IN THIS CHAPTER, you will understand and use the ideas of superposition.

What is superposition?
Waves can pass through each other. When they do, their displacements add together at each point. This is called the principle of superposition. It is a property of waves but not of particles.

LOOKING BACK: Sections 16.1–16.4 Properties of traveling waves
Chapter 17 Preview

What is a standing wave?
A standing wave is created when two waves travel in opposite directions between two boundaries.
- Standing waves have well-defined patterns called nodes.
- Some points on the wave, called nodes, do not oscillate at all.

Chapter 17 Preview

How are standing waves related to music?
The notes played by musical instruments are standing waves.
- Guitars have string standing waves.
- Flutes have pressure standing waves.
Changing the length of a standing wave changes its frequency and the note played.
LOOKING BACK Section 16.5 Sound waves

Chapter 17 Preview

What is interference?
When two sources emit waves with the same wavelength, the overlapped waves create an interference pattern.
- Constructive interference (red) occurs where waves add to produce a wave with a larger amplitude.
- Destructive interference (black) occurs where waves cancel.
Chapter 17 Preview

**What are beats?**
The superposition of two waves with slightly different frequencies produces a loud-soft-loud-soft modulation of the intensity called beats. Beats have important applications in music, ultrasonics, and telecommunications.

---

Chapter 17 Preview

**Why is superposition important?**
Superposition and standing waves occur often in the world around us, especially when there are reflections. Musical instruments, microwave systems, and lasers all depend on standing waves. Standing waves are also important for large structures such as buildings and bridges. Superposition of light waves causes interference, which is used in electro-optic devices and precision measuring techniques.

---

Chapter 17 Reading Questions

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Reading Question 17.1

When a wave pulse on a string reflects from a hard boundary, how is the reflected pulse related to the incident pulse?

A. Shape unchanged, amplitude unchanged  
B. Shape inverted, amplitude unchanged  
C. Shape unchanged, amplitude reduced  
D. Shape inverted, amplitude reduced  
E. Amplitude unchanged, speed reduced

Answer: B. Shape inverted, amplitude unchanged

Reading Question 17.2

There are some points on a standing wave that never move. What are these points called?

A. Harmonics  
B. Normal Modes  
C. Nodes  
D. Anti-nodes  
E. Interference

Answer: C. Nodes
There are some points on a standing wave that never move. What are these points called?

A. Harmonics  
B. Normal Modes  
C. **Nodes**  
D. Anti-nodes  
E. Interference

Two sound waves of nearly equal frequencies are played simultaneously. What is the name of the acoustic phenomena you hear if you listen to these two waves?

A. **Beats**  
B. Diffraction  
C. Harmonics  
D. Chords  
E. Interference
Reading Question 17.4

The various possible standing waves on a string are called the
A. Antinodes.
B. Resonant nodes.
C. Normal modes. ✔
D. Incident waves.

Reading Question 17.5

The frequency of the third harmonic of a string is
A. One-third the frequency of the fundamental.
B. Equal to the frequency of the fundamental.
C. Three times the frequency of the fundamental.
D. Nine times the frequency of the fundamental.
The frequency of the third harmonic of a string is

A. One-third the frequency of the fundamental.
B. Equal to the frequency of the fundamental.
C. Three times the frequency of the fundamental.
D. Nine times the frequency of the fundamental.

Chapter 17 Content, Examples, and QuickCheck Questions

Particles versus Waves

- Two particles flying through the same point at the same time will collide and bounce apart, as in Figure (a).
- But waves, unlike particles, can pass directly through each other, as in Figure (b).
The Principle of Superposition

- If wave 1 displaces a particle in the medium by \( D_1 \) and wave 2 simultaneously displaces it by \( D_2 \), the net displacement of the particle is \( D_1 + D_2 \).

**Principle of superposition** When two or more waves are simultaneously present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

The figure shows the superposition of two waves on a string as they pass through each other.

- The principle of superposition comes into play wherever the waves overlap.
- The solid line is the sum at each point of the two displacements at that point.

QuickCheck 17.1

Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at \( t = 3 \) s?

A. 

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QuickCheck 17.1
Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at t = 3 s?

A. 

B. 

C. 

D.

QuickCheck 17.2
Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at t = 3 s?

A. 

B. 

C. 

D.
Standing Waves

- Shown is a time-lapse photograph of a standing wave on a vibrating string.
- It’s not obvious from the photograph, but this is actually a superposition of two waves.
- To understand this, consider two sinusoidal waves with the same frequency, wavelength, and amplitude traveling in opposite directions.

Standing Waves

- The red wave is traveling to the right.
- The green wave is traveling to the left.
- At this time the waves exactly overlap and the superposition has a maximum amplitude.
- At this time a crest of the red wave meets a trough of the green wave. The waves cancel.

Standing Waves

- The figure has collapsed several graphs into a single graphical representation of a standing wave.
- A striking feature of a standing-wave pattern is the existence of nodes, points that never move!
- The nodes are spaced \( \lambda/2 \) apart.
- Halfway between the nodes are the antinodes where the particles in the medium oscillate with maximum displacement.
In Chapter 16 you learned that the intensity of a wave is proportional to the square of the amplitude: $I \propto A^2$.

- Intensity is maximum at points of constructive interference and zero at points of destructive interference.

QuickCheck 17.3

What is the wavelength of this standing wave?

A. 0.25 m.
B. 0.5 m.
C. 1.0 m.
D. 2.0 m.
E. Standing waves don’t have a wavelength.

QuickCheck 17.3

What is the wavelength of this standing wave?

A. 0.25 m.
B. 0.5 m.
C. 1.0 m. **(Correct Answer)**
D. 2.0 m.
E. Standing waves don’t have a wavelength.
Standing Waves

- This photograph shows the Tacoma Narrows suspension bridge just before it collapsed.
- Aerodynamic forces caused the amplitude of a particular standing wave of the bridge to increase dramatically.
- The red line shows the original line of the deck of the bridge.

The Mathematics of Standing Waves

- A sinusoidal wave traveling to the right along the x-axis with angular frequency $\omega = 2\pi f$, wave number $k = 2\pi/\lambda$ and amplitude $a$ is
  $$D_R = a \sin(\dot{x} - \omega t)$$
- An equivalent wave traveling to the left is
  $$D_L = a \sin(\dot{x} + \omega t)$$
- We previously used the symbol $A$ for the wave amplitude, but here we will use a lowercase $a$ to represent the amplitude of each individual wave and reserve $A$ for the amplitude of the net wave.

The Mathematics of Standing Waves

- According to the principle of superposition, the net displacement of the medium when both waves are present is the sum of $D_R$ and $D_L$:
  $$D(x, t) = D_R + D_L = a \sin(kx - \omega t) + a \sin(kx + \omega t)$$
- We can simplify this by using a trigonometric identity, and arrive at
  $$D(x, t) = A(x) \cos(\omega t)$$
- Where the amplitude function $A(x)$ is defined as
  $$A(x) = 2a \sin(kx)$$
- The amplitude reaches a maximum value of $A_{\text{max}} = 2a$ at points where $\sin kx = 1$. 


The Mathematics of Standing Waves

- Shown is the graph of $D(x,t)$ at several instants of time.
- The nodes occur at $x_m = m\lambda/2$, where $m$ is an integer.

$$D(x,t) = A(x)\cos(at)$$

When $t = 0$, $\cos at = 1$. Thus the outer curve is the amplitude function $A(x)$.

The oscillation amplitude changes with position.

$$A(x) = 2a\sin kx$$

Example 17.1 Node Spacing on a String

**EXAMPLE 17.1** Node spacing on a string

A very long string has a linear density of 5.0 g/m and is stretched with a tension of 0.08 N. 100 Hz waves with amplitudes of 2.0 mm are generated at the ends of the string.

a. What is the node spacing along the resulting standing wave?

b. What is the maximum displacement of the string?

**Nodes.** Two counter-propagating waves of equal frequency create a standing wave.

**Example 17.1 Node Spacing on a String**

**EXAMPLE 17.1** Node spacing on a string

**VISUALIZE** The standing wave will look like Figure 17.5a.

(a) Nodes are spaced $\lambda/2$ apart.
Example 17.1 Node Spacing on a String

**EXAMPLE 17.1** Node spacing on a string

**SOLVE**

a. The speed of the waves on the string is

\[ v = \sqrt{\frac{T}{\mu}} \]

and the wavelength is

\[ \lambda = \frac{\text{length}}{n} = \frac{40 \text{ m}}{100 \text{ Hz}} = 0.4 \text{ m} = 40 \text{ cm} \]

Thus the spacing between adjacent nodes is \( \lambda/2 = 20 \text{ cm} \).

b. The maximum displacement is \( A_{\text{max}} = 2a = 4.0 \text{ cm} \).

---

Waves on a String with a Discontinuity

- A string with a large linear density is connected to one with a smaller linear density.
- The tension is the same in both strings, so the wave speed is slower on the left, faster on the right.
- When a wave encounters such a discontinuity, some of the wave’s energy is transmitted forward and some is reflected.

Before:

![String with slower wave speed]

Discontinuity where the wave speed increases

After:

![String with faster wave speed]

We say that the reflected wave has a phase change of \( \pi \) upon reflection.

---

Waves on a String with a Discontinuity

- Below, a wave encounters discontinuity at which the wave speed decreases.
- In this case, the reflected pulse is inverted.
When a wave reflects from a boundary, the reflected wave is inverted, but has the same amplitude.

The figure shows a string of length $L$ tied at $x = 0$ and $x = L$.

Reflections at the ends of the string cause waves of equal amplitude and wavelength to travel in opposite directions along the string.

These are the conditions that cause a standing wave!

For a string of fixed length $L$, the boundary conditions can be satisfied only if the wavelength has one of the values:

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, 4, \ldots$$

Because $\delta f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength $\lambda_m$ is

$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = \frac{v}{2L} \quad m = 1, 2, 3, 4, \ldots$$

The lowest allowed frequency is called the fundamental frequency: $f_1 = \frac{v}{2L}$. 
Shown are the first four possible standing waves on a string of fixed length $L$.

These possible standing waves are called the **modes** of the string, or sometimes the **normal modes**.

Each mode, numbered by the integer $m$, has a unique wavelength and frequency.

$m$ is the number of antinodes on the standing wave.

The **fundamental mode**, with $m = 1$, has $\lambda_1 = 2L$.

The frequencies of the normal modes form a series: $f_1$, $2f_1$, $3f_1$, …

The fundamental frequency $f_1$ can be found as the difference between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.

Below is a time-exposure photograph of the $m = 3$ standing wave on a string.

What is the mode number of this standing wave?

A. 4
B. 5
C. 6
D. Can’t say without knowing what kind of wave it is.
QuickCheck 17.4

What is the mode number of this standing wave?

A. 4
B. 5  
C. 6
D. Can’t say without knowing what kind of wave it is.

Mode # = number of antinodes

QuickCheck 17.5

A standing wave on a string vibrates as shown. Suppose the string tension is reduced to 1/4 its original value while the frequency and length are kept unchanged. Which standing wave pattern is produced?

A.  
B.  
C.  
D.  

The frequency is \( f_n = \frac{v}{2L} \).
Quartering the tension reduces \( v \) by one half. Thus \( m \) must double to keep the frequency constant.
Standing Electromagnetic Waves

- Standing electromagnetic waves can be established between two parallel mirrors that reflect light back and forth.
- A typical laser cavity has a length $L \approx 30 \text{ cm}$, and visible light has a wavelength $\lambda \approx 600 \text{ nm}$.
- The standing light wave in a typical laser cavity has a mode number $m$ that is $2L/\lambda \approx 1,000,000$!

Example 17.3 The Standing Light Wave Inside a Laser

**Example 17.3** The standing light wave inside a laser

Retroreflective lasers emit the red laser light commonly used in classroom demonstrations and supermarket checkout scanners. A helium-neon laser operates at a wavelength of precisely 632.9954 nm when the spacing between the mirrors is 781.372 mm.

a. In which mode does this laser operate?

b. What is the next longest wavelength that could form a standing wave in this laser cavity?

**Model** The light wave forms a standing wave between the two mirrors.

**Solve**

- a. We can use $m = 2L/n\lambda$ to find that $m$ (the mode) is $m = \frac{2L}{\lambda} = \frac{2 \times 781.372 \text{ mm}}{632.9954 \text{ nm}} = 800.659$

  There are 900.659 antinodes in the standing light wave.

- b. The next longest wavelength that can fit in this laser cavity will have one fewer node. It will be the $m = 900.659$ mode and its wavelength will be $\lambda = \frac{2L}{m} = \frac{2 \times 781.372 \text{ mm}}{900.659} = 632.9950 \text{ nm}$.

**Assess** The wavelength decreases by a mere 0.0004 nm when the mode number is decreased by 1.
A long, narrow column of air, such as the air in a tube or pipe, can support a longitudinal standing sound wave.

A closed end of a column of air must be a displacement node, thus the boundary conditions—nodes at the ends—are the same as for a standing wave on a string.

It is often useful to think of sound as a pressure wave rather than a displacement wave: The pressure oscillates around its equilibrium value.

The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.
Shown is the \( m = 2 \) standing sound wave in a closed-closed tube of air a half-cycle after \( t = 0 \).

The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.

**Example 17.4 Singing in the Shower**

A shower stall, \( L \), of rectangular cross-sectional area \( A \) is filled with air at temperature \( T \) and pressure \( P \). A standing wave can be produced in the air by making sound waves from a source placed at one end of the stall. The sound waves will be reflected from the other end of the stall, and the air in the stall will be driven into standing waves.

The possible standing wave frequencies are integer multiples of the fundamental frequency, \( f_1 \), of the source:

\[
f_n = nf_1 = n(\frac{v}{2L})
\]

where \( n \) is an integer and \( v \) is the speed of sound in air.

The speed of sound in air is given by:

\[
v = \sqrt{\frac{\gamma RT}{M}}
\]

where \( \gamma \) is the ratio of specific heats, \( R \) is the gas constant, \( T \) is the temperature, and \( M \) is the molecular weight of air.

The fundamental frequency, \( f_1 \), is given by:

\[
f_1 = \frac{v}{2L}
\]

The frequency of the sound wave in the stall is given by:

\[
f = nf_1 = n\left(\frac{v}{2L}\right)
\]

where \( n \) is an integer and \( v \) is the speed of sound in air.

The speed of sound in air is given by:

\[
v = \sqrt{\frac{\gamma RT}{M}}
\]

where \( \gamma \) is the ratio of specific heats, \( R \) is the gas constant, \( T \) is the temperature, and \( M \) is the molecular weight of air.

The fundamental frequency, \( f_1 \), is given by:

\[
f_1 = \frac{v}{2L}
\]

The frequency of the sound wave in the stall is given by:

\[
f = nf_1 = n\left(\frac{v}{2L}\right)
\]

where \( n \) is an integer and \( v \) is the speed of sound in air.

The speed of sound in air is given by:

\[
v = \sqrt{\frac{\gamma RT}{M}}
\]

where \( \gamma \) is the ratio of specific heats, \( R \) is the gas constant, \( T \) is the temperature, and \( M \) is the molecular weight of air.

The fundamental frequency, \( f_1 \), is given by:

\[
f_1 = \frac{v}{2L}
\]

The frequency of the sound wave in the stall is given by:

\[
f = nf_1 = n\left(\frac{v}{2L}\right)
\]

where \( n \) is an integer and \( v \) is the speed of sound in air.
Shown are displacement and pressure graphs for the first three standing-wave modes of a tube closed at both ends:

\[ \lambda_n = \frac{2L}{m} \]
\[ f_n = \frac{m}{2L} \sqrt{\frac{\rho}{\mu}} \]
\[ m = 1, 2, 3, 4, \ldots \]

Shown are displacement and pressure graphs for the first three standing-wave modes of a tube open at both ends:

\[ \lambda_n = \frac{2L}{m} \]
\[ f_n = \frac{m}{2L} \sqrt{\frac{\rho}{\mu}} \]
\[ m = 1, 2, 3, 4, \ldots \]

Shown are displacement and pressure graphs for the first three standing-wave modes of a tube open at one end but closed at the other:

\[ \lambda_n = \frac{4L}{m} \]
\[ f_n = \frac{m}{4L} \sqrt{\frac{\rho}{\mu}} \]
\[ m = 1, 3, 5, 7, \ldots \]
QuickCheck 17.6
An open-open tube of air has length \( L \). Which is the displacement graph of the \( m = 3 \) standing wave in this tube?

A. ![Graph A]
B. ![Graph B]
C. ![Graph C]
D. ![Graph D]

QuickCheck 17.6
An open-open tube of air has length \( L \). Which is the displacement graph of the \( m = 3 \) standing wave in this tube?

A. ![Graph A]
B. ![Graph B]
C. ![Graph C]
D. ![Graph D]

3/2 wavelengths
Antinodes at open ends

QuickCheck 17.7
An open-closed tube of air of length \( L \) has the closed end on the right. Which is the displacement graph of the \( m = 3 \) standing wave in this tube?

A. ![Graph A]
B. ![Graph B]
C. ![Graph C]
D. ![Graph D]
QuickCheck 17.7

An open-closed tube of air of length \( L \) has the closed end on the right. Which is the displacement graph of the \( m = 3 \) standing wave in this tube?

A. 

B. 

C. 

D. [Correct Answer]

3/4 wavelengths Node at closed end

Example 17.5 Resonances of the Ear Canal

EXAMPLE 17.5  Resonances of the ear canal

The ear drum, which transmits sound vibrations to the sensory organs of the inner ear, lies at the end of the ear canal. For adults, the ear canal is about 2.5 cm in length. What frequency standing waves can occur in the ear canal that are within the range of human hearing? The speed of sound in the warm air of the ear canal is 350 m/s.

MODEL: The ear canal is open to the air at one end, closed by the ear drum at the other. We can model it as an open-closed tube. The standing waves will be those of Figure 17.15.

Example 17.5 Resonances of the Ear Canal

EXAMPLE 17.5  Resonances of the ear canal

SOLVE: The lowest standing-wave frequency is the fundamental frequency for a 2.5-cm long open-closed tube:

\[
f_1 = \frac{v}{4L} = \frac{350 \text{ m/s}}{4(0.025 \text{ m})} = 3500 \text{ Hz}
\]

Standing waves also occur at the harmonics, but an open-closed tube has only odd harmonics. These are:

\( f_1 = \frac{v}{2L} = 18,300 \text{ Hz} \)
\( f_3 = \frac{3v}{4L} = 17,300 \text{ Hz} \)

Higher harmonics are beyond the range of human hearing, as discussed in Section 16.5.
Instruments such as the harp, the piano, and the violin have strings fixed at the ends and tightened to create tension. A disturbance generated on the string by plucking, striking, or bowing it creates a standing wave on the string. The fundamental frequency is the musical note you hear when the string is sounded:

\[ f_1 = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \]

where \( T \) is the tension in the string and \( \mu \) is its linear density.

With a wind instrument, blowing into the mouthpiece creates a standing sound wave inside a tube of air. The player changes the notes by using her fingers to cover holes or open valves, changing the length of the tube and thus its fundamental frequency:

\[ f_1 = \frac{v}{2L} \quad \text{for an open-open tube instrument,} \]
\[ f_1 = \frac{v}{4L} \quad \text{for an open-closed tube instrument, such as a clarinet} \]

In both of these equations, \( v \) is the speed of sound in the air inside the tube. Overblowing wind instruments can sometimes produce higher harmonics such as \( f_2 = 2f_1 \) and \( f_3 = 3f_1 \).
At room temperature, the fundamental frequency of an open-open tube is 500 Hz. If taken outside on a cold winter day, the fundamental frequency will be

A. Less than 500 Hz.
B. 500 Hz.
C. More than 500 Hz.

QuickCheck 17.8

At room temperature, the fundamental frequency of an open-open tube is 500 Hz. If taken outside on a cold winter day, the fundamental frequency will be

✓A. Less than 500 Hz.
B. 500 Hz.
C. More than 500 Hz.

Example 17.6 Flutes and Clarinets

**EXAMPLE 17.6** Flutes and clarinets

A clarinet is 66.0 cm long. A flute is nearly the same length, with 63.5 cm between the hole the player blows across and the end of the flute. What are the frequencies of the lowest note and the next higher harmonic on a flute and on a clarinet? The speed of sound in warm air is 350 m/s.

**MODEL.** The flute is an open-open tube, open at the end as well as at the hole the player blows across. A clarinet is an open-closed tube because the player's lips and the reed seal the tube at the upper end.
The pattern resulting from the superposition of two waves is often called interference. In this section we will look at the interference of two waves traveling in the same direction.

Two overlapped sound waves

Speaker 2  Speaker 1  Point of detection
Interference in One Dimension

- A sinusoidal wave traveling to the right along the \( x \)-axis has a displacement:
  \[ D = a \sin(kx - \omega t + \phi_0) \]
- The phase constant \( \phi_0 \) tells us what the source is doing at \( t = 0 \).

Constructive Interference

- \( D_1 = a \sin(kx_1 - \omega t + \phi_{10}) \)
- \( D_2 = a \sin(kx_2 - \omega t + \phi_{20}) \)
- \( D = D_1 + D_2 \)
- The two waves are in phase, meaning that \( D_1(x) = D_2(x) \)
- The resulting amplitude is \( A = 2a \) for maximum constructive interference.

Destructive Interference

- \( D_1 = a \sin(kx_1 - \omega t + \phi_{10}) \)
- \( D_2 = a \sin(kx_2 - \omega t + \phi_{20}) \)
- \( D = D_1 + D_2 \)
- The two waves are out of phase, meaning that \( D_1(x) = -D_2(x) \)
- The resulting amplitude is \( A = 0 \) for perfect destructive interference.
The Mathematics of Interference

- As two waves of equal amplitude and frequency travel together along the x-axis, the net displacement of the medium is:
  \[ D = D_1 + D_2 = a \sin(kx_1 - at + \phi_{10}) + a \sin(kx_2 - at + \phi_{20}) = a \sin \phi_1 + a \sin \phi_2 \]

- We can use a trigonometric identity to write the net displacement as
  \[ D = 2a \cos \left( \frac{\Delta \phi}{2} \right) \sin(kx_{eq} - at + \phi_{10}) \]
  where \( \Delta \phi = \phi_1 + \phi_2 \) is the phase difference between the two waves.

The Mathematics of Interference

- The amplitude has a maximum value \( A = 2a \) if \( \cos(\Delta \phi/2) = \pm 1 \). This is maximum constructive interference, when
  \[ \Delta \phi = m \cdot 2\pi \quad \text{(maximum amplitude } A = 2a) \]
  where \( m \) is an integer.

- Similarly, the amplitude is zero if \( \cos(\Delta \phi/2) = 0 \). This is perfect destructive interference, when:
  \[ \Delta \phi = \left( m + \frac{1}{2} \right) \cdot 2\pi \quad \text{(minimum amplitude } A = 0) \]

Interference in One Dimension

- Shown are two identical sources located one wavelength apart:
  \[ \Delta x = \lambda \]

- The two waves are "in step" with \( \Delta \phi = 2\pi \), so we have maximum constructive interference with \( A = 2a \).
Shown are two identical sources located half a wavelength apart:
\[ \Delta x = \lambda/2 \]
The two waves have phase difference \( \Delta \phi = \pi \), so we have perfect destructive interference with \( A = 0 \).

### Example 17.7 Interference Between Two Sound Waves

**Interference between two sound waves**

You are standing in front of two side-by-side loudspeakers playing sounds of the same frequency. Initially there is almost no sound at all. Then one of the speakers is moved slowly away from you. The sound intensity increases as the separation between the speakers increases, reaching a maximum when the speakers are 0.75 m apart. Then, as the speaker continues to move, the intensity starts to decrease. What is the distance between the speakers when the sound intensity is again a minimum?

**Visualizer:** Moving one speaker relative to the other changes the phase difference between the waves.

**Interference between two sound waves**

No. 1. A maximum sound intensity implies that the two sound waves are interfering destructively. Initially the loudspeakers are side by side, so the situation is as shown in Figure 17.2a with \( \Delta x = 0 \) and \( \Delta \phi = \pi \) rad. That is, the speakers themselves are out of phase. Moving one of the speakers does not change \( \Delta \phi \), but it does change the path length difference \( \Delta x \) and thus increases the sound phase difference \( \Delta \phi \). Constructive interference, causing maximum intensity, is reached when

\[ \Delta \phi = 2\pi m + \Delta \phi_0 = 2\pi \frac{\Delta x}{\lambda} = 2n\pi \]

where we used \( m = 1 \) because this is the first separation giving constructive interference.
Example 17.7 Interference Between Two Sound Waves

Example 17.7 Interference Between Two Sound Waves

Assess A separation of \( \lambda \) gives constructive interference for two identical speakers (\( \Delta \phi = 0 \)). Here the phase difference is \( \pi \) rad between the speakers (one is pushing forward as the other pulls back), giving destructive interference at this separation.

Example 17.7 Interference Between Two Sound Waves

It is entirely possible, of course, that the two waves are neither exactly in phase nor exactly out of phase.

• Shown are the calculated interference of two waves that differ in phase by \( 40^\circ \), \( 90^\circ \) and \( 160^\circ \).
Two loudspeakers emit sound waves with the same wavelength and the same amplitude. The waves are shown displaced, for clarity, but assume that both are traveling along the same axis. At the point where the dot is,

A. the interference is constructive.
B. the interference is destructive.
C. the interference is somewhere between constructive and destructive.
D. There's not enough information to tell about the interference.

QuickCheck 17.9

Two loudspeakers emit sound waves with the same wavelength and the same amplitude. Which of the following would cause there to be destructive interference at the position of the dot?

A. Move speaker 2 forward (right) 1.0 m.
B. Move speaker 2 forward (right) 0.5 m.
C. Move speaker 2 backward (left) 0.5 m.
D. Move speaker 2 backward (left) 1.0 m.
E. Nothing. Destructive interference is not possible in this situation.
QuickCheck 17.10

Two loudspeakers emit sound waves with the same wavelength and the same amplitude. Which of the following would cause there to be destructive interference at the position of the dot?

A. Move speaker 2 forward (right) 1.0 m.
B. Move speaker 2 forward (right) 0.5 m.
C. Move speaker 2 backward (left) 0.5 m.
D. Move speaker 2 backward (left) 1.0 m.
E. Nothing. Destructive interference is not possible in this situation.

---

Example 17.8 More Interference of Sound Waves

**EXAMPLE 17.8** More interference of sound waves

Two loudspeakers emit 500 Hz sound waves with an amplitude of 0.00 mm. Speaker 2 is 600 m behind speaker 1, and the phase difference between the speakers is 90°. What is the amplitude of the sound wave at a point 2.00 m in front of speaker 1?

**HINTS:** The amplitude is determined by the interference of the two waves. Assume that the speed of sound has a room-temperature (20°C) value of 343 m/s.

---

Example 17.8 More Interference of Sound Waves

**EXAMPLE 17.8** More interference of sound waves

**SOLVE** The amplitude of the sound wave is

\[ A = \frac{|a_1 \cos(\Delta \phi/2)|}{2} \]

where \( a = 0.00 \text{ mm} \) and the phase difference between the waves is

\[ \Delta \phi = \phi_2 - \phi_1 = 2\pi \frac{\Delta x}{\lambda} + \Delta \phi_0 \]

The sound’s wavelength is

\[ \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{500 \text{ Hz}} = 0.686 \text{ m} \]
Example 17.8 More Interference of Sound Waves

**EXAMPLE 17.8** More interference of sound waves

**SOLVE** Distances $s_1 = 2.00$ m and $s_2 = 3.00$ m are measured from the speakers, so the path-length difference is $\Delta s = 1.00$ m. We're given that the inherent phase difference between the speakers is $\Delta \phi_0 = \pi/2$ rad. Thus the phase difference at the observation point is

$$\Delta \phi = 2\pi \frac{\Delta s}{\lambda} = 2\pi \frac{1.00 \text{ m}}{0.006 \text{ m}} = \frac{\pi}{2} \text{ rad} = 10.73 \text{ rad}$$

and the amplitude of the wave at this point is

$$A = \frac{2A_1 A_2}{\sqrt{A_1^2 + A_2^2}} \left| \frac{\Delta \phi}{2} \right| = \left| \frac{0.200 \text{ mm} \cos \left( \frac{10.73}{2} \right)}{2} \right| = 0.121 \text{ mm}$$

**ASSESS** The interference is constructive because $A > a$, but less than maximum constructive interference.

Application: Thin-Film Optical Coatings

- Thin transparent films, placed on glass surfaces, such as lenses, can control reflections from the glass.
- Antireflection coatings on the lenses in cameras, microscopes, and other optical equipment are examples of thin-film coatings.

![Thin-film coating diagram]

Application: Thin-Film Optical Coatings

- The phase difference between the two reflected waves is

$$\Delta \phi = 2\pi \frac{2d}{\lambda m} = 2\pi \frac{2d}{\lambda}$$

where $n$ is the index of refraction of the coating, $d$ is the thickness, and $\lambda$ is the wavelength of the light in vacuum or air.

- For a particular thin-film, constructive or destructive interference depends on the wavelength of the light:

$$\lambda_C = \frac{2nd}{m} \quad m = 1, 2, 3, \ldots \quad \text{constructive interference}$$

$$\lambda_D = \frac{2nd}{m - \frac{1}{2}} \quad m = 1, 2, 3, \ldots \quad \text{destructive interference}$$
Example 17.9 Designing an Antireflection Coating

**Example 17.9** Designing an antireflection coating

Magnesium fluoride (MgF₂) is used as an antireflection coating on lenses. The index of refraction of MgF₂ is 1.39. What is the thinnest film of MgF₂ that works as an antireflection coating at λ = 510 nm, near the center of the visible spectrum?

**Model:** Reflection is minimized if the two reflected waves interfere destructively.

**Assess:** The reflected light is completely eliminated (perfect destructive interference) only if the two reflected waves have equal amplitudes. In practice, they don’t. Nonetheless, the reflection is reduced from 4% of the incident intensity for “bare glass” to well under 1%. Furthermore, the intensity of reflected light is much reduced across most of the visible spectrum (400–700 nm), even though the phase difference deviates more and more from π rad as the wavelength moves away from 510 nm. It is the increasing reflection at the ends of the visible spectrum (λ ~ 400 nm and λ ~ 700 nm), where ∆k deviates significantly from π rad, that gives a reddish-purple tinge to the lenses on cameras and binoculars. Homework problems will let you explore situations where only one of the two reflections has a reflection phase shift of π rad.

**Solve:** The film thicknesses that cause destructive interference at wavelength λ are

\[ d = \left( \frac{m}{2} \right) \frac{\lambda}{2n} \]

The thinnest film has m = 1. Its thickness is

\[ d = \frac{\lambda}{4n} = \frac{510 \text{ nm}}{4(1.39)} = 82 \text{ nm} \]

The film thickness is significantly less than the wavelength of visible light!
A Circular or Spherical Wave

- A circular or spherical wave can be written
  \[ D(r, t) = a \sin(kr - \omega t + \phi_0) \]
  where \( r \) is the distance measured outward from the source.
- The amplitude \( a \) of a circular or spherical wave diminishes as \( r \) increases.

Interference in Two and Three Dimensions

- Two overlapping water waves create an interference pattern.

Interference in Two and Three Dimensions

- Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.
- Points of destructive interference. A crest is aligned with a trough of another wave.
The mathematical description of interference in two or three dimensions is very similar to that of one-dimensional interference.

The conditions for constructive and destructive interference are

- Maximum constructive interference:
  \[ \Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = m \cdot 2\pi \]
  \[ m = 0, 1, 2, \ldots \]

- Maximum destructive interference:
  \[ \Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = (m + \frac{1}{2}) \cdot 2\pi \]

where \( \Delta r \) is the path-length difference.
QuickCheck 17.11

Two in-phase sources emit sound waves of equal wavelength and intensity. At the position of the dot,
A. The interference is constructive.
B. The interference is destructive.
C. The interference is somewhere between constructive and destructive.
D. There's not enough information to tell about the interference.

QuickCheck 17.12

Two in-phase sources emit sound waves of equal wavelength and intensity. How many antinodal lines (lines of constructive interference) are in the interference pattern?
A. 1
B. 2
C. 3
D. 4
E. 5
Two in-phase sources emit sound waves of equal wavelength and intensity. How many antinodal lines (lines of constructive interference) are in the interference pattern?

A. 1  
B. 2  
C. 3  
D. 4  
E. 5  

Sources are 1.5 \( \lambda \) apart, so no point can have \( \Delta r \) more than 1.5 \( \lambda \).

**Problem-Solving Strategy: Interference of Two Waves**

**PROBLEM-SOLVING STRATEGY 17.1**

**Interference of two waves**

**MODEL** Model the waves as linear, circular, or spherical.

**VISUALIZE** Draw a picture showing the sources of the waves and the point where the waves interfere. Give relevant dimensions. Identify the distances \( r_1 \) and \( r_2 \) from the sources to the point. Note any phase difference \( \Delta \phi \) between the two sources.

**SOLVE** The interference depends on the path-length difference \( \Delta r = r_2 - r_1 \) and the source phase difference \( \Delta \phi \).

Constructive:  
\[
\Delta \phi = 2\pi \frac{\Delta r}{\lambda} = m \cdot 2\pi 
\]

Destructive:  
\[
\Delta \phi = 2\pi \frac{\Delta r}{\lambda} = (m + \frac{1}{2}) \cdot 2\pi 
\]

For identical sources (\( \Delta \phi = 0 \)), the interference is maximum constructive if \( \Delta r = m\lambda \), maximum destructive if \( \Delta r = (m + \frac{1}{2})\lambda \).

**ASSIST** Check that your result has correct units and significant figures, is reasonable, and answers the question.
Example 17.10 Two-Dimensional Interference Between Two Loudspeakers

**EXAMPLE 17.10** Two-dimensional interference between two loudspeakers

Two loudspeakers are 2.0 m apart and in phase with each other. Both emit 700 Hz sound waves into a room where the speed of sound is 340 m/s. A listener stands 3.0 m in front of the loudspeakers and 2.0 m to one side of the center. Is the interference at this point maximum constructive, maximum destructive, or in between? How will the situation differ if the loudspeakers are out of phase?

**MODEL** The two speakers are sources of in-phase, spherical waves. The overlap of these waves causes interference.

---

**VISUALIZE** Figure 17.24 shows the loudspeakers and defines the distances \( r_1 \) and \( r_2 \) to the point of observation. The figure includes dimensions and notes that \( \Delta \theta = \frac{\pi}{2} \text{ rad} \).

---

**SOLVE** It's not \( r_1 \) and \( r_2 \) that matter, but the difference \( \Delta r \) between them. From the geometry of the figure we can calculate that

\[
\begin{align*}
\Delta r_1 &= \sqrt{(3.0 \text{ m})^{2} + (1.0 \text{ m})^{2}} = 3.10 \text{ m} \\
\Delta r_2 &= \sqrt{(5.0 \text{ m})^{2} + (3.0 \text{ m})^{2}} = 5.83 \text{ m}
\end{align*}
\]

Thus the path-length difference is \( \Delta r = r_2 - r_1 = 0.73 \) m. The wavelength of the sound waves is

\[
\lambda = \frac{\text{speed}}{\text{frequency}} = \frac{341 \text{ m/s}}{700 \text{ Hz}} = 0.487 \text{ m}
\]
Example 17.10 Two-Dimensional Interference Between Two Loudspeakers

**Example 17.10** Two-dimensional interference between two loudspeakers

**Solve** In terms of wavelengths, the path-length difference is

\[ \Delta \lambda = \frac{1}{2} \Delta \lambda \]

Because the sources are in phase (\( \Delta \phi = 0 \)), this is the condition for destructive interference. If the sources were out of phase (\( \Delta \phi = \pi \) rad), then the phase difference of the waves at the listener would be

\[ \Delta \phi = 2 \pi \Delta \lambda + \Delta \phi_0 = 2 \pi \left( \frac{1}{2} \right) + \pi \text{ rad} = 4 \pi \text{ rad} \]

This is an integer multiple of \( 2\pi \) rad, so in this case the interference would be constructive.

---

**Example 17.10 Two-Dimensional Interference Between Two Loudspeakers**

**Assess** Both the path-length difference and any inherent phase difference of the sources must be considered when evaluating interference.

---

**Beats**

- The figure shows the history graph for the superposition of the sound from two sources of equal amplitude \( a \), but slightly different frequency.
Beats

- With beats, the sound intensity rises and falls twice during one cycle of the modulation envelope.
- Each "loud-soft-loud" is one beat, so the beat frequency \( f_{\text{beat}} \), which is the number of beats per second, is twice the modulation frequency \( f_{\text{mod}} \).
- The beat frequency is
  \[
  f_{\text{beat}} = 2f_{\text{mod}} = 2 \cdot \frac{\omega_1 - \omega_2}{2\pi} = \frac{1}{\pi} \left( \frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi} \right) = |f_1 - f_2|
  \]
  where, to keep \( f_{\text{beat}} \) from being negative, we will always let \( f_1 \) be the larger of the two frequencies.
- The beat frequency is simply the difference between the two individual frequencies.

Visual Beats

- Shown is a graphical example of beats.
- Two "fences" of slightly different frequencies are superimposed on each other.
- The center part of the figure has two "beats" per inch:
  \[
  f_{\text{beat}} = 27 - 25 = 2
  \]

QuickCheck 17.13

You hear 2 beats per second when two sound sources, both at rest, play simultaneously. The beats disappear if source 2 moves toward you while source 1 remains at rest. The frequency of source 1 is 500 Hz. The frequency of source 2 is

A. 496 Hz  
B. 498 Hz  
C. 500 Hz  
D. 502 Hz  
E. 504 Hz
You hear 2 beats per second when two sound sources, both at rest, play simultaneously. The beats disappear if source 2 moves toward you while source 1 remains at rest. The frequency of source 1 is 500 Hz. The frequency of source 2 is

A. 496 Hz
B. 498 Hz
C. 500 Hz
D. 502 Hz
E. 504 Hz

Example 17.11 Detecting Bats with Beats

The little brown bat is a common species in North America. It emits echolocation pulses at a frequency of 40 kHz, well above the range of human hearing. To allow researchers to “hear” these bats, the bat detector shown in Figure 17.36 combines the bat’s sound wave at frequency $f_b$ with a wave of frequency $f_2$ from a tunable oscillator. The resulting beat frequency is then amplified and sent to a loudspeaker. To what frequency should the tunable oscillator be set to produce an audible beat frequency of 3 kHz?
Example 17.11 Detecting Bats with Beats

**Example 17.11** Detecting bats with beats

SOLVE Combining two waves with different frequencies gives a beat frequency:

\[ f_{\text{beat}} = \frac{|f_1 - f_2|}{2} \]

A beat frequency will be generated at 3 kHz if the oscillator frequency and the bat frequency differ by 3 kHz. An oscillator frequency of either 37 kHz or 43 kHz will work nicely.

**Assess** The electronic circuitry of radios, televisions, and cell phones makes extensive use of mixers to generate difference frequencies.

Chapter 17 Summary Slides

General Principles

**Principle of Superposition**

The displacement of a medium when more than one wave is present is the sum at each point of the displacements due to each individual wave.
**Important Concepts**

**Standing Waves**
Standing waves are due to the superposition of two traveling waves moving in opposite directions.

\[ \begin{align*}
\text{Node} & : y = 0 \\
\text{Antinode} & : y = A
\end{align*} \]

The equation of the nth node is given by:

\[ x = \frac{n\lambda}{2} \]

\[ \lambda = \frac{v}{f} \]

**Boundary Conditions**

- The boundary conditions determine which standing wave patterns are allowed. The allowed standing waves are modes of the system.

**Solving Interference Problems**

- **Constructive Interference** occurs when waves are in phase.
  - \( \Delta \phi = 0 \) or \( \Delta \phi = 2n\pi \)
  - \( \Delta \phi = (2n + 1)\pi \)

- **Destructive Interference** occurs when waves are out of phase.
  - \( \Delta \phi = \pi \) or \( \Delta \phi = (2n + 1)\pi \)

**Applications**

**Boundary Conditions**

- Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends.
  \[ \lambda = \frac{2\pi}{2} \]

- The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.
  - A sound wave in an open-open tube must have a node at the closed end and an antinode at the open end. This leads to:
  \[ \lambda = \frac{4L}{m} \]

  \[ m = 1, 3, 5, \ldots \]
Beats (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.

The beat frequency between waves of frequencies $f_1$ and $f_2$ is

$$f_{beat} = |f_1 - f_2|$$