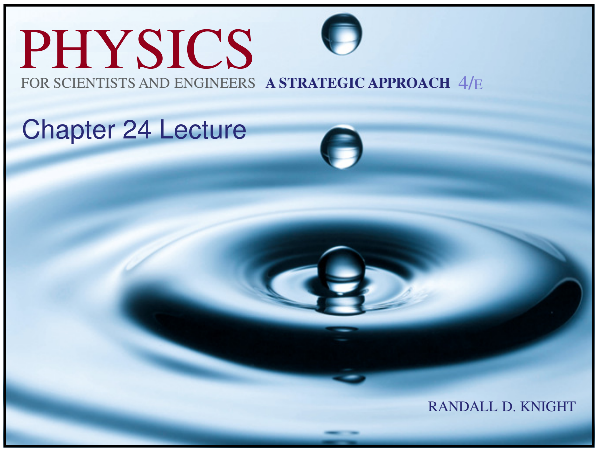
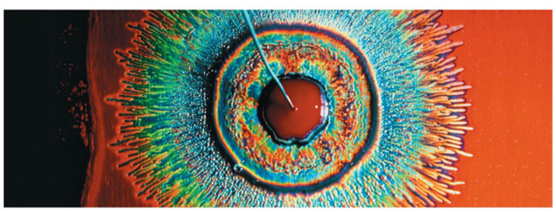


Chapter 24 Gauss's Law



IN THIS CHAPTER, you will learn about and apply Gauss's law.

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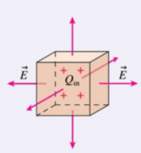
Chapter 24 Preview

What is Gauss's law?

Gauss's law is a general statement about the nature of electric fields. It is more fundamental than Coulomb's law and is the first of what we will later call Maxwell's equations, the governing equations of electricity and magnetism.



Gauss's law says that the electric flux through a closed surface is proportional to the amount of charge Q_{in} enclosed within the surface. This seemingly abstract statement will be the basis of a powerful strategy for finding the electric fields of charge distributions that have a high degree of symmetry.

« LOOKING BACK Section 22.5 The electric field of a point charge Section 23.2 Electric field lines

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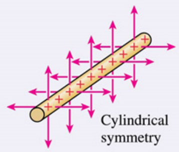
Chapter 24 Preview

What good is symmetry?

For charge distributions with a high degree of symmetry, the symmetry of the electric field must match the symmetry of the charge distribution. Important symmetries are planar symmetry, cylindrical symmetry, and spherical symmetry. The concept of symmetry plays an important role in math and science.



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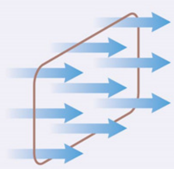
Chapter 24 Preview

What is electric flux?

The amount of electric field passing through a surface is called the electric flux. Electric flux is analogous to the amount of air or water flowing through a loop. You will learn to calculate the flux through open and closed surfaces.

« LOOKING BACK Section 9.3 Vector dot products



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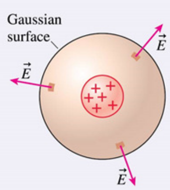
Chapter 24 Preview

How is Gauss's law used?

Gauss's law is easier to use than superposition for finding the electric field both inside and outside of charged spheres, cylinders, and planes. To use Gauss's law, you calculate the electric flux through a Gaussian surface surrounding the charge. This will turn out to be much easier than it sounds!



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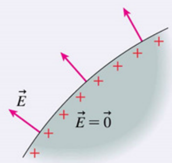
Chapter 24 Preview

What can we learn about conductors?

Gauss's law can be used to establish several properties of conductors in electrostatic equilibrium. In particular:

- Any excess charge is all on the surface.
- The interior electric field is zero.
- The external field is perpendicular to the surface.



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Chapter 24 Reading Questions

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Reading Question 24.1

The amount of electric field passing through a surface is called

A. Electric flux.
B. Gauss's Law.
C. Electricity.
D. Charge surface density.
E. None of the above.

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Reading Question 24.1

The amount of electric field passing through a surface is called

✔ **A. Electric flux.**
B. Gauss's Law.
C. Electricity.
D. Charge surface density.
E. None of the above.

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Reading Question 24.2

Gauss's law is useful for calculating electric fields that are

A. Symmetric.
B. Uniform.
C. Due to point charges.
D. Due to continuous charges.

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Reading Question 24.2

Gauss's law is useful for calculating electric fields that are

✔ **A. Symmetric.**
B. Uniform.
C. Due to point charges.
D. Due to continuous charges.

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Reading Question 24.3

Gauss's law applies to

A. Lines.
B. Flat surfaces.
C. Spheres only.
D. Closed surfaces.

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Reading Question 24.3

Gauss's law applies to

A. Lines.
B. Flat surfaces.
C. Spheres only.
✔ **D. Closed surfaces.**

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Reading Question 24.4

The electric field inside a conductor in electrostatic equilibrium is

A. Uniform.
B. Zero.
C. Radial.
D. Symmetric.

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Reading Question 24.4

The electric field inside a conductor in electrostatic equilibrium is

A. Uniform.
✔**B. Zero.**
C. Radial.
D. Symmetric.

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Chapter 24 Content, Examples, and QuickCheck Questions

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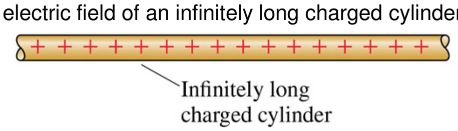
Electric Field of a Charged Cylinder

- Suppose we knew only two things about electric fields:
  1. The field points away from positive charges, toward negative charges.
  2. An electric field exerts a force on a charged particle.
- From this information alone, what can we deduce about the electric field of an infinitely long charged cylinder?



- All we know is that this charge is positive, and that it has *cylindrical symmetry*.

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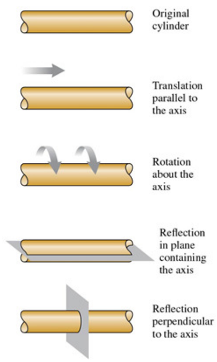
Cylindrical Symmetry

- An infinitely long charged cylinder is symmetric with respect to
  - Translation parallel to the cylinder axis.
  - Rotation by an angle about the cylinder axis.
  - Reflections in any plane containing or perpendicular to the cylinder axis.
- The symmetry of the electric field must match the symmetry of the charge distribution.



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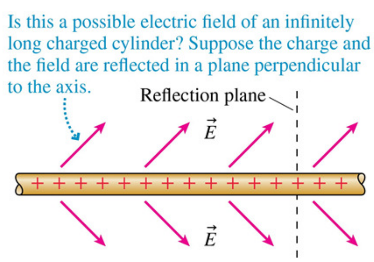
Slide 24-19

Electric Field of a Charged Cylinder

- Could the field look like the figure below? (Imagine this picture rotated about the axis.)
- The next slide shows what the field would look like reflected in a plane perpendicular to the axis (left to right).



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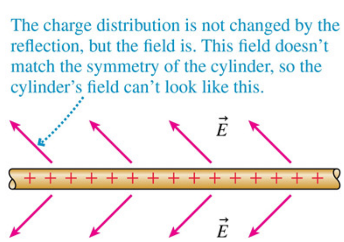
Slide 24-20

Electric Field of a Charged Cylinder

- This reflection, which does not make any change in the charge distribution itself, *does* change the electric field.
- Therefore, the electric field of a cylindrically symmetric charge distribution **cannot have a component parallel to the cylinder axis**.



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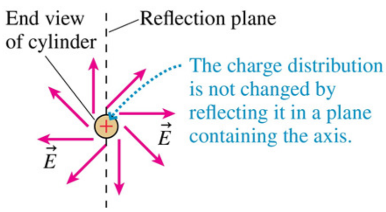
Slide 24-21

Electric Field of a Charged Cylinder

- Could the field look like the figure below? (Here we're looking down the axis of the cylinder.)
- The next slide shows what the field would look like reflected in a plane containing the axis (left to right).



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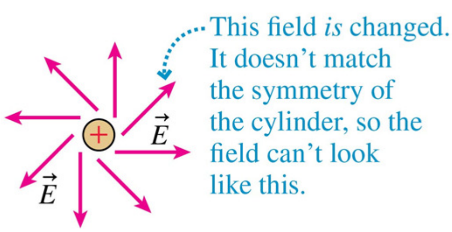
Electric Field of a Charged Cylinder

- This reflection, which does not make any change in the charge distribution itself, *does* change the electric field.
- Therefore, the electric field of a cylindrically symmetric charge distribution **cannot have a component tangent to the circular cross section**.



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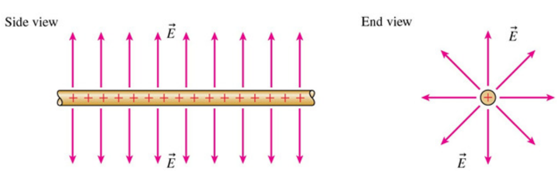
Electric Field of a Charged Cylinder

- Based on symmetry arguments alone, an infinitely long charged cylinder *must* have a radial electric field, as shown below.
- This is the one electric field shape that matches the symmetry of the charge distribution.



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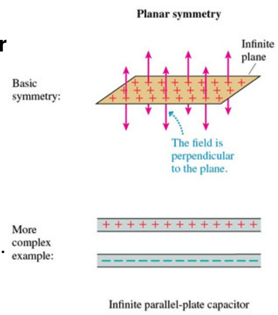
Planar Symmetry

- There are three fundamental symmetries; the first is **planar symmetry**.
- Planar symmetry involves symmetry with respect to:
  - Translation parallel to the plane.
  - Rotation about any line perpendicular to the plane.
  - Reflection in the plane.



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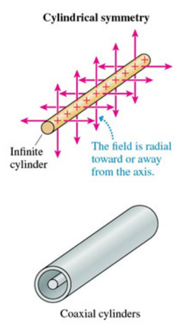
Cylindrical Symmetry

- There are three fundamental symmetries; the second is **cylindrical symmetry**.
- Cylindrical symmetry involves symmetry with respect to
  - Translation parallel to the axis.
  - Rotation about the axis.
  - Reflection in any plane containing or perpendicular to the axis.



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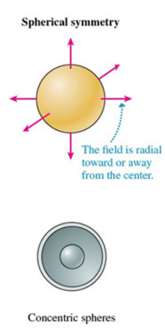
Spherical Symmetry

- There are three fundamental symmetries; the third is **spherical symmetry**.
- Spherical symmetry involves symmetry with respect to
  - Rotation about any axis that passes through the center point.
  - Reflection in any plane containing the center point.



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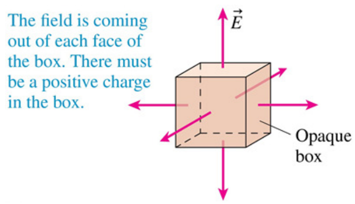
The Concept of Flux

- Consider a box surrounding a region of space.
- We can't see into the box, but we know there is an *outward-pointing* electric field passing through every surface.
- Since electric fields point away from positive charges, we can conclude that the box must contain net *positive* electric charge.



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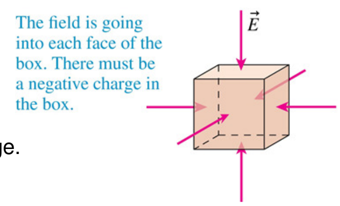
The Concept of Flux

- Consider a box surrounding a region of space.
- We can't see into the box, but we know there is an *inward-pointing* electric field passing through every surface.
- Since electric fields point toward negative charges, we can conclude that the box must contain net *negative* electric charge.



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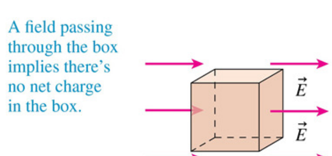
The Concept of Flux

- Consider a box surrounding a region of space.
- We can't see into the box, but we know that the electric field points into the box on the left, and an equal electric field points out of the box on the right.
- Since this external electric field is not altered by the contents of the box, the box must contain *zero* net electric charge.



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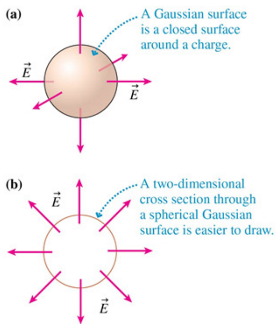
Gaussian Surfaces



- A closed surface through which an electric field passes is called a **Gaussian surface**.
- This is an imaginary, mathematical surface, not a physical surface.

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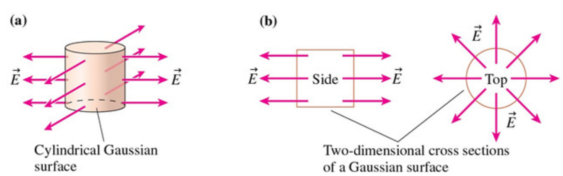
Slide 24-31

Gaussian Surfaces

- A Gaussian surface is most useful when it matches the shape and symmetry of the field.
- Figure (a) below shows a *cylindrical* Gaussian surface.
- Figure (b) simplifies the drawing by showing two-dimensional end and side views.
- The electric field is everywhere *perpendicular* to the side wall and no field passes through the top and bottom surfaces.



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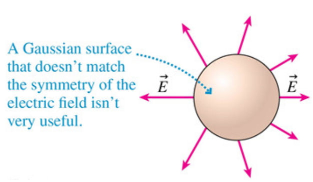
Slide 24-32

Gaussian Surfaces

- Not every surface is useful for learning about charge.
- Consider the spherical surface in the figure.



- This is a Gaussian surface, and the protruding electric field tells us there's a positive charge inside.
- It might be a point charge located on the left side, but we can't really say.
- A Gaussian surface that doesn't match the symmetry of the charge distribution isn't terribly useful.

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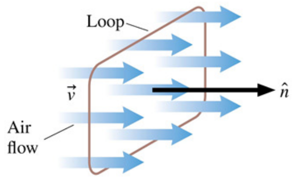
Slide 24-33

The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area A in front of a fan.
- The volume of air flowing through the loop each second depends on the angle between the loop and the direction of flow.
- The flow is *maximum* through a loop that is perpendicular to the airflow.



The air flowing through the loop is maximum when $\theta = 0°$.

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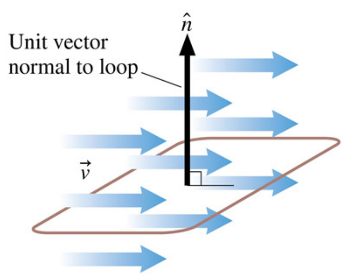
The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area A in front of a fan.
- No air goes through the same loop if it lies parallel to the flow.



No air flows through the loop when $\theta = 90°$.

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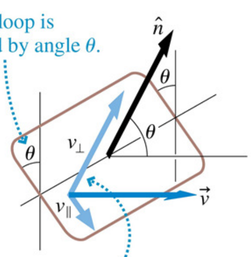
The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area A in front of a fan.
- The volume of air flowing through the loop each second depends on the angle θ between the loop normal and the velocity of the air:



$v_\perp = v\cos\theta$ is the component of the air velocity perpendicular to the loop.

$$\text{volume of air per second (m}^3\text{/s)} = v_\perp A = vA\cos\theta$$

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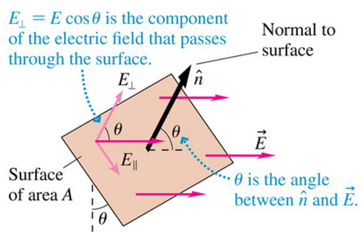
The Electric Flux

- The **electric flux** Φ_e measures the amount of electric field passing through a surface of area A whose normal to the surface is tilted at angle θ from the field.

$$\Phi_e = E_\perp A = EA\cos\theta$$



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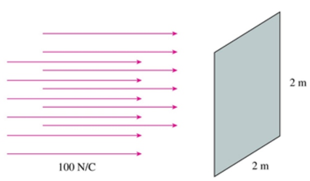
QuickCheck 24.1

The electric flux through the shaded surface is

A. 0
B. 200 N m/C
C. 400 N m^2/C
D. Flux isn't defined for an open surface.



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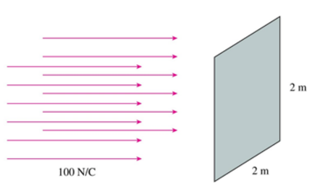
QuickCheck 24.1

The electric flux through the shaded surface is

A. 0
B. 200 N m/C
✔**C. 400 N m^2/C**
D. Flux isn't defined for an open surface.



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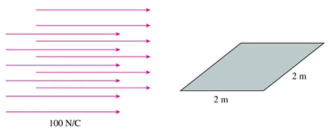
QuickCheck 24.2

The electric flux through the shaded surface is

A. 0
B. 200 N m/C
C. 400 N m^2/C
D. Some other value.



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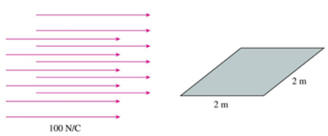
QuickCheck 24.2

The electric flux through the shaded surface is

✔ **A. 0**
B. 200 N m/C
C. 400 N m^2/C
D. Some other value.



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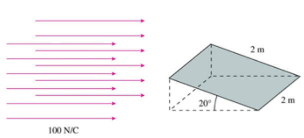
QuickCheck 24.3

The electric flux through the shaded surface is

A. 0
B. 400cos20° N m^2/C
C. 400cos70° N m^2/C
D. 400 N m^2/C
E. Some other value.



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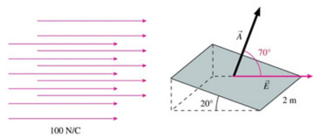
QuickCheck 24.3

The electric flux through the shaded surface is

A. 0
B. $400\cos 20^\circ$ N m^2/C
✔ **C. $400\cos 70^\circ$ N m^2/C**
D. 400 N m^2/C
E. Some other value.



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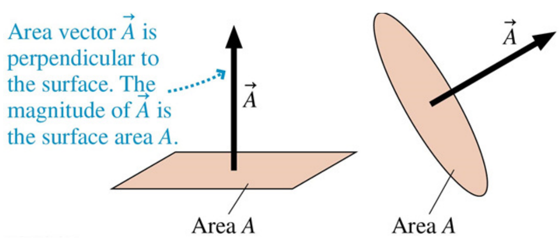
The Area Vector

- Let's define an area vector $\vec{A} = A\hat{n}$ to be a vector in the direction of $\hat{n}$, perpendicular to the surface, with a magnitude A equal to the area of the surface.
- Vector $\vec{A}$ has units of m^2.



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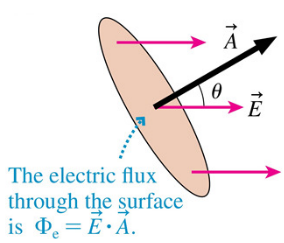
The Electric Flux

- An electric field passes through a surface of area A.
- The electric flux can be defined as the dot-product:

$$\Phi_e = \vec{E} \cdot \vec{A} \quad \text{(electric flux of a constant electric field)}$$



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Example 24.1 The Electric Flux Inside a Parallel-Plate Capacitor

EXAMPLE 24.1 **The electric flux inside a parallel-plate capacitor**

Two 100 cm^2 parallel electrodes are spaced 2.0 cm apart. One is charged to +5.0 nC, the other to −5.0 nC. A 1.0 cm × 1.0 cm surface between the electrodes is tilted to where its normal makes a 45° angle with the electric field. What is the electric flux through this surface?

MODEL Assume the surface is located near the center of the capacitor where the electric field is uniform. The electric flux doesn't depend on the shape of the surface.

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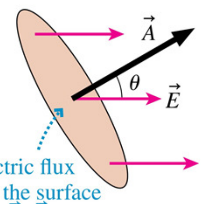
Example 24.1 The Electric Flux Inside a Parallel-Plate Capacitor

EXAMPLE 24.1 **The electric flux inside a parallel-plate capacitor**

VISUALIZE The surface is square, rather than circular, but otherwise the situation looks like Figure 24.13b.



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Example 24.1 The Electric Flux Inside a Parallel-Plate Capacitor

EXAMPLE 24.1 **The electric flux inside a parallel-plate capacitor**

SOLVE In Chapter 23, we found the electric field inside a parallel-plate capacitor to be

$$E = \frac{Q}{\epsilon_0 A_{\text{plates}}} = \frac{5.0 \times 10^{-9}\ \text{C}}{(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)(1.0 \times 10^{-2}\ \text{m}^2)}$$
$$= 5.65 \times 10^4\ \text{N/C}$$

A 1.0 cm × 1.0 cm surface has $A = 1.0 \times 10^{-4}\ \text{m}^2$. The electric flux through this surface is

$$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta$$
$$= (5.65 \times 10^4\ \text{N/C})(1.0 \times 10^{-4}\ \text{m}^2)\cos 45°$$
$$= 4.0\ \text{N m}^2/\text{C}$$

ASSESS The units of electric flux are the product of electric field and area units: N m^2/C.

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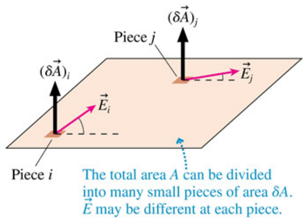
The Electric Flux of a Nonuniform Electric Field

- Consider a surface in a nonuniform electric field.
- Divide the surface into many small pieces of area δA.
- The electric flux through each small piece is

$$\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$$

- The electric flux through the whole surface is the **surface integral**:

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



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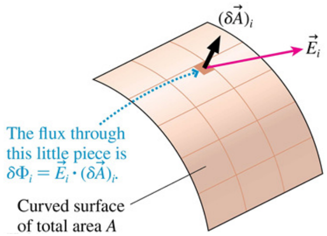
The Flux Through a Curved Surface

- Consider a curved surface in an electric field.
- Divide the surface into many small pieces of area δA.
- The electric flux through each small piece is

$$\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$$

- The electric flux through the whole surface is the **surface integral**:

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



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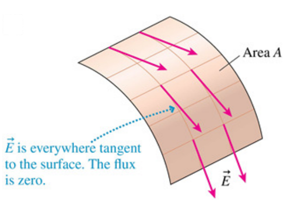
Electric Fields Tangent to a Surface

- Consider an electric field that is everywhere tangent, or parallel, to a curved surface.
- $\vec{E} \cdot d\vec{A}$ is *zero at every point* on the surface, because $\vec{E}$ is perpendicular to $d\vec{A}$ at every point.
- Thus $\Phi_e = 0$.



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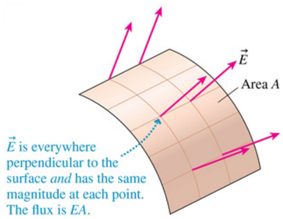
Electric Fields Perpendicular to a Surface

- Consider an electric field that is everywhere perpendicular to the surface *and* has the same magnitude E at every point.
- In this case,



$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_{\text{surface}} E\,dA = E \int_{\text{surface}} dA = EA$$

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Tactics: Evaluating Surface Integrals

TACTICS BOX 24.1

Evaluating surface integrals

1. If the electric field is everywhere tangent to a surface, the electric flux through the surface is $\Phi_e = 0$.
2. If the electric field is everywhere perpendicular to a surface *and* has the same magnitude E at every point, the electric flux through the surface is $\Phi_e = EA$.

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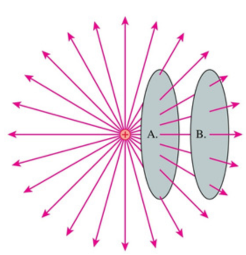
QuickCheck 24.4

Surfaces A and B have the same shape and the same area. Which has the larger electric flux?



A. Surface A has more flux.
B. Surface B has more flux.
C. The fluxes are equal.
D. It's impossible to say without knowing more about the electric field.

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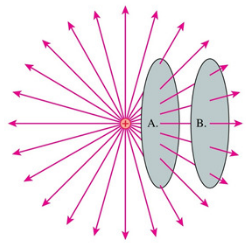
QuickCheck 24.4

Surfaces A and B have the same shape and the same area. Which has the larger electric flux?



✔ **A. Surface A has more flux.**
B. Surface B has more flux.
C. The fluxes are equal.
D. It's impossible to say without knowing more about the electric field.

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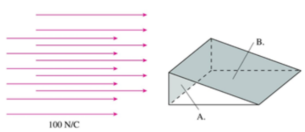
QuickCheck 24.5

Which surface, A or B, has the larger electric flux?



A. Surface A has more flux.
B. Surface B has more flux.
C. The fluxes are equal.
D. It's impossible to say without knowing more about the electric field.

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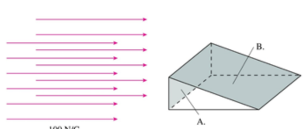
QuickCheck 24.5

Which surface, A or B, has the larger electric flux?



A. Surface A has more flux.
B. Surface B has more flux.
✔ **C. The fluxes are equal.**
D. It's impossible to say without knowing more about the electric field.

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The Electric Flux Through a Closed Surface

- The electric flux through a closed surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

- The electric flux is still the summation of the fluxes through a vast number of tiny pieces, pieces that now cover a closed surface.
- **NOTE:** For a closed surface, we use the convention that the area vector dA is defined to always point *toward the outside*.

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Tactics: Finding the Flux Through a Closed Surface

TACTICS BOX 24.2

Finding the flux through a closed surface

1. Choose a Gaussian surface made up of pieces that are everywhere tangent to the electric field or everywhere perpendicular to the electric field.
2. Use Tactics Box 24.1 to evaluate the surface integrals over these surfaces, then add the results.

Exercise 10

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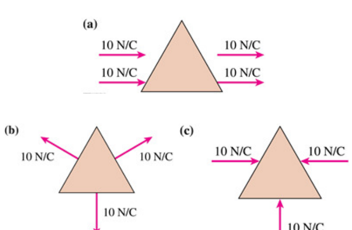
QuickCheck 24.6

These are cross sections of 3D closed surfaces. The top and bottom surfaces, which are flat, are in front of and behind the screen. The electric field is everywhere parallel to the screen. Which closed surface or surfaces have zero electric flux?

A. Surface A
B. Surface B
C. Surface C
D. Surfaces B and C
E. All three surfaces



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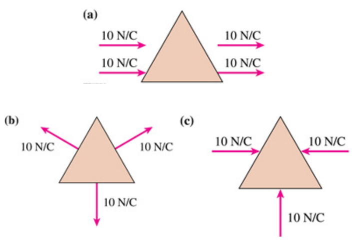
QuickCheck 24.6

These are cross sections of 3D closed surfaces. The top and bottom surfaces, which are flat, are in front of and behind the screen. The electric field is everywhere parallel to the screen. Which closed surface or surfaces have zero electric flux?

✔ **A. Surface A**
B. Surface B
C. Surface C
D. Surfaces B and C
E. All three surfaces



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Electric Flux of a Point Charge

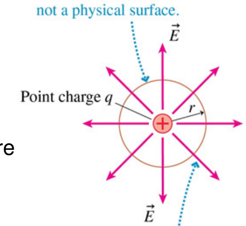
- The flux integral through a spherical Gaussian surface centered on a single point charge is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = EA_{\text{sphere}}$$

- The surface area of a sphere is $A_{\text{sphere}} = 4\pi r^2$.
- Using Coulomb's law for E, we find

$$\Phi_e = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

Cross section of a Gaussian sphere of radius r. This is a mathematical surface, not a physical surface.



The electric field is everywhere perpendicular to the surface *and* has the same magnitude at every point.

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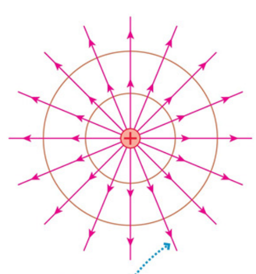
Electric Flux of a Point Charge

- The electric flux through a spherical surface centered on a single positive point charge is $\Phi_e = q/\epsilon_0$.
- This depends on the amount of charge, but *not* on the radius of the sphere.
- For a point charge, electric flux is *independent* of r.



Every field line passes through the smaller *and* the larger sphere. The flux through the two spheres is the same.

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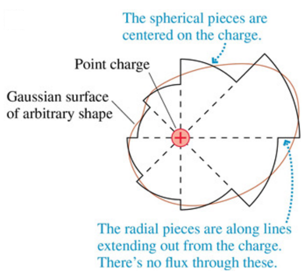
Electric Flux of a Point Charge

- The electric flux through any arbitrary closed surface surrounding a point charge q may be broken up into spherical and radial pieces.
- The total flux through the spherical pieces must be the same as through a single sphere:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$



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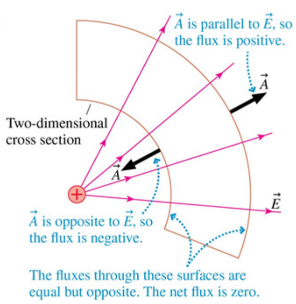
Electric Flux of a Point Charge

- The electric flux through any arbitrary closed surface entirely *outside* a point charge q may also be broken up into spherical and radial pieces.
- The total flux through the concave and convex spherical pieces must cancel each other.
- The net electric flux is *zero* through a closed surface that does not contain any net charge.



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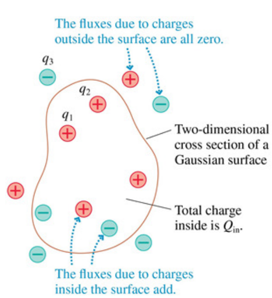
Electric Flux of Multiple Charges

- Consider an arbitrary Gaussian surface and a group of charges $q_1, q_2, q_3, \ldots$
- The contribution to the total flux for any charge q_i inside the surface is q_i/ϵ_0.
- The contribution for any charge outside the surface is zero.
- Defining Q_{in} to be the sum of all the charge inside the surface, we find $\Phi_e = Q_{in}/\epsilon_0$.



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Gauss's Law

- For any *closed* surface enclosing total charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

- This result for the electric flux is known as **Gauss's Law**.

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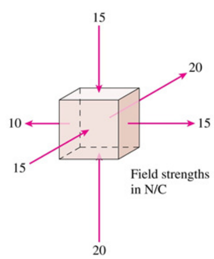
QuickCheck 24.7

The electric field is constant over each face of the box. The box contains



A. Positive charge.
B. Negative charge.
C. No net charge.
D. Not enough information to tell.

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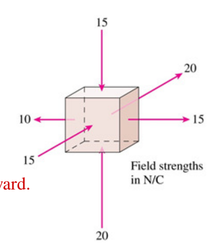
QuickCheck 24.7

The electric field is constant over each face of the box. The box contains



✔ **A. Positive charge.** Net flux is outward.
B. Negative charge.
C. No net charge.
D. Not enough information to tell.

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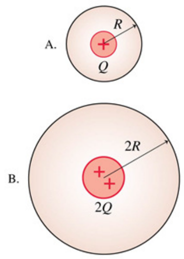
QuickCheck 24.8

Which spherical Gaussian surface has the larger electric flux?

A. Surface A
B. Surface B
C. They have the same flux.
D. Not enough information to tell.



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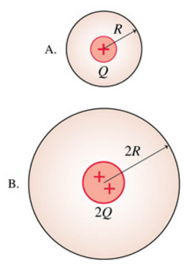
QuickCheck 24.8

Which spherical Gaussian surface has the larger electric flux?

A. Surface A
✔ **B. Surface B**
C. They have the same flux.
D. Not enough information to tell.



Flux depends only on the enclosed charge, not the radius.

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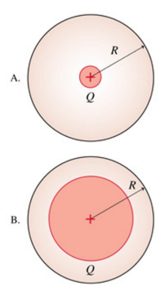
QuickCheck 24.9

Spherical Gaussian surfaces of equal radius R surround two spheres of equal charge Q. Which Gaussian surface has the larger electric field?

A. Surface A
B. Surface B
C. They have the same electric field.
D. Not enough information to tell.



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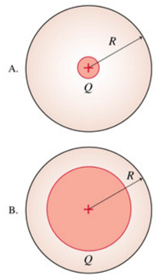
QuickCheck 24.9

Spherical Gaussian surfaces of equal radius R surround two spheres of equal charge Q. Which Gaussian surface has the larger electric field?

A. Surface A
B. Surface B
✔ **C. They have the same electric field.**
D. Not enough information to tell.



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Using Gauss's Law

1. Gauss's law applies only to a *closed* surface, called a Gaussian surface.
2. A Gaussian surface is not a physical surface. It need not coincide with the boundary of any physical object (although it could if we wished). It is an imaginary, mathematical surface in the space surrounding one or more charges.
3. We can't find the electric field from Gauss's law alone. We need to apply Gauss's law in situations where, from symmetry and superposition, we already can guess the *shape* of the field.

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Problem-Solving Strategy: Gauss's Law

PROBLEM-SOLVING STRATEGY 24.1 MP

Gauss's law

MODEL Model the charge distribution as a distribution with symmetry.

VISUALIZE Draw a picture of the charge distribution.
- Determine the symmetry of its electric field.
- Choose and draw a Gaussian surface with the *same symmetry*.
- You need not enclose all the charge within the Gaussian surface.
- Be sure every part of the Gaussian surface is either tangent to or perpendicular to the electric field.

SOLVE The mathematical representation is based on Gauss's law

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

- Use Tactics Boxes 24.1 and 24.2 to evaluate the surface integral.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 19

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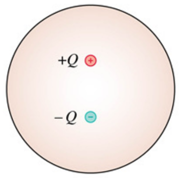
QuickCheck 24.10

A spherical Gaussian surface surrounds an electric dipole. The net enclosed charge is zero. Which is true?



A. The electric field is zero everywhere on the Gaussian surface.
B. The electric field is not zero everywhere on the Gaussian surface.
C. Whether or not the field is zero on the surface depends on where the dipole is inside the sphere.

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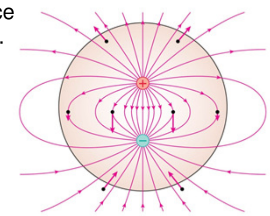
QuickCheck 24.10

A spherical Gaussian surface surrounds an electric dipole. The net enclosed charge is zero. Which is true?



A. The electric field is zero everywhere on the Gaussian surface.
✔**B. The electric field is not zero everywhere on the Gaussian surface.**
C. Whether or not the field is zero on the surface depends on where the dipole is inside the sphere.

The flux is zero, but that doesn't require the field to be zero.

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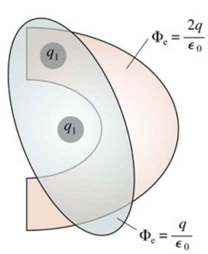
QuickCheck 24.11

The electric flux is shown through two Gaussian surfaces. In terms of q, what are charges q_1 and q_2?



A. $q_1 = 2q$; $q_2 = q$
B. $q_1 = q$; $q_2 = 2q$
C. $q_1 = 2q$; $q_2 = -q$
D. $q_1 = 2q$; $q_2 = -2q$
E. $q_1 = q/2$; $q_2 = q/2$

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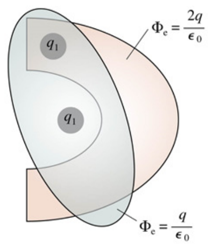
QuickCheck 24.11

The electric flux is shown through two Gaussian surfaces. In terms of q, what are charges q_1 and q_2?



A. $q_1 = 2q$; $q_2 = q$
B. $q_1 = q$; $q_2 = 2q$
✔C. **$q_1 = 2q$; $q_2 = -q$**
D. $q_1 = 2q$; $q_2 = -2q$
E. $q_1 = q/2$; $q_2 = q/2$

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Example 24.3 Outside a Sphere of Charge

EXAMPLE 24.3 **Outside a sphere of charge**

In Chapter 23 we asserted, without proof, that the electric field outside a sphere of total charge Q is the same as the field of a point charge Q at the center. Use Gauss's law to prove this result.

MODEL The charge distribution within the sphere need not be uniform (i.e., the charge density might increase or decrease with r), but it must have spherical symmetry in order for us to use Gauss's law. We will assume that it does.

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Example 24.3 Outside a Sphere of Charge

EXAMPLE 24.3 **Outside a sphere of charge**

VISUALIZE **FIGURE 24.23** shows a sphere of charge Q and radius R. We want to find $\vec{E}$ outside this sphere, for distances $r > R$. The spherical symmetry of the charge distribution tells us that the electric field must point *radially outward* from the sphere. Although Gauss's law is true for any surface surrounding the charged sphere, it is useful only if we choose a Gaussian surface to match the spherical symmetry of the charge distribution and the field. Thus a spherical surface of radius *r > R concentric with* the charged sphere will be our Gaussian surface. Because this surface surrounds the entire sphere of charge, the enclosed charge is simply $Q_{in} = Q$.

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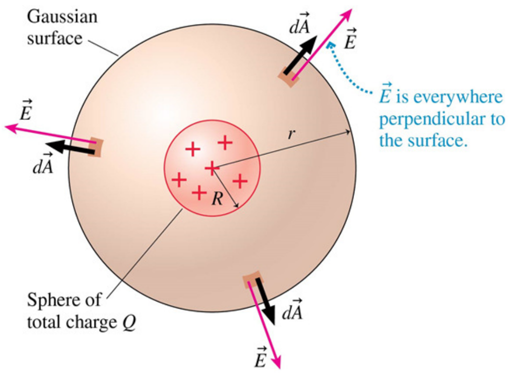
Example 24.3 Outside a Sphere of Charge



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Example 24.3 Outside a Sphere of Charge

EXAMPLE 24.3 Outside a sphere of charge

SOLVE Gauss's law is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

To calculate the flux, notice that the electric field is everywhere perpendicular to the spherical surface. And although we don't know the electric field magnitude E, spherical symmetry dictates that E must have the same value at all points equally distant from the center of the sphere.

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Example 24.3 Outside a Sphere of Charge

EXAMPLE 24.3 Outside a sphere of charge

SOLVE Thus we have the simple result that the net flux through the Gaussian surface is

$$\Phi_e = EA_{sphere} = 4\pi r^2 E$$

where we used the fact that the surface area of a sphere is $A_{sphere} = 4\pi r^2$. With this result for the flux, Gauss's law is

$$4\pi r^2 E = \frac{Q}{\epsilon_0}$$

Thus the electric field at distance r outside a sphere of charge is

$$E_{outside} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

Or in vector form, making use of the fact that $\vec{E}$ is radially outward,

$$\vec{E}_{outside} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

where $\hat{r}$ is a radial unit vector.

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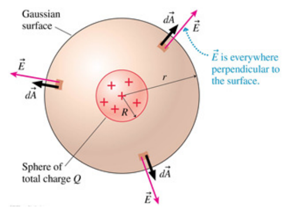
Example 24.3 Outside a Sphere of Charge

EXAMPLE 24.3 | **Outside a sphere of charge**

ASSESS The field is exactly that of a point charge Q, which is what we wanted to show.



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Example 24.6 The Electric Field of a Plane of Charge

EXAMPLE 24.6 | **The electric field of a plane of charge**

Use Gauss's law to find the electric field of an infinite plane of charge with surface charge density η (C/m^2).

MODEL A uniformly charged flat electrode can be modeled as an infinite plane of charge.

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Example 24.6 The Electric Field of a Plane of Charge

EXAMPLE 24.6 | **The electric field of a plane of charge**

VISUALIZE FIGURE 24.27 on the next page shows a uniformly charged plane with surface charge density η. We will assume that the plane extends infinitely far in all directions, although we obviously have to show "edges" in our drawing. The planar symmetry allows the electric field to point only straight toward or away from the plane. With this in mind, choose as a Gaussian surface a cylinder with length L and faces of area A centered on the plane of charge. Although we've drawn them as circular, the shape of the faces is not relevant.

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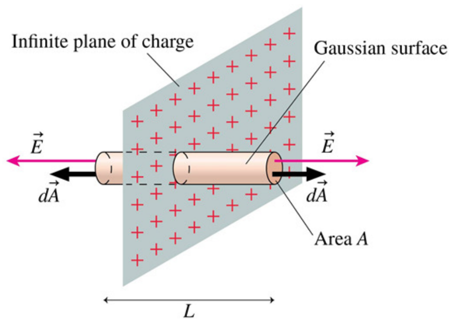
Example 24.6 The Electric Field of a Plane of Charge



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Example 24.6 The Electric Field of a Plane of Charge

EXAMPLE 24.6 **The electric field of a plane of charge**

SOLVE The electric field is perpendicular to both faces of the cylinder, so the total flux through both faces is $\Phi_{faces} = 2EA$. (The fluxes add rather than cancel because the area vector $\vec{A}$ points *outward* on each face.) There's *no* flux through the wall of the cylinder because the field vectors are tangent to the wall. Thus the net flux is simply

$$\Phi_e = 2EA$$

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Example 24.6 The Electric Field of a Plane of Charge

EXAMPLE 24.6 **The electric field of a plane of charge**

SOLVE The charge inside the cylinder is the charge contained in area A of the plane. This is

$$Q_{in} = \eta A$$

With these expressions for Q_{in} and Φ_e, Gauss's law is

$$\Phi_e = 2EA = \frac{Q_{in}}{\epsilon_0} = \frac{\eta A}{\epsilon_0}$$

Thus the electric field of an infinite charged plane is

$$E_{plane} = \frac{\eta}{2\epsilon_0}$$

This agrees with the result in Chapter 23.

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Example 24.6 The Electric Field of a Plane of Charge

EXAMPLE 24.6 The electric field of a plane of charge

ASSESS This is another example of a Gaussian surface enclosing only some of the charge. Most of the plane's charge is outside the Gaussian surface and does not contribute to the flux, but it does affect the shape of the field. We wouldn't have planar symmetry, with the electric field exactly perpendicular to the plane, without all the rest of the charge on the plane.

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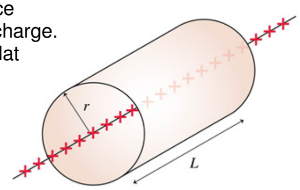
QuickCheck 24.12

A cylindrical Gaussian surface surrounds an infinite line of charge. The flux Φ_e through the two flat ends of the cylinder is



A. 0
B. $2 \times 2\pi rE$
C. $2 \times \pi r^2 E$
D. $2 \times rLE$
E. It will require an integration to find out.

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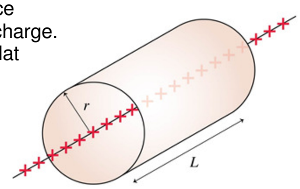
QuickCheck 24.12

A cylindrical Gaussian surface surrounds an infinite line of charge. The flux Φ_e through the two flat ends of the cylinder is



✔**A. 0**
B. $2 \times 2\pi rE$
C. $2 \times \pi r^2 E$
D. $2 \times rLE$
E. It will require an integration to find out.

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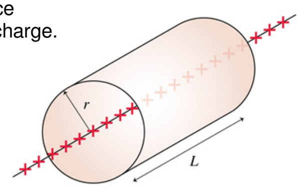
QuickCheck 24.13

A cylindrical Gaussian surface surrounds an infinite line of charge. The flux Φ_e through the wall of the cylinder is



A. 0
B. $2\pi rLE$
C. πr^2LE
D. rLE
E. It will require an integration to find out.

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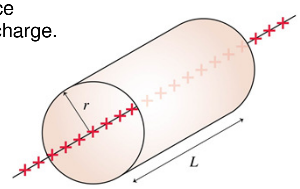
QuickCheck 24.13

A cylindrical Gaussian surface surrounds an infinite line of charge. The flux Φ_e through the wall of the cylinder is



A. 0
✔**B. $2\pi rLE$**
C. πr^2LE
D. rLE
E. It will require an integration to find out.

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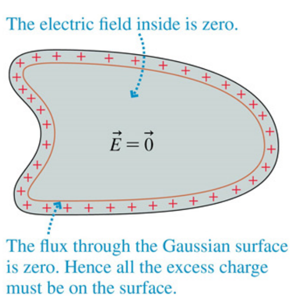
Conductors in Electrostatic Equilibrium

- The figure shows a Gaussian surface just inside a conductor's surface.
- The electric field must be zero at all points within the conductor, or else the electric field would cause the charge carriers to move and it wouldn't be in equilibrium.
- By Gauss's Law, $Q_{in} = 0$



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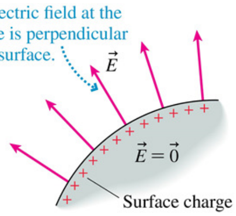
Conductors in Electrostatic Equilibrium

- **The external electric field right at the surface of a conductor must be perpendicular to that surface.**
- If it were to have a tangential component, it would exert a force on the surface charges and cause a surface current, and the conductor would not be in electrostatic equilibrium.



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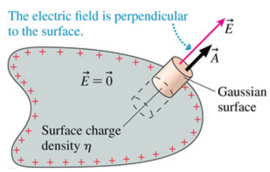
Electric Field at the Surface of a Conductor

- A Gaussian surface extending through the surface of a conductor has a flux only through the outer face.
- The net flux is $\Phi_e = AE_{surface} = Q_{in}/\epsilon_0$.
- Here $Q_{in} = \eta A$, so the electric field at the surface of a conductor is



$$\vec{E}_{surface} = \left(\frac{\eta}{\epsilon_0}, \text{ perpendicular to surface}\right)$$

where η is the surface charge density of the conductor.

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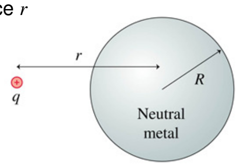
QuickCheck 24.14

A point charge q is located distance r from the center of a neutral metal sphere. The electric field at the center of the sphere is



A. $\dfrac{q}{4\pi\epsilon_0 r^2}$

B. $\dfrac{q}{4\pi\epsilon_0 R^2}$

C. $\dfrac{q}{4\pi\epsilon_0 (R-r)^2}$

D. 0

E. It depends on what the metal is.

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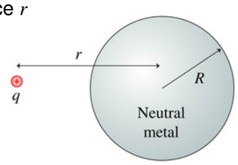
QuickCheck 24.14

A point charge q is located distance r from the center of a neutral metal sphere. The electric field at the center of the sphere is



A. $\dfrac{q}{4\pi\epsilon_0 r^2}$

B. $\dfrac{q}{4\pi\epsilon_0 R^2}$

C. $\dfrac{q}{4\pi\epsilon_0 (R-r)^2}$

✔ **D. 0**

E. It depends on what the metal is.

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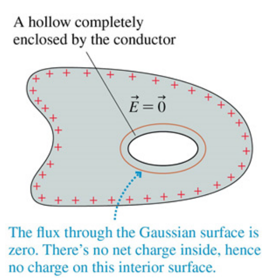
Conductors in Electrostatic Equilibrium

- The figure shows a charged conductor with a hole inside.
- Since the electric field is zero inside the conductor, we must conclude that $Q_{in} = 0$ for the interior surface.
- Furthermore, since there's no electric field inside the conductor and no charge inside the hole, the electric field in the hole must be zero.



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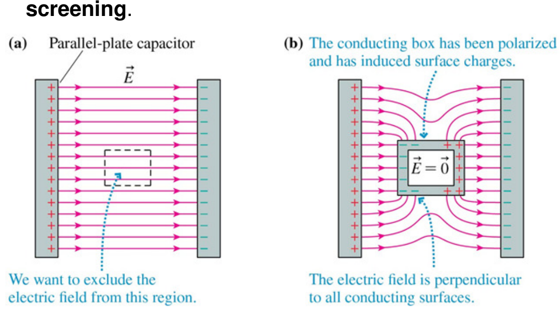
Faraday Cages

- The use of a conducting box, or *Faraday cage*, to exclude electric fields from a region of space is called **screening**.



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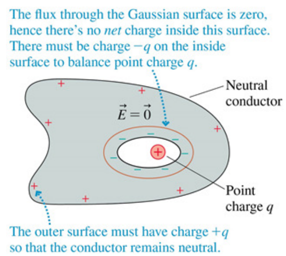
Conductors in Electrostatic Equilibrium

- The figure shows a charge q inside a hole within a neutral conductor.
- Net charge $-q$ moves to the inner surface and net charge $+q$ is left behind on the exterior surface.



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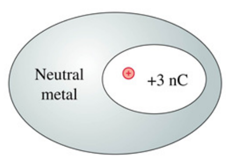
QuickCheck 24.15

Charge $+3$ nC is in a hollow cavity inside a large chunk of metal that is electrically neutral. The total charge on the exterior surface of the metal is



A. 0 nC
B. +3 nC
C. –3 nC
D. Can't say without knowing the shape and location of the hollow cavity.

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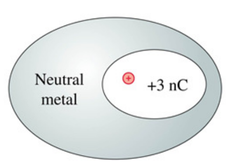
QuickCheck 24.15

Charge $+3$ nC is in a hollow cavity inside a large chunk of metal that is electrically neutral. The total charge on the exterior surface of the metal is



A. 0 nC
✔**B. +3 nC**
C. –3 nC
D. Can't say without knowing the shape and location of the hollow cavity.

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Tactics: Finding the Electric Field of a Conductor in Electrostatic Equilibrium

TACTICS BOX 24.3

Finding the electric field of a conductor in electrostatic equilibrium

1. The electric field is zero at all points within the volume of the conductor.
2. Any excess charge resides entirely on the *exterior* surface.
3. The external electric field at the surface of a charged conductor is perpendicular to the surface and of magnitude η/ϵ_0, where η is the surface charge density at that point.
4. The electric field is zero inside any hole within a conductor unless there is a charge in the hole.

Exercises 20–24

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Example 24.7 The Electric Field at the Surface of a Charged Metal Sphere

EXAMPLE 24.7 **The electric field at the surface of a charged metal sphere**

A 2.0-cm-diameter brass sphere has been given a charge of 2.0 nC. What is the electric field strength at the surface?

MODEL Brass is a conductor. The excess charge resides on the surface.

VISUALIZE The charge distribution has spherical symmetry. The electric field points radially outward from the surface.

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Example 24.7 The Electric Field at the Surface of a Charged Metal Sphere

EXAMPLE 24.7 **The electric field at the surface of a charged metal sphere**

SOLVE We can solve this problem in two ways. One uses the fact that a sphere, because of its complete symmetry, is the one shape for which any excess charge will spread out to a *uniform* surface charge density. Thus

$$\eta = \frac{q}{A_{\text{sphere}}} = \frac{q}{4\pi R^2} = \frac{2.0 \times 10^{-9}\ \text{C}}{4\pi(0.010\ \text{m})^2} = 1.59 \times 10^{-6}\ \text{C/m}^2$$

From Equation 24.20, we know the electric field at the surface has strength

$$E_{\text{surface}} = \frac{\eta}{\epsilon_0} = \frac{1.59 \times 10^{-6}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2} = 1.8 \times 10^5\ \text{N/C}$$

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Example 24.7 The Electric Field at the Surface of a Charged Metal Sphere

EXAMPLE 24.7 **The electric field at the surface of a charged metal sphere**

SOLVE Alternatively, we could have used the result, obtained earlier in the chapter, that the electric field strength outside a sphere of charge Q is $E_{\text{outside}} = Q_{\text{in}}/(4\pi\epsilon_0 r^2)$. But $Q_{\text{in}} = q$ and, at the surface, $r = R$. Thus

$$E_{\text{surface}} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2} = (9.0 \times 10^9\ \text{N m}^2/\text{C}^2)\frac{2.0 \times 10^{-9}\ \text{C}}{(0.010\ \text{m})^2}$$

$$= 1.8 \times 10^5\ \text{N/C}$$

As we can see, both methods lead to the same result.

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Chapter 24 Summary Slides

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General Principles

Gauss's Law

For any *closed* surface enclosing net charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

The electric flux Φ_e is the same for *any* closed surface enclosing charge Q_{in}.

To solve electric field problems with Gauss's law:

MODEL Model the charge distribution as one with symmetry.

VISUALIZE Draw a picture of the charge distribution. Draw a Gaussian surface with the same symmetry as the electric field, every part of which is either tangent to or perpendicular to the electric field.

SOLVE Apply Gauss's law and Tactics Boxes 24.1 and 24.2 to evaluate the surface integral.

ASSESS Is the result reasonable?

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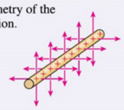
General Principles

Symmetry

The symmetry of the electric field must match the symmetry of the charge distribution.



In practice, Φ_e is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.

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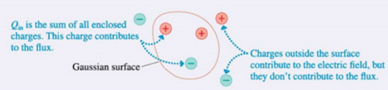
Important Concepts

Charge creates the electric field that is responsible for the electric flux.



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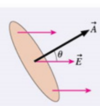
Important Concepts

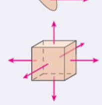
Flux is the amount of electric field passing through a surface of area A:

$$\Phi_e = \vec{E} \cdot \vec{A}$$

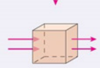
where $\vec{A}$ is the **area vector.**



For closed surfaces:
A net flux in or out indicates that the surface encloses a net charge.



Field lines through but with no *net* flux mean that the surface encloses no *net* charge.



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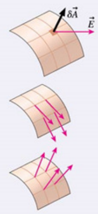
Important Concepts

Surface integrals calculate the flux by summing the fluxes through many small pieces of the surface:

$$\Phi_e = \sum \vec{E} \cdot \delta\vec{A}$$
$$\rightarrow \int \vec{E} \cdot d\vec{A}$$



Two important situations:

If the electric field is everywhere tangent to the surface, then

$$\Phi_e = 0$$

If the electric field is everywhere perpendicular to the surface *and* has the same strength E at all points, then

$$\Phi_e = EA$$

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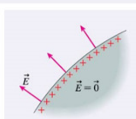
Applications

Conductors in electrostatic equilibrium

- The electric field is zero at all points within the conductor.
- Any excess charge resides entirely on the exterior surface.
- The external electric field is perpendicular to the surface and of magnitude η/ϵ_0, where η is the surface charge density.
- The electric field is zero inside any hole within a conductor unless there is a charge in the hole.



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