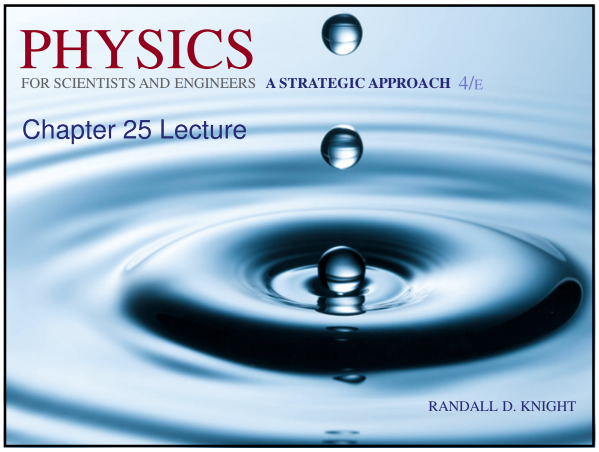


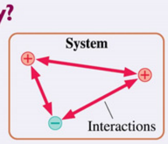
Chapter 25 The Electric Potential

IN THIS CHAPTER, you will learn to use the electric potential and electric potential energy.

Chapter 25 Preview

What is electric potential energy?

Recall that potential energy is an interaction energy. Charged particles that interact via the electric force have electric potential energy U. You'll learn that there's a close analogy with gravitational potential energy.



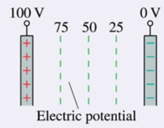
« LOOKING BACK Section 10.1 Potential energy
« LOOKING BACK Section 10.5 Energy diagrams

Chapter 25 Preview

What is the electric potential?

You've seen that source charges create an electric field. Source charges also create an electric potential. The electric potential V

- Exists everywhere in space.
- Is a scalar.
- Causes charges to have potential energy.
- Is measured in volts.

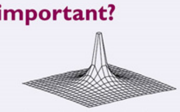


Slide 25-4

Chapter 25 Preview

What potentials are especially important?

We'll calculate the electric potential of four important charge distributions: a point charge, a charged sphere, a ring of charge, and a parallel-plate capacitor. Finding the potential of a continuous charge distribution is similar to calculating electric fields, but easier because potential is a scalar.


Potential of a point charge

« LOOKING BACK Section 23.3 The electric field

Slide 25-5

Chapter 25 Preview

How is potential represented?

Electric potential is a fairly abstract idea, so it will be important to visualize how the electric potential varies in space. One way of doing so is with equipotential surfaces. These are mathematical surfaces, not physical surfaces, with the same value of the potential V at every point.

Slide 25-6

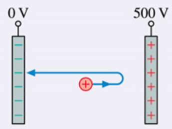
Chapter 25 Preview

How is electric potential used?

A charged particle q in an electric potential V has electric potential energy $U = qV$.

- Charged particles accelerate as they move through a potential difference.
- Mechanical energy is conserved:

$$K_f + qV_f = K_i + qV_i$$



« LOOKING BACK Section 10.4 Energy conservation

 Slide 25-7

Chapter 25 Preview

Why is energy important in electricity?

Energy allows things to happen. You want your lights to light, your computer to compute, and your music to play. All these require energy—*electric* energy. This is the first of two chapters that explore electric energy and its connection to electric forces and fields. You'll then be prepared to understand electric circuits—which are all about how energy is transformed and transferred from sources, such as batteries, to devices that utilize and dissipate the energy.

 Slide 25-8

Chapter 25 Reading Questions

 Slide 25-9

Reading Question 25.1

The electric potential energy of a system of two point charges is proportional to

A. The distance between the two charges.
B. The square of the distance between the two charges.
C. The inverse of the distance between the two charges.
D. The inverse of the square of the distance between the two charges.

Slide 25-10

Reading Question 25.1

The electric potential energy of a system of two point charges is proportional to

A. The distance between the two charges.
B. The square of the distance between the two charges.
✔**C. The inverse of the distance between the two charges.**
D. The inverse of the square of the distance between the two charges.

Slide 25-11

Reading Question 25.2

What are the units of *potential difference*?

A. Amperes
B. Potentiometers
C. Farads
D. Volts
E. Henrys

Slide 25-12

Reading Question 25.2

What are the units of *potential difference*?

A. Amperes
B. Potentiometers
C. Farads
✔**D. Volts**
E. Henrys

Slide 25-13

Reading Question 25.3

New units of the electric field were introduced in this chapter. They are

A. V/C
B. N/C
C. V/m
D. J/m^2
E. Ω/m

Slide 25-14

Reading Question 25.3

New units of the electric field were introduced in this chapter. They are

A. V/C
B. N/C
✔**C. V/m**
D. J/m^2
E. Ω/m

Slide 25-15

Reading Question 25.4

Which of the following statements about equipotential surfaces is true?

A. Tangent lines to equipotential surfaces are always parallel to the electric field vectors.
B. Equipotential surfaces are surfaces that have the same value of potential energy at every point.
C. Equipotential surfaces are surfaces that have the same value of potential at every point.
D. Equipotential surfaces are always parallel planes.
E. Equipotential surfaces are real physical surfaces that exist in space.

Reading Question 25.4

Which of the following statements about equipotential surfaces is true?

A. Tangent lines to equipotential surfaces are always parallel to the electric field vectors.
B. Equipotential surfaces are surfaces that have the same value of potential energy at every point.
✔**C. Equipotential surfaces are surfaces that have the same value of potential at every point.**
D. Equipotential surfaces are always parallel planes.
E. Equipotential surfaces are real physical surfaces that exist in space.

Reading Question 25.5

The electric potential inside a capacitor

A. Is constant.
B. Increases linearly from the negative to the positive plate.
C. Decreases linearly from the negative to the positive plate.
D. Decreases inversely with distance from the negative plate.
E. Decreases inversely with the square of the distance from the negative plate.

Reading Question 25.5

The electric potential inside a capacitor

A. Is constant.

✔ **B. Increases linearly from the negative to the positive plate.**

C. Decreases linearly from the negative to the positive plate.

D. Decreases inversely with distance from the negative plate.

E. Decreases inversely with the square of the distance from the negative plate.

Chapter 25 Content, Examples, and QuickCheck Questions

Energy

- The kinetic energy of a system, K, is the sum of the kinetic energies $K_i = 1/2m_iv_i^2$ of all the particles in the system.
- The potential energy of a system, U, is the *interaction energy* of the system.
- The change in potential energy, ΔU, is -1 times the work done by the interaction forces:

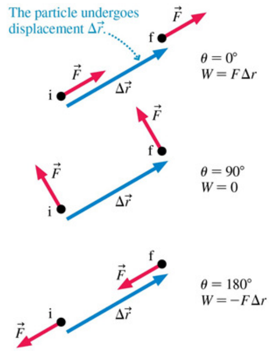
$$\Delta U = -W_{\text{interaction}}(\text{i} \rightarrow \text{f})$$

- If all of the forces involved are *conservative forces* (such as gravity or the electric force) then the total energy $K + U$ is *conserved*; it does not change with time.

Work Done by a Constant Force

- Recall that the work done by a constant force depends on the angle θ between the force F and the displacement Δr:

$$W = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r \cos\theta$$



A Gravitational Analogy

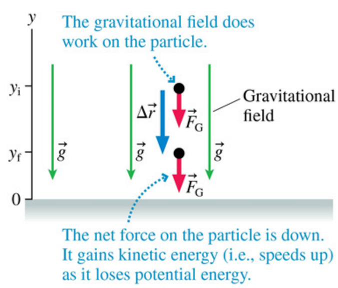
- Every conservative force is associated with a potential energy.
- In the case of gravity, the work done is

  $W_{grav} = mgy_i - mgy_f$
- The change in gravitational potential energy is

  $\Delta U_{grav} = -W_{grav}$

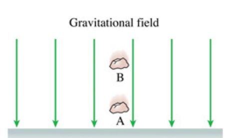
  where

  $U_{grav} = U_0 + mgy$



QuickCheck 25.1

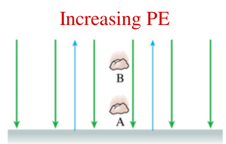
Two rocks have equal mass. Which has more gravitational potential energy?



A. Rock A
B. Rock B
C. They have the same potential energy.
D. Both have zero potential energy.

QuickCheck 25.1

Two rocks have equal mass. Which has more gravitational potential energy?



A. Rock A

✔**B. Rock B**

C. They have the same potential energy.

D. Both have zero potential energy.

 Slide 25-25

A Uniform Electric Field

- A positive charge q inside a capacitor speeds up as it "falls" toward the negative plate.
- There is a constant force $F = qE$ in the direction of the displacement.
- The work done is

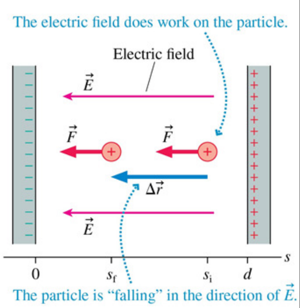
$$W_{elec} = qEs_i - qEs_f$$

- The change in **electric potential energy** is

$$\Delta U_{elec} = -W_{elec}$$

where

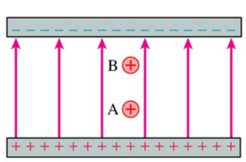
$$U_{elec} = U_0 + qEs$$



 Slide 25-26

QuickCheck 25.2

Two positive charges are equal. Which has more electric potential energy?



A. Charge A

B. Charge B

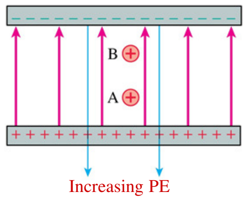
C. They have the same potential energy.

D. Both have zero potential energy.

 Slide 25-27

QuickCheck 25.2

Two positive charges are equal. Which has more electric potential energy?



✔ **A. Charge A**

B. Charge B

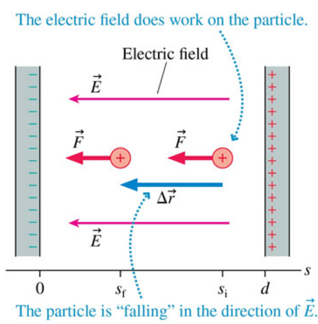
C. They have the same potential energy.

D. Both have zero potential energy.

Electric Potential Energy in a Uniform Field

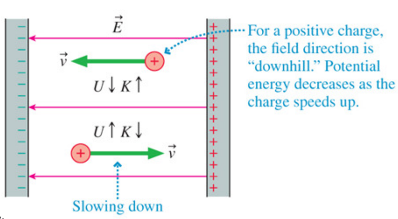
- A positive charge inside a capacitor speeds up and gains kinetic energy as it "falls" toward the negative plate.
- The charge is losing potential energy as it gains kinetic energy.

$U_{elec} = U_0 + qEs$



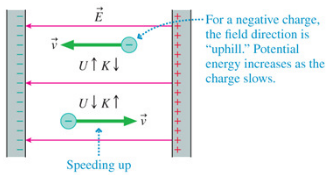
Electric Potential Energy in a Uniform Field

- For a positive charge, U decreases and K increases as the charge moves toward the negative plate.
- A positive charge moving opposite the field direction is going "uphill," slowing as it transforms kinetic energy into electric potential energy.



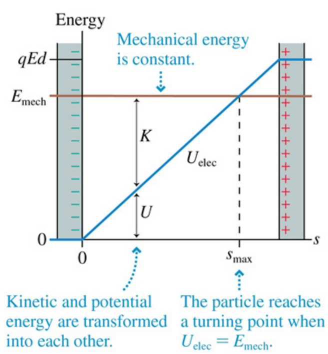
Electric Potential Energy in a Uniform Field

- A negative charged particle has *negative* potential energy.
- U increases (becomes less negative) as the negative charge moves toward the negative plate.
- A negative charge moving in the field direction is going "uphill," transforming $K \rightarrow U$ as it slows.



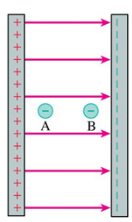
Electric Potential Energy in a Uniform Field

- The figure shows the **energy diagram** for a positively charged particle in a uniform electric field.
- The potential energy increases linearly with distance, but the total mechanical energy E_{mech} is fixed.



QuickCheck 25.3

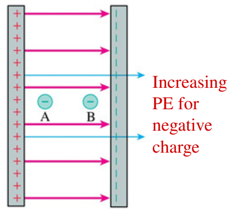
Two negative charges are equal. Which has more electric potential energy?



A. Charge A
B. Charge B
C. They have the same potential energy.
D. Both have zero potential energy.

QuickCheck 25.3

Two negative charges are equal. Which has more electric potential energy?



A. Charge A

✔ **B. Charge B**

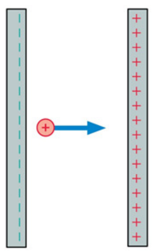
C. They have the same potential energy.

D. Both have zero potential energy.

Slide 25-34

QuickCheck 25.4

A positive charge moves as shown. Its kinetic energy



A. Increases.

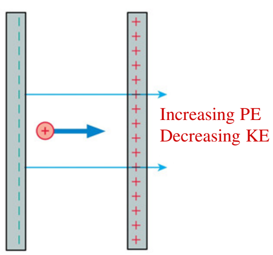
B. Remains constant.

C. Decreases.

Slide 25-35

QuickCheck 25.4

A positive charge moves as shown. Its kinetic energy



A. Increases.

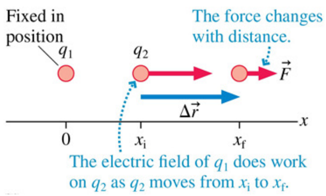
B. Remains constant.

✔ **C. Decreases.**

Slide 25-36

The Potential Energy of Two Point Charges

- Consider two like charges q_1 and q_2.
- The electric field of q_1 *pushes* q_2 as it moves from x_i to x_f.
- The work done is



$$W_{elec} = \int_{x_i}^{x_f} F_{1 \text{ on } 2}\, dx = \int_{x_i}^{x_f} \frac{Kq_1q_2}{x^2}\, dx = Kq_1q_2 \frac{-1}{x}\bigg|_{x_i}^{x_f} = -\frac{Kq_1q_2}{x_f} + \frac{Kq_1q_2}{x_i}$$

- The change in electric potential energy of the system is $\Delta U_{elec} = -W_{elec}$ if

$$U_{elec} = \frac{Kq_1q_2}{x}$$

 Slide 25-37

The Potential Energy of Two Point Charges

- Consider two point charges, q_1 and q_2, separated by a distance r. The electric potential energy is

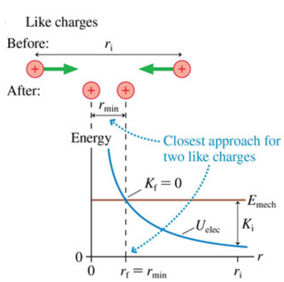
$$U_{elec} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \quad \text{(two point charges)}$$

- This is explicitly the energy of *the system*, not the energy of just q_1 or q_2.
- Note that the potential energy of two charged particles approaches zero as $r \to \infty$.

 Slide 25-38

The Potential Energy of Two Point Charges

- Two like charges approach each other.
- Their total energy is $E_{mech} > 0$.
- They gradually slow down until the distance separating them is r_{min}.
- This is the *distance of closest approach.*

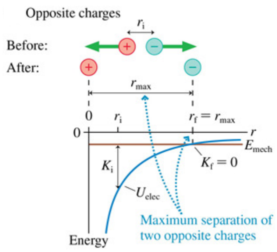


$$U_{elec} = \frac{Kq_1q_2}{x}$$

 Slide 25-39

The Potential Energy of Two Point Charges

- Two opposite charges are shot apart from one another with equal and opposite momenta.
- Their total energy is $E_{mech} < 0$.
- They gradually slow down until the distance separating them is r_{max}.
- This is their *maximum separation.*



$$U_{elec} = \frac{Kq_1q_2}{x}$$

QuickCheck 25.5

A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do,

A. A positive potential energy becomes more positive.
B. A positive potential energy becomes less positive.
C. A negative potential energy becomes more negative.
D. A negative potential energy becomes less negative.
E. A positive potential energy becomes a negative potential energy.

QuickCheck 25.5

A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do,

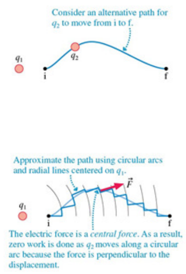
A. A positive potential energy becomes more positive.
B. A positive potential energy becomes less positive.
✔ **C. A negative potential energy becomes more negative.**
D. A negative potential energy becomes less negative.
E. A positive potential energy becomes a negative potential energy.

$$U_{elec} = \frac{Kq_1q_2}{r}$$

Kq_1q_2 — Opposite signs, so U is Negative.
r — U increases in magnitude as r decreases.

The Electric Force Is a Conservative Force

- Any path away from q_1 can be approximated using circular arcs and radial lines.
- All the work is done along the radial line segments, which is equivalent to a straight line from i to f.
- Therefore the work done by the electric force depends only on initial and final position, not the path followed.



Example 25.2 Approaching a Charged Sphere

EXAMPLE 25.2 | **Approaching a charged sphere**

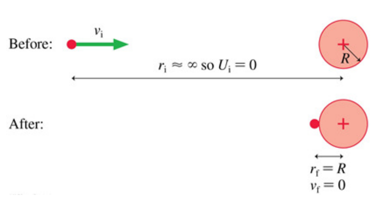
A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to +100 nC. What initial speed must the proton have to just reach the surface of the glass?

MODEL Energy is conserved. The glass sphere can be modeled as a charged particle, so the potential energy is that of two point charges. The glass is so much more massive than the proton that it remains at rest as the proton moves. The proton starts "far away," which we interpret as sufficiently far to make $U_i \approx 0$.

Example 25.2 Approaching a Charged Sphere

EXAMPLE 25.2 | **Approaching a charged sphere**

VISUALIZE FIGURE 25.10 shows the before-and-after pictorial representation. To "just reach" the glass sphere means that the proton comes to rest, $v_f = 0$, as it reaches $r_f = 0.50$ mm, the *radius* of the sphere.



Example 25.2 Approaching a Charged Sphere

EXAMPLE 25.2 Approaching a charged sphere

SOLVE Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + \frac{K q_p q_{sphere}}{r_f} = \frac{1}{2} m v_i^2 + 0$$

The proton charge is $q_p = e$. With this, we can solve for the proton's initial speed:

$$v_i = \sqrt{\frac{2Keq_{sphere}}{m r_f}} = 1.86 \times 10^7 \text{ m/s}$$

Example 25.3 Escape Speed

EXAMPLE 25.3 Escape speed

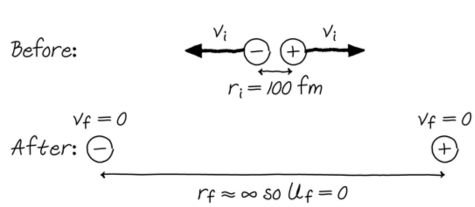
An interaction between two elementary particles causes an electron and a positron (a positive electron) to be shot out back to back with equal speeds. What minimum speed must each have when they are 100 fm apart in order to escape each other?

MODEL Energy is conserved. The particles end "far apart," which we interpret as sufficiently far to make $U_f \approx 0$.

Example 25.3 Escape Speed

EXAMPLE 25.3 Escape speed

VISUALIZE FIGURE 25.11 shows the before-and-after pictorial representation. The minimum speed to escape is the speed that allows the particles to reach $r_f = \infty$ with $v_f = 0$.



Example 25.3 Escape Speed

EXAMPLE 25.3 Escape speed

SOLVE U_{elec} is the potential energy of the electron + positron system. Similarly, K is the *total* kinetic energy of the system. The electron and the positron, with equal masses and equal speeds, have equal kinetic energies. Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + 0 + 0 = \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}mv_i^2 + \frac{Kq_e q_p}{r_i} = mv_i^2 - \frac{Ke^2}{r_i}$$

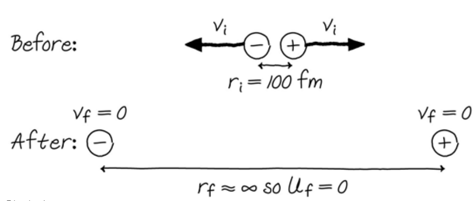
Using $r_i = 100$ fm $= 1.0 \times 10^{-13}$ m, we can calculate the minimum initial speed to be

$$v_i = \sqrt{\frac{Ke^2}{mr_i}} = 5.0 \times 10^7 \text{ m/s}$$

Example 25.3 Escape Speed

EXAMPLE 25.3 Escape speed

ASSESS v_i is a little more than 10% the speed of light, just about the limit of what a "classical" calculation can predict. We would need to use the theory of relativity if v_i were any larger.



The Potential Energy of Multiple Point Charges

- Consider more than two point charges. The potential energy is the sum of the potential energies due to all pairs of charges:

$$U_{elec} = \sum_{i<j} \frac{Kq_i q_j}{r_{ij}}$$

  where r_{ij} is the distance between q_i and q_j.

- The summation contains the $i < j$ restriction to ensure that each pair of charges is counted only once.

Example 25.4 Launching an Electron

EXAMPLE 25.4 **Launching an electron**

Three electrons are spaced 1.0 mm apart along a vertical line. The outer two electrons are fixed in position.

a. Is the center electron at a point of stable or unstable equilibrium?

b. If the center electron is displaced horizontally by a small distance, what will its speed be when it is very far away?

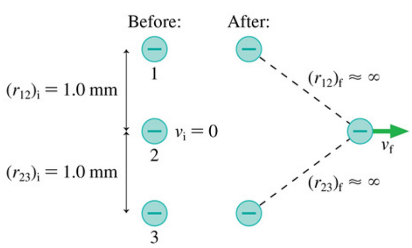
MODEL Energy is conserved. The outer two electrons don't move, so we don't need to include the potential energy of their interaction.

 Slide 25-52

Example 25.4 Launching an Electron

EXAMPLE 25.4 **Launching an electron**

VISUALIZE **FIGURE 25.12** shows the situation.



 Slide 25-53

Example 25.4 Launching an Electron

EXAMPLE 25.4 **Launching an electron**

SOLVE a. The center electron is in equilibrium *exactly* in the center because the two electric forces on it balance. But if it moves a little to the right or left, no matter how little, then the horizontal components of the forces from both outer electrons will push the center electron farther away. This is an unstable equilibrium for horizontal displacements, like being on the top of a hill.

 Slide 25-54

Example 25.4 Launching an Electron

EXAMPLE 25.4 | **Launching an electron**

SOLVE b. A small displacement will cause the electron to move away. If the displacement is only infinitesimal, the initial conditions are $(r_{12})_i = (r_{23})_i = 1.0$ mm and $v_i = 0$. "Far away" is interpreted as $r_f \to \infty$, where $U_f \approx 0$. There are now *two* terms in the potential energy, so conservation of energy $K_f + U_f = K_i + U_i$ gives

$$\tfrac{1}{2}mv_f^2 + 0 + 0 = 0 + \left[\frac{Kq_1q_2}{(r_{12})_i} + \frac{Kq_2q_3}{(r_{23})_i}\right]$$
$$= \left[\frac{Ke^2}{(r_{12})_i} + \frac{Ke^2}{(r_{23})_i}\right]$$

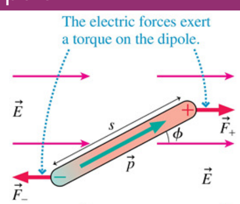
This is easily solved to give

$$v_f = \sqrt{\frac{2}{m}\left[\frac{Ke^2}{(r_{12})_i} + \frac{Ke^2}{(r_{23})_i}\right]} = 1000 \text{ m/s}$$

 Slide 25-55

The Potential Energy of a Dipole

- Consider a dipole in a uniform electric field.
- The forces F_+ and F_- exert a torque on the dipole.
- The work done is



$$W_{elec} = -pE\int_{\phi_i}^{\phi_f} \sin\phi \, d\phi = pE\cos\phi_f - pE\cos\phi_i$$

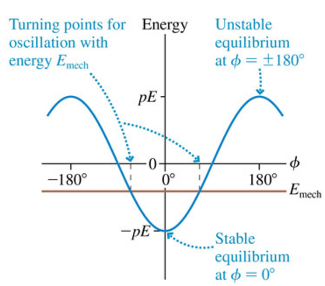
- The change in electric potential energy of the system is $\Delta U_{elec} = -W_{elec}$ if

$$U_{dipole} = -pE\cos\phi = -\vec{p}\cdot\vec{E}$$

 Slide 25-56

The Potential Energy of a Dipole

- The potential energy of a dipole is $\phi = 0°$ minimum at where the dipole is aligned with the electric field.
- A frictionless dipole with mechanical energy E_{mech} will oscillate back and forth between turning points on either side of $\phi = 0°$.



$$U_{dipole} = -pE\cos\phi = -\vec{p}\cdot\vec{E}$$

 Slide 25-57

Example 25.5 Rotating a Molecule

EXAMPLE 25.5 **Rotating a molecule**

The water molecule is a permanent electric dipole with dipole moment 6.2×10^{-30} C m. A water molecule is aligned in an electric field with field strength 1.0×10^{7} N/C. How much energy is needed to rotate the molecule 90°?

MODEL The molecule is at the point of minimum energy. It won't spontaneously rotate 90°. However, an external force that supplies energy, such as a collision with another molecule, can cause the water molecule to rotate.

Example 25.5 Rotating a Molecule

EXAMPLE 25.5 **Rotating a molecule**

SOLVE The molecule starts at $\phi_i = 0°$ and ends at $\phi_f = 90°$. The increase in potential energy is

$$\Delta U_{\text{dipole}} = U_f - U_i = -pE\cos 90° - (-pE\cos 0°)$$
$$= pE = 6.2 \times 10^{-23}\ \text{J}$$

This is the energy needed to rotate the molecule 90°.

ASSESS ΔU_{dipole} is significantly less than $k_B T$ at room temperature. Thus collisions with other molecules can easily supply the energy to rotate the water molecules and keep them from staying aligned with the electric field.

The Electric Potential

- We define the electric potential V (or, for brevity, just the potential) as

$$V \equiv \frac{U_{q+\text{sources}}}{q}$$

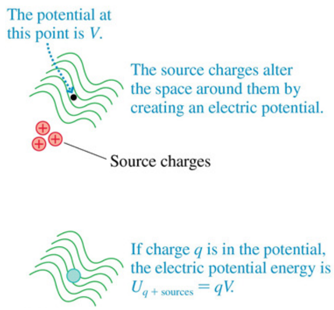
- The unit of electric potential is the joule per coulomb, which is called the **volt V**:

$$1\ \text{volt} = 1\ \text{V} \equiv 1\ \text{J/C}$$

This battery is a source of *electric potential.* The electric potential difference between the + and – sides is 1.5 V.

The Electric Potential

- Test charge q is used as a probe to determine the electric potential, but the value of V is *independent of* q.
- **The electric potential, like the electric field, is a property of the source charges.**



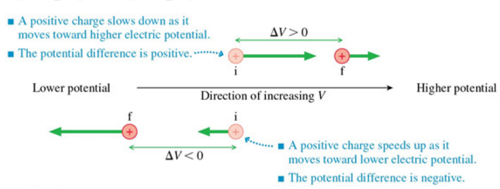
Slide 25-61

Using the Electric Potential

- As a charged particle moves through a changing electric potential, energy is conserved:

$$K_f + qV_f = K_i + qV_i$$

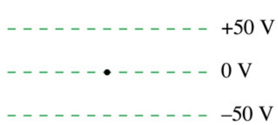
	Electric potential	
	Increasing ($\Delta V > 0$)	Decreasing ($\Delta V < 0$)
+ charge	Slows down	Speeds up
− charge	Speeds up	Slows down



Slide 25-62

QuickCheck 25.6

A proton is released from rest at the dot. Afterward, the proton

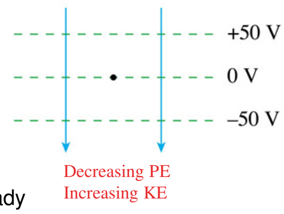


A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
E. Moves downward with an increasing speed.

Slide 25-63

QuickCheck 25.6

A proton is released from rest at the dot. Afterward, the proton



A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
✔**E. Moves downward with an increasing speed.**

Slide 25-64

QuickCheck 25.7

If a positive charge is released from rest, it moves in the direction of

A. A stronger electric field.
B. A weaker electric field.
C. Higher electric potential.
D. Lower electric potential.
E. Both B and D.

Slide 25-65

QuickCheck 25.7

If a positive charge is released from rest, it moves in the direction of

A. A stronger electric field.
B. A weaker electric field.
C. Higher electric potential.
✔**D. Lower electric potential.**
E. Both B and D.

Slide 25-66

Problem-Solving Strategy: Conservation of Energy in Charge Interactions

PROBLEM-SOLVING STRATEGY 25.1

Conservation of energy in charge interactions

MODEL Define the system. If possible, model it as an isolated system for which mechanical energy is conserved.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols, list known values, and identify what you're trying to find.

Problem-Solving Strategy: Conservation of Energy in Charge Interactions

PROBLEM-SOLVING STRATEGY 25.1

Conservation of energy in charge interactions

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + qV_f = K_i + qV_i$$

- Is the electric potential given in the problem statement? If not, you'll need to use a known potential, such as that of a point charge, or calculate the potential using the procedure given later, in Problem-Solving Strategy 25.2.
- K_i and K_f are the sums of the kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 22

Example 25.6 Moving Through a Potential Difference

EXAMPLE 25.6 Moving through a potential difference

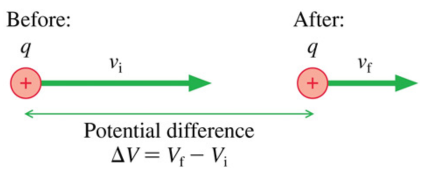
A proton with a speed of 2.0×10^5 m/s enters a region of space in which there is an electric potential. What is the proton's speed after it moves through a potential difference of 100 V? What will be the final speed if the proton is replaced by an electron?

MODEL The system is the charge plus the unseen source charges creating the potential. This is an isolated system, so mechanical energy is conserved.

Example 25.6 Moving Through a Potential Difference

EXAMPLE 25.6 **Moving through a potential difference**

VISUALIZE FIGURE 25.17 is a before-and-after pictorial representation of a charged particle moving through a potential difference. A positive charge *slows down* as it moves into a region of higher potential ($K \rightarrow U$). A negative charge *speeds up* ($U \rightarrow K$).



Slide 25-70

Example 25.6 Moving Through a Potential Difference

EXAMPLE 25.6 **Moving through a potential difference**

SOLVE The potential energy of charge q is $U = qV$. Conservation of energy, now expressed in terms of the electric potential V, is $K_f + qV_f = K_i + qV_i$, or

$$K_f = K_i - q\Delta V$$

where $\Delta V = V_f - V_i$ is the potential difference through which the particle moves. In terms of the speeds, energy conservation is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - q\Delta V$$

We can solve this for the final speed:

$$v_f = \sqrt{v_i^2 - \frac{2q}{m}\Delta V}$$

Slide 25-71

Example 25.6 Moving Through a Potential Difference

EXAMPLE 25.6 **Moving through a potential difference**

SOLVE For a proton, with $q = e$, the final speed is

$$(v_f)_p = \sqrt{(2.0 \times 10^5 \text{ m/s})^2 - \frac{2(1.60 \times 10^{-19} \text{ C})(100 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}}$$
$$= 1.4 \times 10^5 \text{ m/s}$$

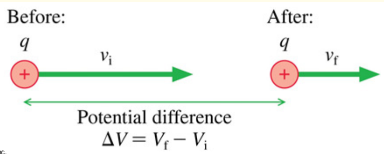
An electron, though, with $q = -e$ and a different mass, reaches speed $(v_f)_e = 5.9 \times 10^6$ m/s.

Slide 25-72

Example 25.6 Moving Through a Potential Difference

EXAMPLE 25.6 **Moving through a potential difference**

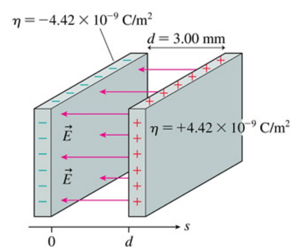
ASSESS The proton slowed down and the electron sped up, as we expected. Note that the electric potential *already existed* in space due to other charges that are not explicitly seen in the problem. The electron and proton have nothing to do with creating the potential. Instead, they *respond* to the potential by having potential energy $U = qV$.



 Slide 25-73

The Electric Field Inside a Parallel-Plate Capacitor

- This is a review of Chapter 23.



$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{from positive toward negative}\right)$$
$$= (500 \text{ N/C, from right to left})$$

 Slide 25-74

The Electric Potential Inside a Parallel-Plate Capacitor

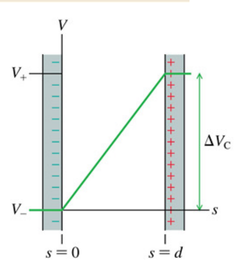
- The electric potential inside a parallel-plate capacitor is

$V = Es$ (electric potential inside a parallel-plate capacitor)

where s **is the distance from the** ***negative*** **electrode**.

- The *potential difference* ΔV_C, or "voltage" between the two capacitor plates is

$$\Delta V_C = V_+ - V_- = Ed$$



 Slide 25-75

Units of Electric Field

- If we know a capacitor's voltage ΔV and the distance between the plates d, then the electric field strength within the capacitor is

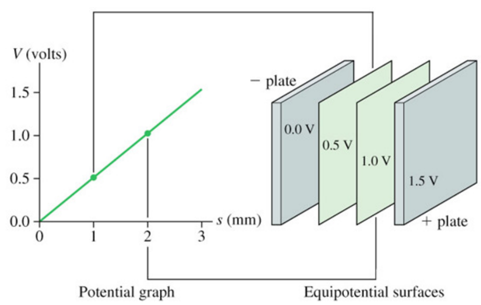
$$E = \frac{\Delta V_C}{d}$$

- This implies that the units of electric field are volts per meter, or V/m.
- Previously, we have been using electric field units of newtons per coulomb.
- In fact, as you can show as a homework problem, these units are equivalent to each other:

$$1 \text{ N/C} = 1 \text{ V/m}$$

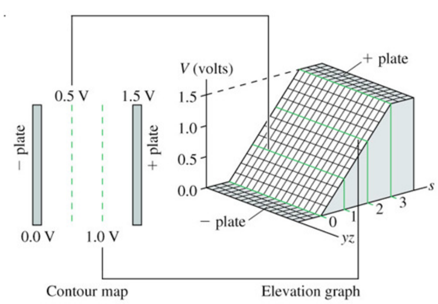
 Slide 25-76

The Electric Potential Inside a Parallel-Plate Capacitor



 Slide 25-77

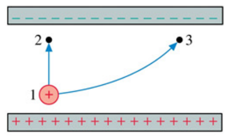
The Electric Potential Inside a Parallel-Plate Capacitor



 Slide 25-78

QuickCheck 25.8

Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons' speeds at points 2 and 3 are related by

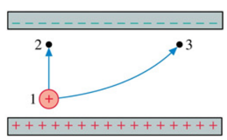


A. $v_2 > v_3$
B. $v_2 = v_3$
C. $v_2 < v_3$
D. Not enough information to compare their speeds.

Slide 25-79

QuickCheck 25.8

Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons' speeds at points 2 and 3 are related by

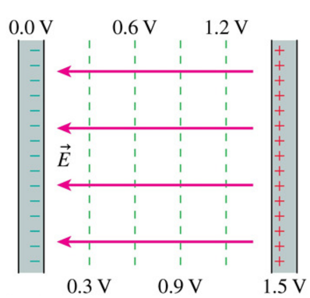


A. $v_2 > v_3$
✔**B.** $\mathbf{v_2 = v_3}$ Energy conservation
C. $v_2 < v_3$
D. Not enough information to compare their speeds.

Slide 25-80

The Parallel-Plate Capacitor

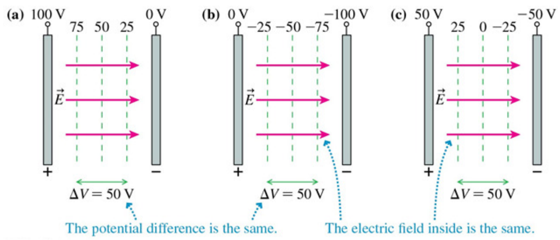
- The figure shows the contour lines of the electric potential and the electric field vectors inside a parallel-plate capacitor.
- The electric field vectors are *perpendicular* to the equipotential surfaces.
- The electric field points in the direction of *decreasing* potential.



Slide 25-81

The Zero Point of Electric Potential

- Where you choose $V = 0$ is arbitrary. The three contour maps below represent the *same physical situation*.

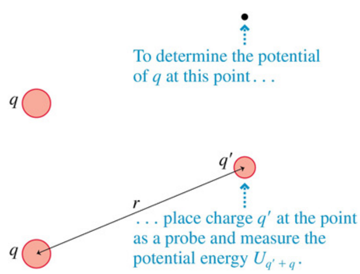


 Slide 25-82

The Electric Potential of a Point Charge

- Let q in the figure be the source charge, and let a second charge q', a distance r away, probe the electric potential of q.
- The potential energy of the two point charges is

$$U_{q'+q} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r}$$



 Slide 25-83

The Electric Potential of a Point Charge

- The electric potential due to a point charge q is

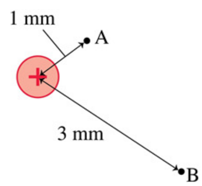
$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{(electric potential of a point charge)}$$

- The potential extends through all of space, showing the influence of charge q, but it weakens with distance as $1/r$.
- This expression for V assumes that we have chosen $V = 0$ to be at $r = \infty$.

 Slide 25-84

QuickCheck 25.9

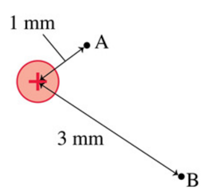
What is the ratio V_B/V_A of the electric potentials at the two points?



A. 9
B. 3
C. 1/3
D. 1/9
E. Undefined without knowing the charge

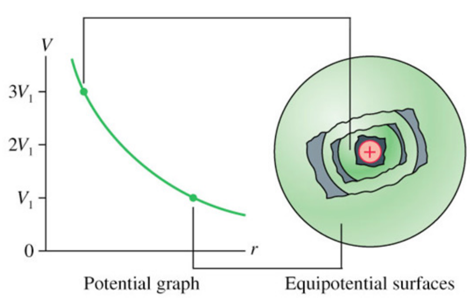
QuickCheck 25.9

What is the ratio V_B/V_A of the electric potentials at the two points?

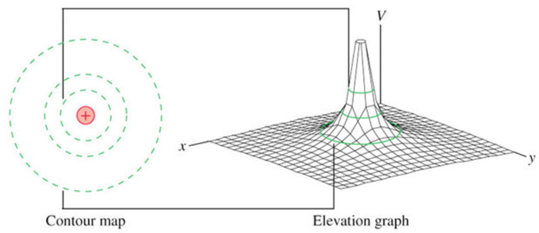


A. 9
B. 3
✔ **C. 1/3** Potential of a point charge decreases inversely with distance.
D. 1/9
E. Undefined without knowing the charge

The Electric Potential of a Point Charge



The Electric Potential of a Point Charge



 Slide 25-88

The Electric Potential of a Charged Sphere

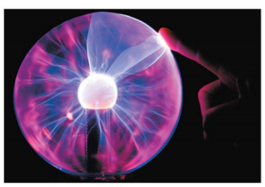
- Outside a uniformly charged sphere of radius R, the electric potential is identical to that of a point charge Q at the center:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

where $r \geq R$.

- If the potential at the surface V_0 is known, then the potential at $r \geq R$ is

$$V = \frac{R}{r}V_0$$

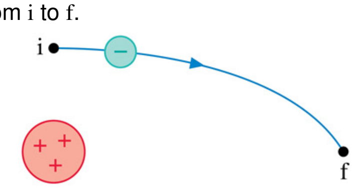


A *plasma ball* consists of a small metal ball inside a hollow glass sphere filled with low-pressure neon gas. The high voltage of the ball creates “lightning bolts” between the ball and the glass sphere.

 Slide 25-89

QuickCheck 25.10

An electron follows the trajectory shown from i to f. At point f,

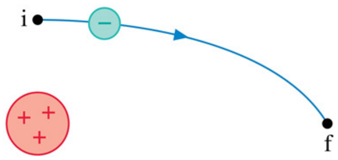


A. $v_f > v_i$
B. $v_f = v_i$
C. $v_f < v_i$
D. Not enough information to compare the speeds at these points.

 Slide 25-90

QuickCheck 25.10

An electron follows the trajectory shown from i to f. At point f,



A. $v_f > v_i$
B. $v_f = v_i$
✔C. $v_f < v_i$
D. Not enough information to compare the speeds at these points.

Increasing PE (becoming less negative) so decreasing KE

Slide 25-91

Example 25.8 A Proton and a Charged Sphere

EXAMPLE 25.8 | A proton and a charged sphere

A proton is released from rest at the surface of a 1.0-cm-diameter sphere that has been charged to +1000 V.

a. What is the charge of the sphere?

b. What is the proton's speed at 1.0 cm from the sphere?

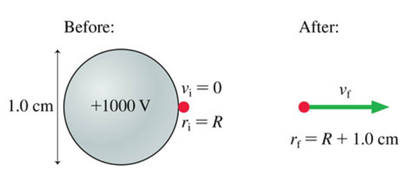
MODEL Energy is conserved. The potential outside the charged sphere is the same as the potential of a point charge at the center.

Slide 25-92

Example 25.8 A Proton and a Charged Sphere

EXAMPLE 25.8 | A proton and a charged sphere

VISUALIZE



SOLVE a. The charge of the sphere is

$$Q = 4\pi\epsilon_0 R V_0 = 0.56 \times 10^{-9}\ \text{C} = 0.56\ \text{nC}$$

Slide 25-93

Example 25.8 A Proton and a Charged Sphere

EXAMPLE 25.8 A proton and a charged sphere

SOLVE b. A sphere charged to $V_0 = +1000$ V is positively charged. The proton will be repelled by this charge and move away from the sphere. The conservation of energy equation $K_f + eV_f = K_i + eV_i$, with Equation 25.34 for the potential of a sphere, is

$$\frac{1}{2}mv_f^2 + \frac{eR}{r_f}V_0 = \frac{1}{2}mv_i^2 + \frac{eR}{r_i}V_0$$

The proton starts from the surface of the sphere, $r_i = R$, with $v_i = 0$. When the proton is 1.0 cm from the *surface* of the sphere, it has $r_f = 1.0\text{ cm} + R = 1.5$ cm. Using these, we can solve for v_f:

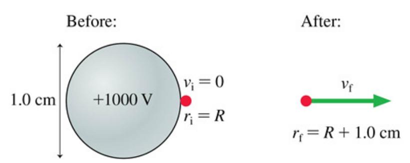
$$v_f = \sqrt{\frac{2eV_0}{m}\left(1 - \frac{R}{r_f}\right)} = 3.6 \times 10^5 \text{ m/s}$$

 Slide 25-94

Example 25.8 A Proton and a Charged Sphere

EXAMPLE 25.8 A proton and a charged sphere

ASSESS This example illustrates how the ideas of electric potential and potential energy work together, yet they are *not* the same thing.



 Slide 25-95

The Electric Potential of Many Charges

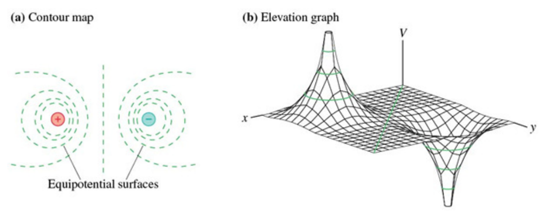
- The electric potential V at a point in space is the sum of the potentials due to each charge:

$$V = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i}$$

  where r_i is the distance from charge q_i to the point in space where the potential is being calculated.
- **The electric potential, like the electric field, obeys the principle of superposition.**

 Slide 25-96

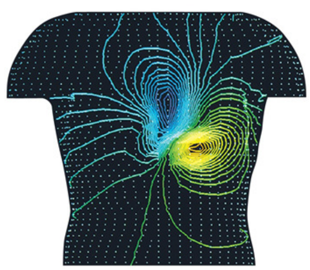
The Electric Potential of an Electric Dipole



Slide 25-97

The Electric Potential of a Human Heart

- Electrical activity within the body can be monitored by measuring equipotential lines on the skin.
- The equipotentials near the heart are a slightly distorted but recognizable *electric dipole*.



Slide 25-98

QuickCheck 25.11

At the midpoint between these two equal but opposite charges,

A. $\vec{E} = \vec{0}$; $V = 0$
B. $\vec{E} = \vec{0}$; $V > 0$
C. $\vec{E} = \vec{0}$; $V < 0$
D. $\vec{E}$ points right; $V = 0$
E. $\vec{E}$ points left; $V = 0$

Slide 25-99

QuickCheck 25.11

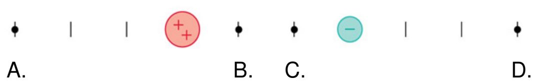
At the midpoint between these two equal but opposite charges,

A. $\vec{E} = \vec{0}$; $V = 0$
B. $\vec{E} = \vec{0}$; $V > 0$
C. $\vec{E} = \vec{0}$; $V < 0$
✔**D. $\vec{E}$ points right; $V = 0$**
E. $\vec{E}$ points left; $V = 0$

Slide 25-100

QuickCheck 25.12

At which point or points is the electric potential zero?

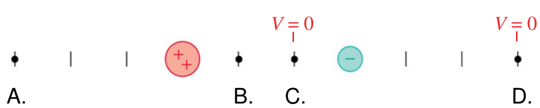


E. More than one of these.

Slide 25-101

QuickCheck 25.12

At which point or points is the electric potential zero?



✔**E. More than one of these.**

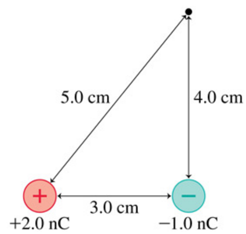
Slide 25-102

Example 25.9 The Potential of Two Charges

EXAMPLE 25.9 The potential of two charges

What is the electric potential at the point indicated in **FIGURE 25.29**?

MODEL The potential is the sum of the potentials due to each charge.



Example 25.9 The Potential of Two Charges

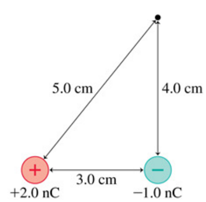
EXAMPLE 25.9 The potential of two charges

SOLVE The potential at the indicated point is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$

$$= (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)\left(\frac{2.0 \times 10^{-9}\text{ C}}{0.050\text{ m}} + \frac{-1.0 \times 10^{-9}\text{ C}}{0.040\text{ m}}\right)$$

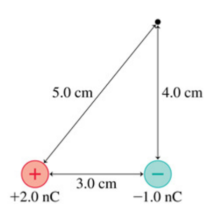
$$= 135 \text{ V}$$



Example 25.9 The Potential of Two Charges

EXAMPLE 25.9 The potential of two charges

ASSESS The potential is a *scalar,* so we found the net potential by adding two numbers. We don't need any angles or components to calculate the potential.



Problem-Solving Strategy: The Electric Potential of a Continuous Distribution of Charge

PROBLEM-SOLVING STRATEGY 25.2

The electric potential of a continuous distribution of charge

MODEL Model the charge distribution as a simple shape.

VISUALIZE For the pictorial representation:

- Draw a picture, establish a coordinate system, and identify the point P at which you want to calculate the electric potential.
- Divide the total charge Q into small pieces of charge ΔQ, using shapes for which you *already know* how to determine V. This division is often, but not always, into point charges.
- Identify distances that need to be calculated.

Problem-Solving Strategy: The Electric Potential of a Continuous Distribution of Charge

PROBLEM-SOLVING STRATEGY 25.2

The electric potential of a continuous distribution of charge

SOLVE The mathematical representation is $V = \sum V_i$.

- Use superposition to form an algebraic expression for the potential at P. Let the (x, y, z) coordinates of the point remain as variables.
- Replace the small charge ΔQ with an equivalent expression involving a *charge density* and a *coordinate*, such as dx. **This is the critical step in making the transition from a sum to an integral** because you need a coordinate to serve as the integration variable.
- All distances must be expressed in terms of the coordinates.
- Let the sum become an integral. The integration limits will depend on the coordinate system you have chosen.

ASSESS Check that your result is consistent with any limits for which you know what the potential should be.

Exercise 29

Example 25.10 The Potential of a Ring of Charge

EXAMPLE 25.10 | The potential of a ring of charge

A thin, uniformly charged ring of radius R has total charge Q. Find the potential at distance z on the axis of the ring.

MODEL Because the ring is thin, we'll assume the charge lies along a circle of radius R.

Example 25.10 The Potential of a Ring of Charge

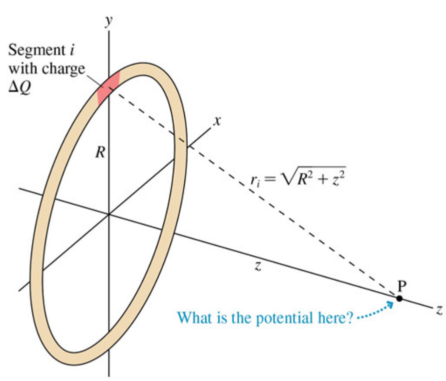
EXAMPLE 25.10 **The potential of a ring of charge**

VISUALIZE **FIGURE 25.30** on the next page illustrates the problem-solving strategy. We've chosen a coordinate system in which the ring lies in the xy-plane and point P is on the z-axis. We've then divided the ring into N small segments of charge ΔQ, each of which can be modeled as a point charge. The distance r_i between segment i and point P is

$$r_i = \sqrt{R^2 + z^2}$$

Note that r_i is a constant distance, the same for every charge segment.

Example 25.10 The Potential of a Ring of Charge



Example 25.10 The Potential of a Ring of Charge

EXAMPLE 25.10 **The potential of a ring of charge**

SOLVE The potential V at P is the sum of the potentials due to each segment of charge:

$$V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{R^2+z^2}}\sum_{i=1}^{N}\Delta Q$$

We were able to bring all terms involving z to the front because z is a constant as far as the summation is concerned. Surprisingly, we don't need to convert the sum to an integral to complete this calculation. The sum of all the ΔQ charge segments around the ring is simply the ring's total charge, $\sum(\Delta Q) = Q$; hence the electric potential on the axis of a charged ring is

$$V_{\text{ring on axis}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+z^2}}$$

Example 25.10 The Potential of a Ring of Charge

EXAMPLE 25.10 **The potential of a ring of charge**

ASSESS From far away, the ring appears as a point charge Q in the distance. Thus we expect the potential of the ring to be that of a point charge when $z \gg R$. You can see that $V_{ring} \approx Q/4\pi\epsilon_0 z$ when $z \gg R$, which is, indeed, the potential of a point charge Q.

Chapter 25 Summary Slides

General Principles

Sources of Potential

The **electric potential** V, like the electric field, is created by source charges. Two major tools for calculating the potential are:

- The potential of a point charge, $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$
- The principle of superposition

For multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \cdots$

For a continuous distribution of charge

MODEL Model as a simple charge distribution.

VISUALIZE Draw a pictorial representation.

- Establish a coordinate system.
- Identify where the potential will be calculated.

SOLVE Set up a sum.

- Divide the charge into point-like ΔQ.
- Find the potential due to each ΔQ.
- Use the charge density (λ or η) to replace ΔQ with an integration coordinate, then sum by integrating.

V is easier to calculate than $\vec{E}$ because potential is a scalar.

General Principles

Electric Potential Energy

If charge q is placed in an electric potential V, the system's **electric potential energy** (interaction energy) is

$$U = qV$$

Point charges and dipoles

The electric potential energy of two **point charges** is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

The potential energy of two opposite charges is negative.
The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Solving conservation of energy problems

MODEL Model as an isolated system.

VISUALIZE Draw a before-and-after representation.

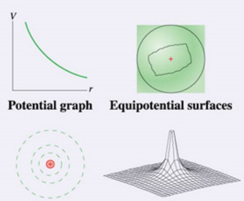
SOLVE Mechanical energy is conserved.

- Mathematically $K_f + qV_f = K_i + qV_i$.
- K is the sum of the kinetic energies of all particles.
- V is the potential due to the source charges.

 Slide 25-115

Applications

Graphical representations of the potential:



 Slide 25-116

Applications

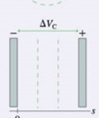
Sphere of charge Q

Same as a point charge if $r \geq R$

Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \frac{\Delta V_C}{d}$$



Units

Electric potential: 1 V = 1 J/C

Electric field: 1 V/m = 1 N/C

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