

$$\Delta V_{bat} = \mathcal{E}$$

where the emf $\mathcal{E}$ is the work per charge done by the charge escalator.



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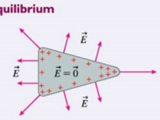
Important Concepts

For a **conductor in electrostatic equilibrium**

- The interior electric field is zero.
- The exterior electric field is perpendicular to the surface.
- The surface is an equipotential.
- The interior is at the same potential as the surface.



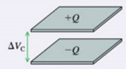
Slide 26-108

Applications

Capacitors

The **capacitance** of two conductors charged to $\pm Q$ is

$$C = \frac{Q}{\Delta V_C}$$



A parallel-plate capacitor has

$$C = \frac{\epsilon_0 A}{d}$$

Filling the space between the plates with a **dielectric** of dielectric constant κ increases the capacitance to $C = \kappa C_0$.

The energy stored in a capacitor is $u_C = \frac{1}{2}C(\Delta V_C)^2$.

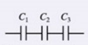
This energy is stored in the electric field at density $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$.

Slide 26-109

Applications

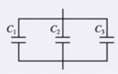
Combinations of capacitors

Series capacitors



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

Parallel capacitors



$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

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