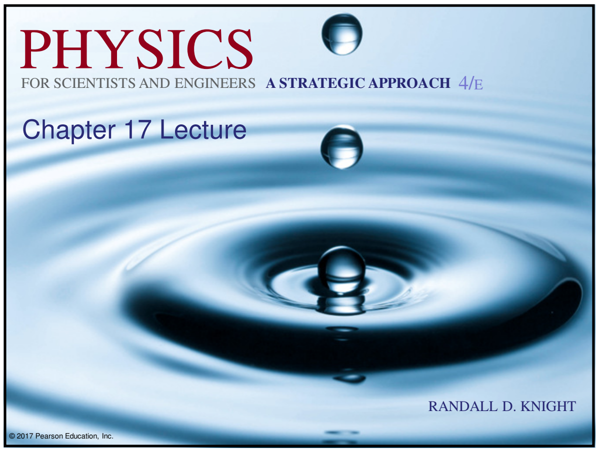


Chapter 17 Superposition

IN THIS CHAPTER, you will understand and use the ideas of superposition.

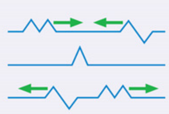
 Slide 17-2

Chapter 17 Preview

What is superposition?

Waves can pass through each other. When they do, their displacements add together at each point. This is called the principle of superposition. It is a property of waves but not of particles.

« LOOKING BACK Sections 16.1–16.4 Properties of traveling waves



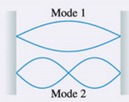
 Slide 17-3

Chapter 17 Preview

What is a standing wave?

A standing wave is created when two waves travel in opposite directions between two boundaries.

- Standing waves have well-defined patterns called modes.
- Some points on the wave, called nodes, do not oscillate at all.



 Slide 17-4

Chapter 17 Preview

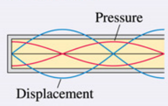
How are standing waves related to music?

The notes played by musical instruments are standing waves.

- Guitars have string standing waves.
- Flutes have pressure standing waves.

Changing the length of a standing wave changes its frequency and the note played.

« LOOKING BACK Section 16.5 Sound waves



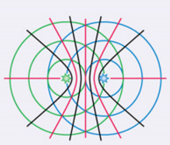
 Slide 17-5

Chapter 17 Preview

What is interference?

When two sources emit waves with the same wavelength, the overlapped waves create an interference pattern.

- Constructive interference (red) occurs where waves add to produce a wave with a larger amplitude.
- Destructive interference (black) occurs where waves cancel.

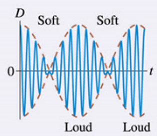


 Slide 17-6

Chapter 17 Preview

What are beats?

The superposition of two waves with slightly different frequencies produces a loud-soft-loud-soft modulation of the intensity called beats. Beats have important applications in music, ultrasonics, and telecommunications.



Slide 17-7

Chapter 17 Preview

Why is superposition important?

Superposition and standing waves occur often in the world around us, especially when there are reflections. Musical instruments, microwave systems, and lasers all depend on standing waves. Standing waves are also important for large structures such as buildings and bridges. Superposition of light waves causes interference, which is used in electro-optic devices and precision measuring techniques.

Slide 17-8

Chapter 17 Reading Questions

Slide 17-9

Reading Question 17.1

When a wave pulse on a string reflects from a hard boundary, how is the reflected pulse related to the incident pulse?

A. Shape unchanged, amplitude unchanged
B. Shape inverted, amplitude unchanged
C. Shape unchanged, amplitude reduced
D. Shape inverted, amplitude reduced
E. Amplitude unchanged, speed reduced

Slide 17-10

Reading Question 17.1

When a wave pulse on a string reflects from a hard boundary, how is the reflected pulse related to the incident pulse?

A. Shape unchanged, amplitude unchanged
✔ **B. Shape inverted, amplitude unchanged**
C. Shape unchanged, amplitude reduced
D. Shape inverted, amplitude reduced
E. Amplitude unchanged, speed reduced

Slide 17-11

Reading Question 17.2

There are some points on a standing wave that never move. What are these points called?

A. Harmonics
B. Normal Modes
C. Nodes
D. Anti-nodes
E. Interference

Slide 17-12

Reading Question 17.2

There are some points on a standing wave that never move. What are these points called?

A. Harmonics
B. Normal Modes
✔**C. Nodes**
D. Anti-nodes
E. Interference

 Slide 17-13

Reading Question 17.3

Two sound waves of nearly equal frequencies are played simultaneously. What is the name of the acoustic phenomena you hear if you listen to these two waves?

A. Beats
B. Diffraction
C. Harmonics
D. Chords
E. Interference

 Slide 17-14

Reading Question 17.3

Two sound waves of nearly equal frequencies are played simultaneously. What is the name of the acoustic phenomena you hear if you listen to these two waves?

✔**A. Beats**
B. Diffraction
C. Harmonics
D. Chords
E. Interference

 Slide 17-15

Reading Question 17.4

The various possible standing waves on a string are called the

A. Antinodes.
B. Resonant nodes.
C. Normal modes.
D. Incident waves.

Slide 17-16

Reading Question 17.4

The various possible standing waves on a string are called the

A. Antinodes.
B. Resonant nodes.
✔**C. Normal modes.**
D. Incident waves.

Slide 17-17

Reading Question 17.5

The frequency of the third harmonic of a string is

A. One-third the frequency of the fundamental.
B. Equal to the frequency of the fundamental.
C. Three times the frequency of the fundamental.
D. Nine times the frequency of the fundamental.

Slide 17-18

Reading Question 17.5

The frequency of the third harmonic of a string is

A. One-third the frequency of the fundamental.
B. Equal to the frequency of the fundamental.
✔ **C. Three times the frequency of the fundamental.**
D. Nine times the frequency of the fundamental.

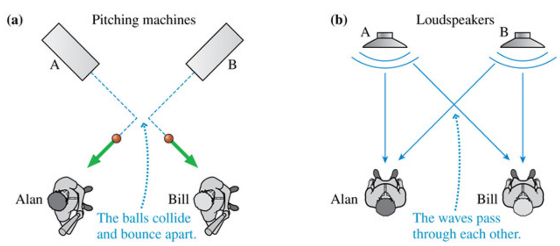
Slide 17-19

Chapter 17 Content, Examples, and QuickCheck Questions

Slide 17-20

Particles versus Waves

- Two particles flying through the same point at the same time will collide and bounce apart, as in Figure (a).
- But waves, unlike particles, can pass directly through each other, as in Figure (b).



Slide 17-21

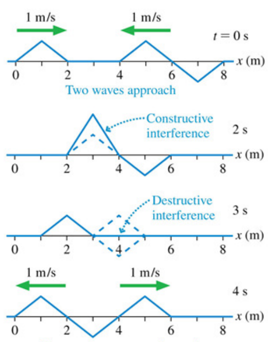
The Principle of Superposition

- If wave 1 displaces a particle in the medium by D_1 and wave 2 simultaneously displaces it by D_2, the net displacement of the particle is $D_1 + D_2$.

Principle of superposition When two or more waves are *simultaneously* present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

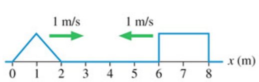
The Principle of Superposition

- The figure shows the superposition of two waves on a string as they pass through each other.
- The principle of superposition comes into play wherever the waves overlap.
- The solid line is the sum at each point of the two displacements at that point.

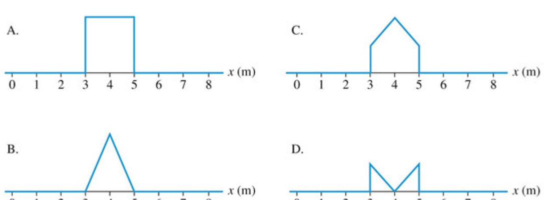


QuickCheck 17.1

Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at $t = 3$ s?

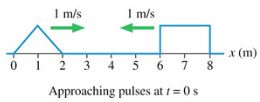


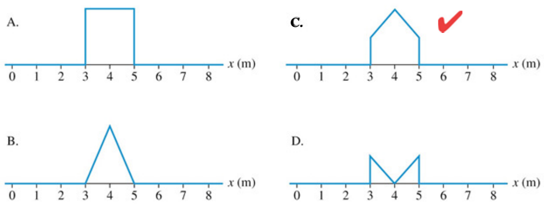
Approaching pulses at $t = 0$ s



QuickCheck 17.1

Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at $t = 3$ s?

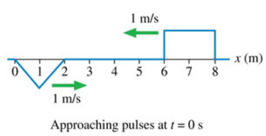


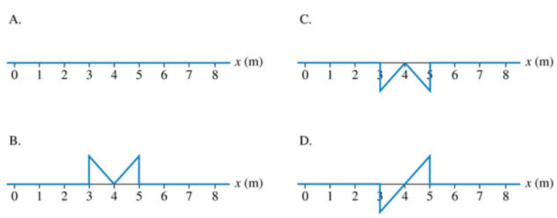


Slide 17-25

QuickCheck 17.2

Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at $t = 3$ s?

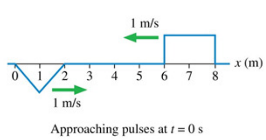


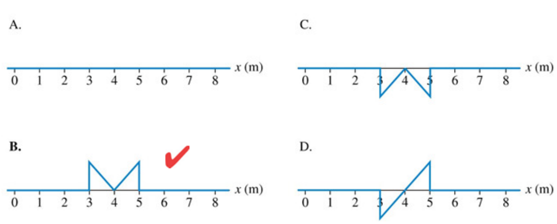


Slide 17-26

QuickCheck 17.2

Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at $t = 3$ s?





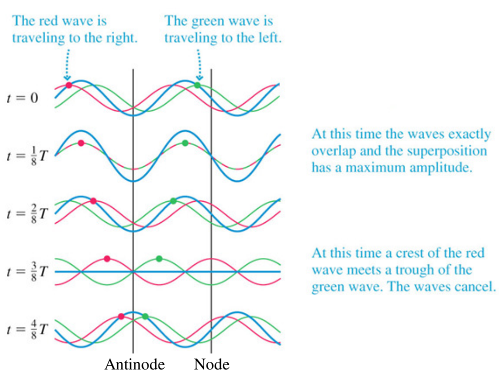
Slide 17-27

Standing Waves

- Shown is a time-lapse photograph of a *standing wave* on a vibrating string.

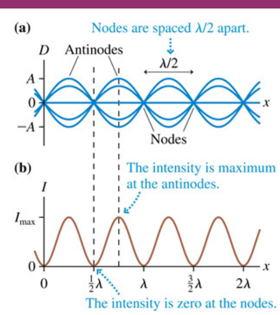
- It's not obvious from the photograph, but this is actually a superposition of two waves.
- To understand this, consider two sinusoidal waves with the **same frequency**, **wavelength**, **and amplitude** traveling in opposite directions.

Standing Waves



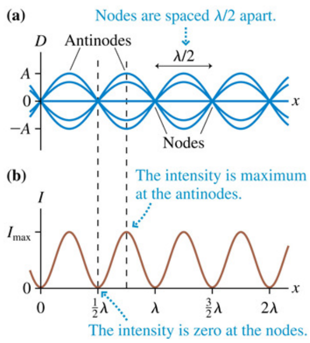
Standing Waves

- The figure has collapsed several graphs into a single graphical representation of a standing wave.
- A striking feature of a standing-wave pattern is the existence of **nodes**, points that *never move*!
- The nodes are spaced $\lambda/2$ apart.



- Halfway between the nodes are the **antinodes** where the particles in the medium oscillate with maximum displacement.

Standing Waves

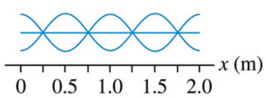


- In Chapter 16 you learned that the *intensity* of a wave is proportional to the square of the amplitude: $I \propto A^2$.
- Intensity is maximum at points of constructive interference and zero at points of destructive interference.

 Slide 17-31

QuickCheck 17.3

What is the wavelength of this standing wave?

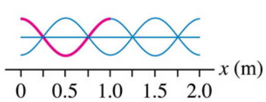


A. 0.25 m.
B. 0.5 m.
C. 1.0 m.
D. 2.0 m.
E. Standing waves don't have a wavelength.

 Slide 17-32

QuickCheck 17.3

What is the wavelength of this standing wave?



A. 0.25 m.
B. 0.5 m.
✔ **C. 1.0 m.**
D. 2.0 m.
E. Standing waves don't have a wavelength.

 Slide 17-33

Standing Waves

- This photograph shows the Tacoma Narrows suspension bridge just before it collapsed.
- Aerodynamic forces caused the amplitude of a particular standing wave of the bridge to increase dramatically.

- The red line shows the original line of the deck of the bridge.

Slide 17-34

The Mathematics of Standing Waves

- A sinusoidal wave traveling to the right along the x-axis with angular frequency $\omega = 2\pi f$, wave number $k = 2\pi/\lambda$ and amplitude a is

$$D_R = a\sin(kx - \omega t)$$

- An equivalent wave traveling to the left is

$$D_L = a\sin(kx + \omega t)$$

- We previously used the symbol A for the wave amplitude, but here we will use a lowercase a to represent the amplitude of each individual wave and reserve A for the amplitude of the net wave.

Slide 17-35

The Mathematics of Standing Waves

- According to the principle of superposition, the net displacement of the medium when both waves are present is the sum of D_R and D_L:

$$D(x, t) = D_R + D_L = a\sin(kx - \omega t) + a\sin(kx + \omega t)$$

- We can simplify this by using a trigonometric identity, and arrive at

$$D(x,t) = A(x)\cos\omega t$$

- Where the **amplitude function** $A(x)$ is defined as

$$A(x) = 2a\sin kx$$

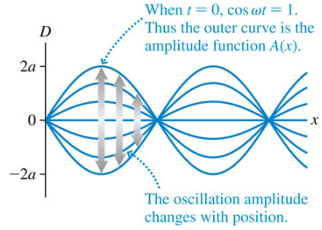
- The amplitude reaches a maximum value of $A_{max} = 2a$ at points where $\sin kx = 1$.

Slide 17-36

The Mathematics of Standing Waves

- Shown is the graph of $D(x,t)$ at several instants of time.
- The nodes occur at $x_m = m\lambda/2$, where m is an integer.

$D(x, t) = A(x)\cos\omega t$



$A(x) = 2a\sin kx$

Example 17.1 Node Spacing on a String

EXAMPLE 17.1 Node spacing on a string

A very long string has a linear density of 5.0 g/m and is stretched with a tension of 8.0 N. 100 Hz waves with amplitudes of 2.0 mm are generated at the ends of the string.

a. What is the node spacing along the resulting standing wave?

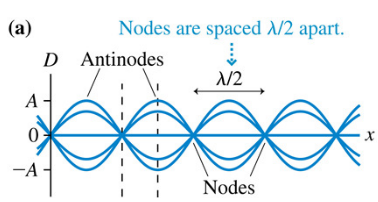
b. What is the maximum displacement of the string?

MODEL Two counter-propagating waves of equal frequency create a standing wave.

Example 17.1 Node Spacing on a String

EXAMPLE 17.1 Node spacing on a string

VISUALIZE The standing wave will look like Figure 17.5a.



Example 17.1 Node Spacing on a String

EXAMPLE 17.1 **Node spacing on a string**

SOLVE a. The speed of the waves on the string is

$$v = \sqrt{\frac{T_s}{\mu}} = \sqrt{\frac{8.0\ \text{N}}{0.0050\ \text{kg/m}}} = 40\ \text{m/s}$$

and the wavelength is

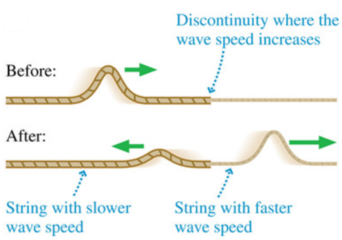
$$\lambda = \frac{v}{f} = \frac{40\ \text{m/s}}{100\ \text{Hz}} = 0.40\ \text{m} = 40\ \text{cm}$$

Thus the spacing between adjacent nodes is $\lambda/2 = 20$ cm.

b. The maximum displacement is $A_{max} = 2a = 4.0$ mm.

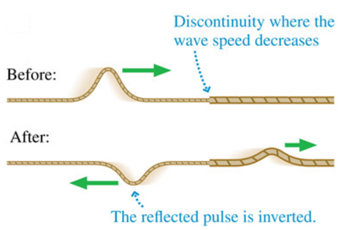
Waves on a String with a Discontinuity

- A string with a large linear density is connected to one with a smaller linear density.
- The tension is the same in both strings, so the wave speed is slower on the left, faster on the right.
- When a wave encounters such a discontinuity, some of the wave's energy is transmitted forward and some is reflected.



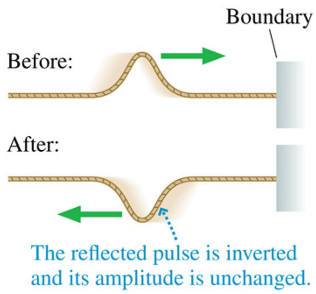
Waves on a String with a Discontinuity

- Below, a wave encounters discontinuity at which the wave speed decreases.
- In this case, the reflected pulse is *inverted*.
- We say that the reflected wave has a *phase change of π upon reflection*.



Waves on a String with a Boundary

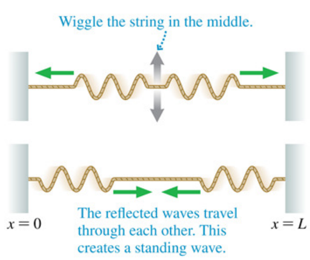
- When a wave reflects from a boundary, the reflected wave is inverted, but has the same amplitude.



 Slide 17-43

Creating Standing Waves

- The figure shows a string of length L tied at $x = 0$ and $x = L$.
- Reflections at the ends of the string cause waves of *equal amplitude and wavelength* to travel in opposite directions along the string.
- These are the conditions that cause a standing wave!



 Slide 17-44

Standing Waves on a String

- For a string of fixed length L, the boundary conditions can be satisfied only if the wavelength has one of the values:

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, 4, \ldots$$

- Because $\lambda f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength λ_m is

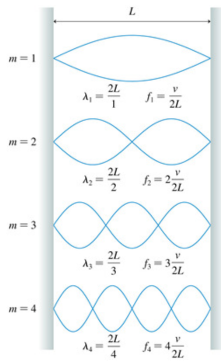
$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m\frac{v}{2L} \qquad m = 1, 2, 3, 4, \ldots$$

- The lowest allowed frequency is called the **fundamental frequency**: $f_1 = v/2L$.

 Slide 17-45

Standing Waves on a String

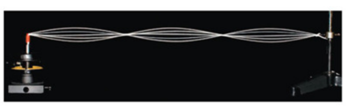
- Shown are the first four possible standing waves on a string of fixed length L.
- These possible standing waves are called the **modes** of the string, or sometimes the *normal modes*.
- Each mode, numbered by the integer m, has a unique wavelength and frequency.



 Slide 17-46

Standing Waves on a String

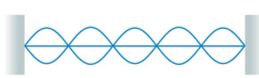
- m is the number of *antinodes* on the standing wave.
- The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
- The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \ldots$
- The fundamental frequency f_1 can be found as the *difference* between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.
- Below is a time-exposure photograph of the $m = 3$ standing wave on a string.



 Slide 17-47

QuickCheck 17.4

What is the mode number of this standing wave?



A. 4
B. 5
C. 6
D. Can't say without knowing what kind of wave it is.

 Slide 17-48

QuickCheck 17.4

What is the mode number of this standing wave?

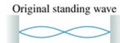
A. 4
✔ **B. 5**
C. 6
D. Can't say without knowing what kind of wave it is.

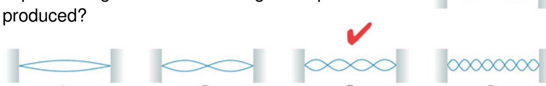
QuickCheck 17.5

A standing wave on a string vibrates as shown. Suppose the string tension is reduced to 1/4 its original value while the frequency and length are kept unchanged. Which standing wave pattern is produced?

QuickCheck 17.5

A standing wave on a string vibrates as shown. Suppose the string tension is reduced to 1/4 its original value while the frequency and length are kept unchanged. Which standing wave pattern is produced?



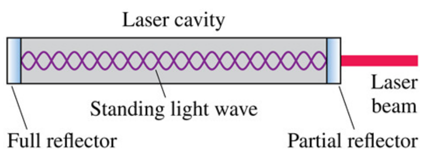


The frequency is $f_m = m\frac{v}{2L}$.

Quartering the tension reduces v by one half.
Thus m must double to keep the frequency constant.

Standing Electromagnetic Waves

- Standing electromagnetic waves can be established between two parallel mirrors that reflect light back and forth.
- A typical laser cavity has a length $L \approx 30$ cm, and visible light has a wavelength $\lambda \approx 600$ nm.
- The standing light wave in a typical laser cavity has a mode number m that is $2L/\lambda \approx 1{,}000{,}000$!



Slide 17-52

Example 17.3 The Standing Light Wave Inside a Laser

EXAMPLE 17.3 The standing light wave inside a laser

Helium-neon lasers emit the red laser light commonly used in classroom demonstrations and supermarket checkout scanners. A helium-neon laser operates at a wavelength of precisely 632.9924 nm when the spacing between the mirrors is 310.372 mm.

a. In which mode does this laser operate?

b. What is the next longest wavelength that could form a standing wave in this laser cavity?

MODEL The light wave forms a standing wave between the two mirrors.

Slide 17-53

Example 17.3 The Standing Light Wave Inside a Laser

EXAMPLE 17.3 The standing light wave inside a laser

VISUALIZE The standing wave looks like Figure 17.12.

SOLVE a. We can use $\lambda_m = 2L/m$ to find that m (the mode) is

$$m = \frac{2L}{\lambda_m} = \frac{2(0.310372\text{ m})}{6.329924 \times 10^{-7}\text{ m}} = 980{,}650$$

There are 980,650 antinodes in the standing light wave.

b. The next longest wavelength that can fit in this laser cavity will have one fewer node. It will be the $m = 980{,}649$ mode and its wavelength will be

$$\lambda = \frac{2L}{m} = \frac{2(0.310372\text{ m})}{980{,}649} = 632.9930\text{ nm}$$

ASSESS The wavelength increases by a mere 0.0006 nm when the mode number is decreased by 1.

Slide 17-54

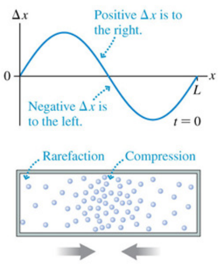
Standing Sound Waves

- A long, narrow column of air, such as the air in a tube or pipe, can support a longitudinal standing sound wave.
- A closed end of a column of air must be a displacement node, thus the boundary conditions—nodes at the ends—are the same as for a standing wave on a string.
- It is often useful to think of sound as a pressure wave rather than a displacement wave: The pressure oscillates around its equilibrium value.
- The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.

 Slide 17-55

Standing Sound Wave Time Sequence Slide 1 of 3

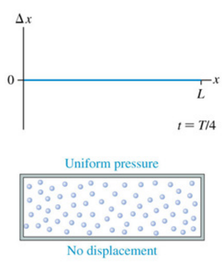
- Shown is the $m = 2$ standing sound wave in a closed-closed tube of air at $t = 0$.



 Slide 17-56

Standing Sound Wave Time Sequence Slide 2 of 3

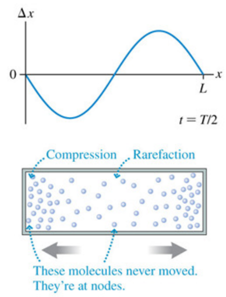
- Shown is the $m = 2$ standing sound wave in a closed-closed tube of air a quarter-cycle after $t = 0$.



 Slide 17-57

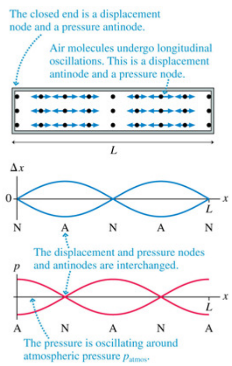
Standing Sound Wave Time Sequence Slide 3 of 3

- Shown is the $m = 2$ standing sound wave in a closed-closed tube of air a half-cycle after $t = 0$.



Standing Sound Waves

- Shown are the displacement Δx and pressure graphs for the $m = 2$ mode of standing sound waves in a closed-closed tube.
- The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.



Example 17.4 Singing in the Shower

EXAMPLE 17.4 | Singing in the shower

A shower stall is 2.45 m (8 ft) tall. For what frequencies less than 500 Hz are there standing sound waves in the shower stall?

MODEL The shower stall, to a first approximation, is a column of air 2.45 m long. It is closed at the ends by the ceiling and floor. Assume a 20°C speed of sound.

VISUALIZE A standing sound wave will have nodes at the ceiling and the floor. The $m = 2$ mode will look like Figure 17.14 rotated 90°.

SOLVE The fundamental frequency for a standing sound wave in this air column is

$$f_1 = \frac{v}{2L} = \frac{343\text{ m/s}}{2(2.45\text{ m})} = 70\text{ Hz}$$

The possible standing-wave frequencies are integer multiples of the fundamental frequency. These are 70 Hz, 140 Hz, 210 Hz, 280 Hz, 350 Hz, 420 Hz, and 490 Hz.

ASSESS The many possible standing waves in a shower cause the sound to *resonate,* which helps explain why some people like to sing in the shower. Our approximation of the shower stall as a one-dimensional tube is actually a bit too simplistic. A three-dimensional analysis would find additional modes, making the "sound spectrum" even richer.

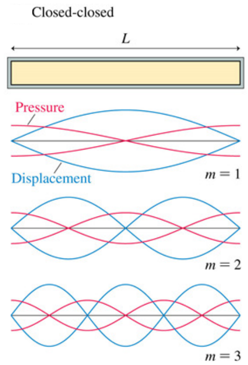
Standing Sound Waves

- Shown are displacement and pressure graphs for the first three standing-wave modes of a tube closed at both ends:

$$\lambda_m = \frac{2L}{m}$$

$$f_m = m\frac{v}{2L}$$

$$m = 1, 2, 3, 4, \ldots$$



Slide 17-61

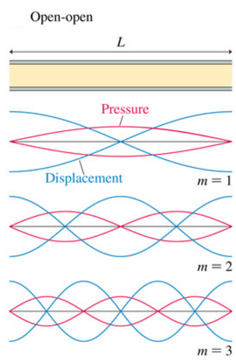
Standing Sound Waves

- Shown are displacement and pressure graphs for the first three standing-wave modes of a tube open at both ends:

$$\lambda_m = \frac{2L}{m}$$

$$f_m = m\frac{v}{2L}$$

$$m = 1, 2, 3, 4, \ldots$$



Slide 17-62

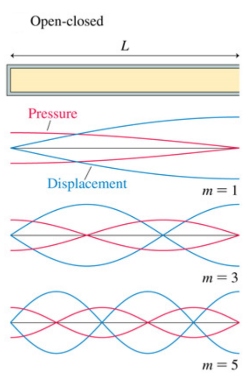
Standing Sound Waves

- Shown are displacement and pressure graphs for the first three standing-wave modes of a tube open at one end but closed at the other:

$$\lambda_m = \frac{4L}{m}$$

$$f_m = m\frac{v}{4L}$$

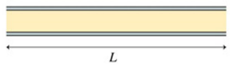
$$m = 1, 3, 5, 7, \ldots$$

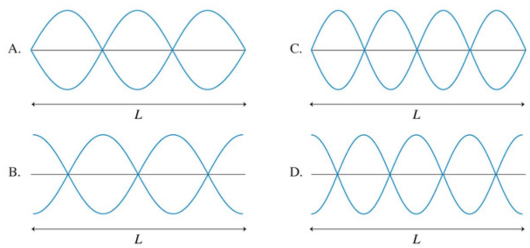


Slide 17-63

QuickCheck 17.6

An open-open tube of air has length L. Which is the displacement graph of the $m = 3$ standing wave in this tube?

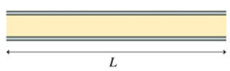


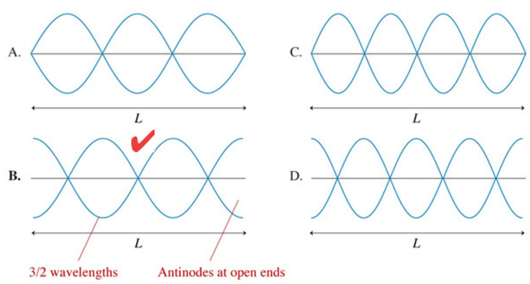


Slide 17-64

QuickCheck 17.6

An open-open tube of air has length L. Which is the displacement graph of the $m = 3$ standing wave in this tube?

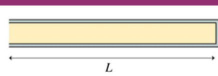


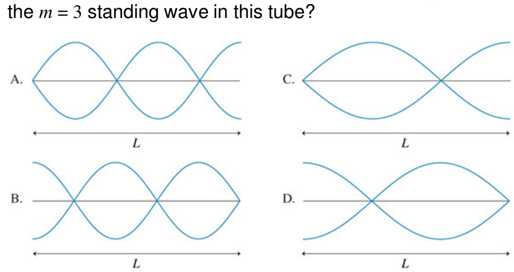


Slide 17-65

QuickCheck 17.7

An open-closed tube of air of length L has the closed end on the right. Which is the displacement graph of the $m = 3$ standing wave in this tube?





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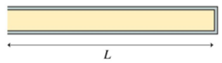
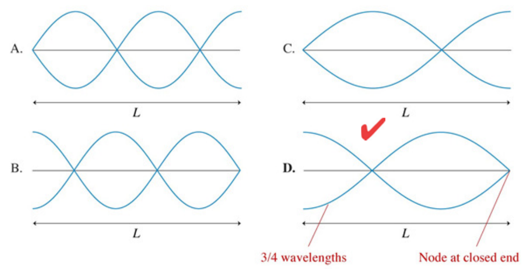
Slide 17-66

QuickCheck 17.7

An open-closed tube of air of length L has the closed end on the right. Which is the displacement graph of the $m = 3$ standing wave in this tube?





Slide 17-67

Example 17.5 Resonances of the Ear Canal

EXAMPLE 17.5 **Resonances of the ear canal**

The eardrum, which transmits sound vibrations to the sensory organs of the inner ear, lies at the end of the ear canal. For adults, the ear canal is about 2.5 cm in length. What frequency standing waves can occur in the ear canal that are within the range of human hearing? The speed of sound in the warm air of the ear canal is 350 m/s.

MODEL The ear canal is open to the air at one end, closed by the eardrum at the other. We can model it as an open-closed tube. The standing waves will be those of Figure 17.15c.

Slide 17-68

Example 17.5 Resonances of the Ear Canal

EXAMPLE 17.5 **Resonances of the ear canal**

SOLVE The lowest standing-wave frequency is the fundamental frequency for a 2.5-cm-long open-closed tube:

$$f_1 = \frac{v}{4L} = \frac{350\text{ m/s}}{4(0.025\text{ m})} = 3500\text{ Hz}$$

Standing waves also occur at the harmonics, but an open-closed tube has only odd harmonics. These are

$$f_3 = 3f_1 = 10{,}500\text{ Hz}$$
$$f_5 = 5f_1 = 17{,}500\text{ Hz}$$

Higher harmonics are beyond the range of human hearing, as discussed in Section 16.5.

Slide 17-69

Example 17.5 Resonances of the Ear Canal

EXAMPLE 17.5 **Resonances of the ear canal**

ASSESS The ear canal is short, so we expected the standing-wave frequencies to be relatively high. The air in your ear canal responds readily to sounds at these frequencies—what we call a *resonance* of the ear canal—and transmits theses sounds to the eardrum. Consequently, your ear actually is slightly more sensitive to sounds with frequencies around 3500 Hz and 10,500 Hz than to sounds at nearby frequencies.

 Slide 17-70

Musical Instruments

- Instruments such as the harp, the piano, and the violin have strings fixed at the ends and tightened to create tension.
- A disturbance generated on the string by plucking, striking, or bowing it creates a **standing wave** on the string.
- The fundamental frequency is the musical note you hear when the string is sounded:

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T_s}{\mu}}$$

where T_s is the tension in the string and μ is its linear density.

 Slide 17-71

Musical Instruments

- With a wind instrument, blowing into the mouthpiece creates a standing sound wave inside a tube of air.
- The player changes the notes by using her fingers to cover holes or open valves, changing the length of the tube and thus its fundamental frequency:

$$f_1 = \frac{v}{2L}$$ for an open-open tube instrument, such as a flute

$$f_1 = \frac{v}{4L}$$ for an open-closed tube instrument, such as a clarinet

- In both of these equations, v is the speed of sound in the air *inside* the tube.
- Overblowing wind instruments can sometimes produce higher harmonics such as $f_2 = 2f_1$ and $f_3 = 3f_1$.

 Slide 17-72

QuickCheck 17.8

At room temperature, the fundamental frequency of an open-open tube is 500 Hz. If taken outside on a cold winter day, the fundamental frequency will be

A. Less than 500 Hz.
B. 500 Hz.
C. More than 500 Hz.

QuickCheck 17.8

At room temperature, the fundamental frequency of an open-open tube is 500 Hz. If taken outside on a cold winter day, the fundamental frequency will be

✔ **A. Less than 500 Hz.**
B. 500 Hz.
C. More than 500 Hz.

Example 17.6 Flutes and Clarinets

EXAMPLE 17.6 **Flutes and clarinets**

A clarinet is 66.0 cm long. A flute is nearly the same length, with 63.6 cm between the hole the player blows across and the end of the flute. What are the frequencies of the lowest note and the next higher harmonic on a flute and on a clarinet? The speed of sound in warm air is 350 m/s.

MODEL The flute is an open-open tube, open at the end as well as at the hole the player blows across. A clarinet is an open-closed tube because the player's lips and the reed seal the tube at the upper end.

Example 17.6 Flutes and Clarinets

EXAMPLE 17.6 Flutes and clarinets

SOLVE The lowest frequency is the fundamental frequency. For the flute, an open-open tube, this is

$$f_1 = \frac{v}{2L} = \frac{350 \text{ m/s}}{2(0.636 \text{ m})} = 275 \text{ Hz}$$

The clarinet, an open-closed tube, has

$$f_1 = \frac{v}{4L} = \frac{350 \text{ m/s}}{4(0.660 \text{ m})} = 133 \text{ Hz}$$

The next higher harmonic on the flute's open-open tube is $m = 2$ with frequency $f_2 = 2f_1 = 550$ Hz. An open-closed tube has only odd harmonics, so the next higher harmonic of the clarinet is $f_3 = 3f_1 = 399$ Hz.

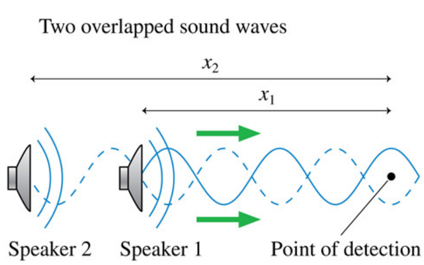
Example 17.6 Flutes and Clarinets

EXAMPLE 17.6 Flutes and clarinets

ASSESS The clarinet plays a much lower note than the flute—musically, about an octave lower—because it is an open-closed tube. It's worth noting that neither of our fundamental frequencies is exactly correct because our open-open and open-closed tube models are a bit too simplified to adequately describe a real instrument. However, both calculated frequencies are close because our models do capture the essence of the physics.

Interference in One Dimension

- The pattern resulting from the superposition of two waves is often called interference. In this section we will look at the interference of two waves traveling in the *same* direction.

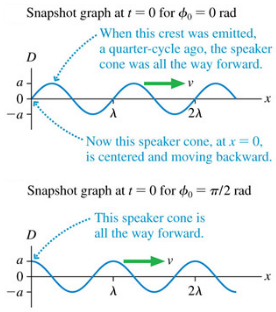


Interference in One Dimension

- A sinusoidal wave traveling to the right along the x-axis has a displacement:

  $D = a\sin(kx - \omega t + \phi_0)$

- The phase constant ϕ_0 tells us what the source is doing at $t = 0$.



Constructive Interference

- $D_1 = a\,sin(kx_1 - \omega t + \phi_{10})$

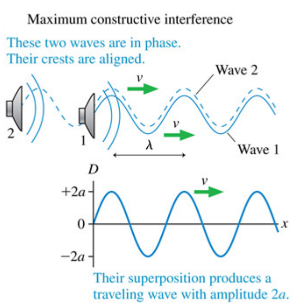
  $D_2 = a\,sin(kx_2 - \omega t + \phi_{20})$

  $D = D_1 + D_2$

- The two waves are **in phase**, meaning that

  $D_1(x) = D_2(x)$

- The resulting amplitude is $A = 2a$ for *maximum constructive interference*.



Destructive Interference

- $D_1 = a\,sin(kx_1 - \omega t + \phi_{10})$

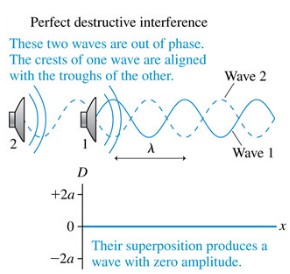
  $D_2 = a\,sin(kx_2 - \omega t + \phi_{20})$

  $D = D_1 + D_2$

- The two waves are **out of phase**, meaning that

  $D_1(x) = -D_2(x)$

- The resulting amplitude is $A = 0$ for *perfect destructive interference*.



The Mathematics of Interference

- As two waves of equal amplitude and frequency travel together along the x-axis, the net displacement of the medium is:

$$D = D_1 + D_2 = a\sin(kx_1 - \omega t + \phi_{10}) + a\sin(kx_2 - \omega t + \phi_{20})$$
$$= a\sin\phi_1 + a\sin\phi_2$$

- We can use a trigonometric identity to write the net displacement as

$$D = \left[2a\cos\left(\frac{\Delta\phi}{2}\right)\right]\sin(kx_{avg} - \omega t + (\phi_0)_{avg})$$

where $\Delta\phi = \phi_1 + \phi_2$ is the phase difference between the two waves.

 Slide 17-82

The Mathematics of Interference

- The amplitude has a maximum value $A = 2a$ if $\cos(\Delta\phi/2) = \pm 1$.
- This is **maximum constructive interference**, when

$$\Delta\phi = m \cdot 2\pi \qquad \text{(maximum amplitude } A = 2a\text{)}$$

where m is an integer.

- Similarly, the amplitude is zero if $\cos(\Delta\phi/2) = 0$.
- This is **perfect destructive interference**, when:

$$\Delta\phi = \left(m + \tfrac{1}{2}\right) \cdot 2\pi \qquad \text{(minimum amplitude } A = 0\text{)}$$

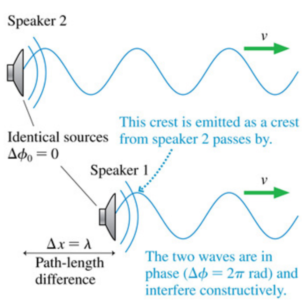
 Slide 17-83

Interference in One Dimension

- Shown are two identical sources located one wavelength apart:

$$\Delta x = \lambda$$

- The two waves are “in step” with $\Delta\phi = 2\pi$, so we have maximum constructive interference with $A = 2a$.



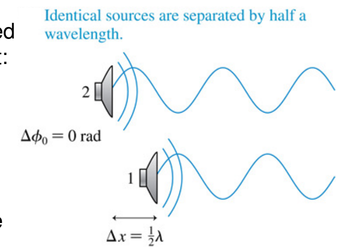
 Slide 17-84

Interference in One Dimension

- Shown are two identical sources located half a wavelength apart:

  $\Delta x = \lambda/2$

- The two waves have phase difference $\Delta\phi = \pi$, so we have perfect destructive interference with $A = 0$.



Slide 17-85

Example 17.7 Interference Between Two Sound Waves

EXAMPLE 17.7 **Interference between two sound waves**

You are standing in front of two side-by-side loudspeakers playing sounds of the same frequency. Initially there is almost no sound at all. Then one of the speakers is moved slowly away from you. The sound intensity increases as the separation between the speakers increases, reaching a maximum when the speakers are 0.75 m apart. Then, as the speaker continues to move, the intensity starts to decrease. What is the distance between the speakers when the sound intensity is again a minimum?

MODEL The changing sound intensity is due to the interference of two overlapped sound waves.

VISUALIZE Moving one speaker relative to the other changes the phase difference between the waves.

Slide 17-86

Example 17.7 Interference Between Two Sound Waves

EXAMPLE 17.7 **Interference between two sound waves**

SOLVE A minimum sound intensity implies that the two sound waves are interfering destructively. Initially the loudspeakers are side by side, so the situation is as shown in Figure 17.20a with $\Delta x = 0$ and $\Delta\phi_0 = \pi$ rad. That is, the speakers themselves are out of phase. Moving one of the speakers does not change $\Delta\phi_0$, but it does change the path-length difference Δx and thus increases the overall phase difference $\Delta\phi$. Constructive interference, causing maximum intensity, is reached when

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\pi\frac{\Delta x}{\lambda} + \pi = 2\pi \text{ rad}$$

where we used $m = 1$ because this is the first separation giving constructive interference.

Slide 17-87

Example 17.7 Interference Between Two Sound Waves

EXAMPLE 17.7 | **Interference between two sound waves**

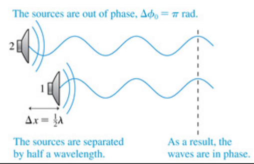
SOLVE The speaker separation at which this occurs is $\Delta x = \lambda/2$. This is the situation shown in **FIGURE 17.21**.

Because $\Delta x = 0.75$ m is $\lambda/2$, the sound's wavelength is $\lambda = 1.50$ m. The next point of destructive interference, with $m = 1$, occurs when

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\pi\frac{\Delta x}{\lambda} + \pi = 3\pi \text{ rad}$$

Thus the distance between the speakers when the sound intensity is again a minimum is

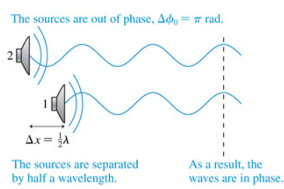
$$\Delta x = \lambda = 1.50 \text{ m}$$



Example 17.7 Interference Between Two Sound Waves

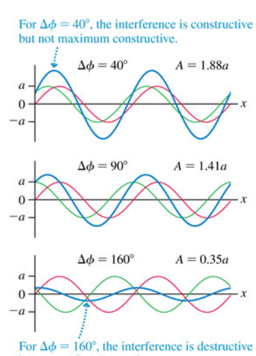
EXAMPLE 17.7 | **Interference between two sound waves**

ASSESS A separation of λ gives constructive interference for two *identical* speakers ($\Delta\phi_0 = 0$). Here the phase difference of π rad between the speakers (one is pushing forward as the other pulls back) gives destructive interference at this separation.



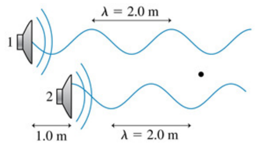
The Mathematics of Interference

- It is entirely possible, of course, that the two waves are neither exactly in phase nor exactly out of phase.
- Shown are the calculated interference of two waves that differ in phase by 40º, 90º and 160º.



QuickCheck 17.9

Two loudspeakers emit sound waves with the same wavelength and the same amplitude. The waves are shown displaced, for clarity, but assume that both are traveling along the same axis. At the point where the dot is,

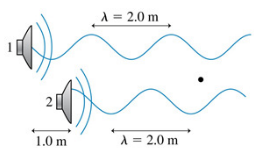


A. the interference is constructive.
B. the interference is destructive.
C. the interference is somewhere between constructive and destructive.
D. There's not enough information to tell about the interference.

Slide 17-91

QuickCheck 17.9

Two loudspeakers emit sound waves with the same wavelength and the same amplitude. The waves are shown displaced, for clarity, but assume that both are traveling along the same axis. At the point where the dot is,

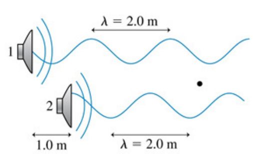


A. the interference is constructive.
B. the interference is destructive.
✔ **C. the interference is somewhere between constructive and destructive.**
D. There's not enough information to tell about the interference.

Slide 17-92

QuickCheck 17.10

Two loudspeakers emit sound waves with the same wavelength and the same amplitude. Which of the following would cause there to be destructive interference at the position of the dot?

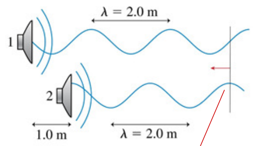


A. Move speaker 2 forward (right) 1.0 m.
B. Move speaker 2 forward (right) 0.5 m.
C. Move speaker 2 backward (left) 0.5 m.
D. Move speaker 2 backward (left) 1.0 m.
E. Nothing. Destructive interference is not possible in this situation.

Slide 17-93

QuickCheck 17.10

Two loudspeakers emit sound waves with the same wavelength and the same amplitude. Which of the following would cause there to be destructive interference at the position of the dot?



A. Move speaker 2 forward (right) 1.0 m.
B. Move speaker 2 forward (right) 0.5 m.
✔ **C. Move speaker 2 backward (left) 0.5 m.**
D. Move speaker 2 backward (left) 1.0 m.
E. Nothing. Destructive interference is not possible in this situation.

Move this peak back 1/4 wavelength to align with the trough of wave 1.

Example 17.8 More Interference of Sound Waves

EXAMPLE 17.8 More interference of sound waves

Two loudspeakers emit 500 Hz sound waves with an amplitude of 0.10 mm. Speaker 2 is 1.00 m behind speaker 1, and the phase difference between the speakers is 90°. What is the amplitude of the sound wave at a point 2.00 m in front of speaker 1?

MODEL The amplitude is determined by the interference of the two waves. Assume that the speed of sound has a room-temperature (20°C) value of 343 m/s.

Example 17.8 More Interference of Sound Waves

EXAMPLE 17.8 More interference of sound waves

SOLVE The amplitude of the sound wave is

$$A = \left|2a\cos(\Delta\phi/2)\right|$$

where $a = 0.10$ mm and the phase difference between the waves is

$$\Delta\phi = \phi_2 - \phi_1 = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0$$

The sound's wavelength is

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{500\text{ Hz}} = 0.686\text{ m}$$

Example 17.8 More Interference of Sound Waves

EXAMPLE 17.8 More interference of sound waves

SOLVE Distances $x_1 = 2.00$ m and $x_2 = 3.00$ m are measured from the speakers, so the path-length difference is $\Delta x = 1.00$ m. We're given that the inherent phase difference between the speakers is $\Delta\phi_0 = \pi/2$ rad. Thus the phase difference at the observation point is

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\pi\frac{1.00\text{ m}}{0.686\text{ m}} + \frac{\pi}{2}\text{ rad} = 10.73\text{ rad}$$

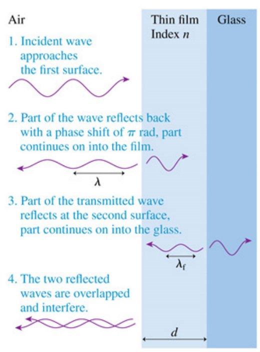
and the amplitude of the wave at this point is

$$A = \left|2a\cos\left(\frac{\Delta\phi}{2}\right)\right| = \left|(0.200\text{ mm})\cos\left(\frac{10.73}{2}\right)\right| = 0.121\text{ mm}$$

ASSESS The interference is constructive because $A > a$, but less than maximum constructive interference.

Application: Thin-Film Optical Coatings

- Thin transparent films, placed on glass surfaces, such as lenses, can control reflections from the glass.
- Antireflection coatings on the lenses in cameras, microscopes, and other optical equipment are examples of thin-film coatings.



Application: Thin-Film Optical Coatings

- The phase difference between the two reflected waves is

$$\Delta\phi = 2\pi\frac{2d}{\lambda/n} = 2\pi\frac{2nd}{\lambda}$$

where n is the index of refraction of the coating, d is the thickness, and λ is the wavelength of the light in vacuum or air.

- For a particular thin-film, constructive or destructive interference depends on the wavelength of the light:

$$\lambda_C = \frac{2nd}{m} \qquad m = 1, 2, 3, \ldots \qquad \text{(constructive interference)}$$

$$\lambda_D = \frac{2nd}{m - \frac{1}{2}} \qquad m = 1, 2, 3, \ldots \qquad \text{(destructive interference)}$$

Example 17.9 Designing an Antireflection Coating

EXAMPLE 17.9 | **Designing an antireflection coating**

Magnesium fluoride (MgF_2) is used as an antireflection coating on lenses. The index of refraction of MgF_2 is 1.39. What is the thinnest film of MgF_2 that works as an antireflection coating at $\lambda = 510$ nm, near the center of the visible spectrum?

MODEL Reflection is minimized if the two reflected waves interfere destructively.

Example 17.9 Designing an Antireflection Coating

EXAMPLE 17.9 | **Designing an antireflection coating**

SOLVE The film thicknesses that cause destructive interference at wavelength λ are

$$d = \left(m - \frac{1}{2}\right)\frac{\lambda}{2n}$$

The thinnest film has $m = 1$. Its thickness is

$$d = \frac{\lambda}{4n} = \frac{510\text{ nm}}{4(1.39)} = 92\text{ nm}$$

The film thickness is significantly less than the wavelength of visible light!

Example 17.9 Designing an Antireflection Coating

EXAMPLE 17.9 | **Designing an antireflection coating**

ASSESS The reflected light is completely eliminated (perfect destructive interference) only if the two reflected waves have equal amplitudes. In practice, they don't. Nonetheless, the reflection is reduced from $\approx$ 4% of the incident intensity for "bare glass" to well under 1%. Furthermore, the intensity of reflected light is much reduced across most of the visible spectrum (400–700 nm), even though the phase difference deviates more and more from π rad as the wavelength moves away from 510 nm. It is the increasing reflection at the ends of the visible spectrum ($\lambda \approx 400$ nm and $\lambda \approx 700$ nm), where $\Delta\phi$ deviates significantly from π rad, that gives a reddish-purple tinge to the lenses on cameras and binoculars. Homework problems will let you explore situations where only one of the two reflections has a reflection phase shift of π rad.

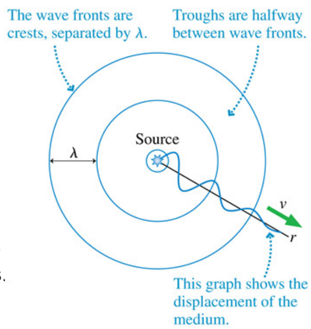
A Circular or Spherical Wave

- A circular or spherical wave can be written

$D(r, t) = a \sin(kr - \omega t + \phi_0)$

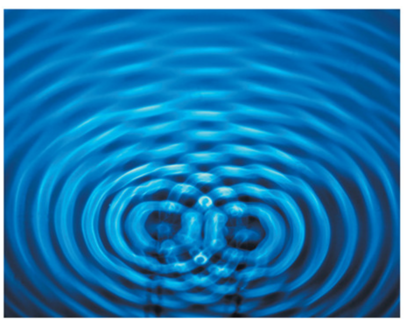
where r is the distance measured outward from the source.

- The amplitude a of a circular or spherical wave diminishes as r increases.



 Slide 17-103

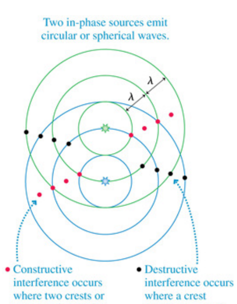
Interference in Two and Three Dimensions



- Two overlapping water waves create an interference pattern.

 Slide 17-104

Interference in Two and Three Dimensions



• = Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.

• = Points of destructive interference. A crest is aligned with a trough of another wave.

 Slide 17-105

Interference in Two and Three Dimensions

- The mathematical description of interference in two or three dimensions is very similar to that of one-dimensional interference.
- The conditions for constructive and destructive interference are

Maximum constructive interference:

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = m \cdot 2\pi$$

$$m = 0, 1, 2, \ldots$$

Maximum destructive interference:

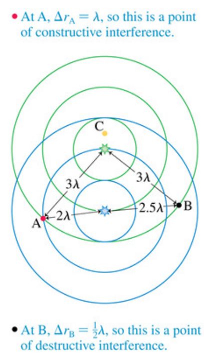
$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = (m + \tfrac{1}{2}) \cdot 2\pi$$

where Δr is the *path-length difference*.

Slide 17-106

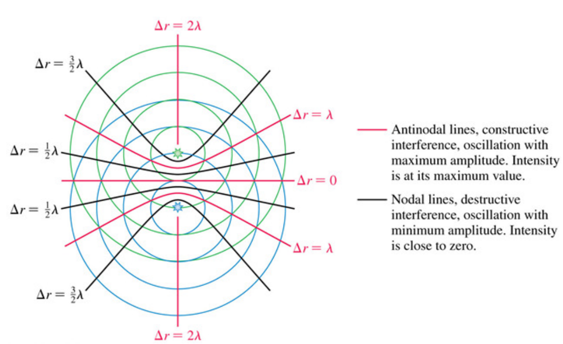
Interference in Two and Three Dimensions

- The figure shows two identical sources that are in phase.
- The path-length difference Δr determines whether the interference at a particular point is constructive or destructive.



Slide 17-107

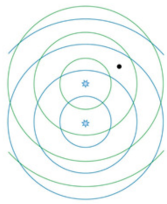
Interference in Two and Three Dimensions



Slide 17-108

QuickCheck 17.11

Two in-phase sources emit sound waves of equal wavelength and intensity. At the position of the dot,

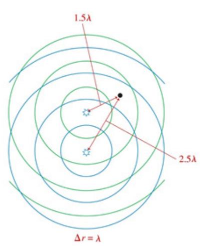


A. The interference is constructive.
B. The interference is destructive.
C. The interference is somewhere between constructive and destructive.
D. There's not enough information to tell about the interference.

Slide 17-109

QuickCheck 17.11

Two in-phase sources emit sound waves of equal wavelength and intensity. At the position of the dot,

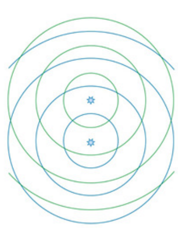


✔ **A. The interference is constructive.**
B. The interference is destructive.
C. The interference is somewhere between constructive and destructive.
D. There's not enough information to tell about the interference.

Slide 17-110

QuickCheck 17.12

Two in-phase sources emit sound waves of equal wavelength and intensity. How many antinodal lines (lines of constructive interference) are in the interference pattern?



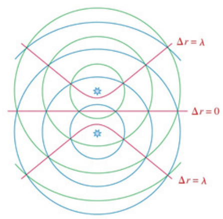
A. 1
B. 2
C. 3
D. 4
E. 5

Slide 17-111

QuickCheck 17.12

Two in-phase sources emit sound waves of equal wavelength and intensity. How many antinodal lines (lines of constructive interference) are in the interference pattern?

A. 1
B. 2
✔**C. 3**
D. 4
E. 5



Sources are 1.5 λ apart, so no point can have Δr more than 1.5 λ.

Slide 17-112

Problem-Solving Strategy: Interference of Two Waves

PROBLEM-SOLVING STRATEGY 17.1

Interference of two waves

MODEL Model the waves as linear, circular, or spherical.

VISUALIZE Draw a picture showing the sources of the waves and the point where the waves interfere. Give relevant dimensions. Identify the distances r_1 and r_2 from the sources to the point. Note any phase difference $\Delta\phi_0$ between the two sources.

Slide 17-113

Problem-Solving Strategy: Interference of Two Waves

PROBLEM-SOLVING STRATEGY 17.1

Interference of two waves

SOLVE The interference depends on the path-length difference $\Delta r = r_2 - r_1$ and the source phase difference $\Delta\phi_0$.

Constructive: $$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = m \cdot 2\pi$$

Destructive: $$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \left(m + \tfrac{1}{2}\right) \cdot 2\pi$$

$m = 0, 1, 2, \ldots$

For identical sources ($\Delta\phi_0 = 0$), the interference is maximum constructive if $\Delta r = m\lambda$, maximum destructive if $\Delta r = (m + \frac{1}{2})\lambda$.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 18

Slide 17-114

Example 17.10 Two-Dimensional Interference Between Two Loudspeakers

EXAMPLE 17.10 **Two-dimensional interference between two loudspeakers**

Two loudspeakers in a plane are 2.0 m apart and in phase with each other. Both emit 700 Hz sound waves into a room where the speed of sound is 341 m/s. A listener stands 5.0 m in front of the loudspeakers and 2.0 m to one side of the center. Is the interference at this point maximum constructive, maximum destructive, or in between? How will the situation differ if the loudspeakers are out of phase?

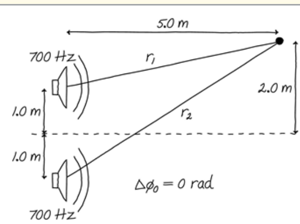
MODEL The two speakers are sources of in-phase, spherical waves. The overlap of these waves causes interference.

 Slide 17-115

Example 17.10 Two-Dimensional Interference Between Two Loudspeakers

EXAMPLE 17.10 **Two-dimensional interference between two loudspeakers**

VISUALIZE FIGURE 17.28 shows the loudspeakers and defines the distances r_1 and r_2 to the point of observation. The figure includes dimensions and notes that $\Delta\phi_0 = 0$ rad.



 Slide 17-116

Example 17.10 Two-Dimensional Interference Between Two Loudspeakers

EXAMPLE 17.10 **Two-dimensional interference between two loudspeakers**

SOLVE It's not r_1 and r_2 that matter, but the *difference* Δr between them. From the geometry of the figure we can calculate that

$$r_1 = \sqrt{(5.0\text{ m})^2 + (1.0\text{ m})^2} = 5.10\text{ m}$$

$$r_2 = \sqrt{(5.0\text{ m})^2 + (3.0\text{ m})^2} = 5.83\text{ m}$$

Thus the path-length difference is $\Delta r = r_2 - r_1 = 0.73$ m. The wavelength of the sound waves is

$$\lambda = \frac{v}{f} = \frac{341\text{ m/s}}{700\text{ Hz}} = 0.487\text{ m}$$

 Slide 17-117

Example 17.10 Two-Dimensional Interference Between Two Loudspeakers

EXAMPLE 17.10 Two-dimensional interference between two loudspeakers

SOLVE In terms of wavelengths, the path-length difference is

$$\Delta r = \tfrac{3}{2}\lambda$$

Because the sources are in phase ($\Delta\phi_0 = 0$), this is the condition for *destructive* interference. If the sources were out of phase ($\Delta\phi_0 = \pi$ rad), then the phase difference of the waves at the listener would be

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = 2\pi\left(\frac{3}{2}\right) + \pi \text{ rad} = 4\pi \text{ rad}$$

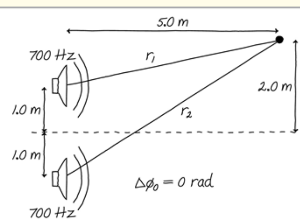
This is an integer multiple of 2π rad, so in this case the interference would be *constructive*.

Slide 17-118

Example 17.10 Two-Dimensional Interference Between Two Loudspeakers

EXAMPLE 17.10 Two-dimensional interference between two loudspeakers

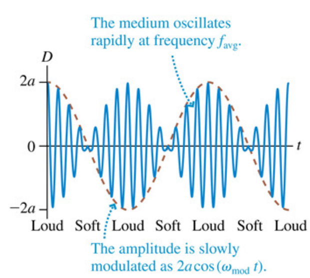
ASSESS Both the path-length difference and any inherent phase difference of the sources must be considered when evaluating interference.



Slide 17-119

Beats

- The figure shows the history graph for the superposition of the sound from two sources of equal amplitude a, but slightly different frequency.



Slide 17-120

Beats

- With beats, the sound intensity rises and falls *twice* during one cycle of the modulation envelope.
- Each "loud-soft-loud" is one beat, so the **beat frequency** f_{beat}, which is the number of beats per second, is *twice* the modulation frequency f_{mod}.
- The beat frequency is

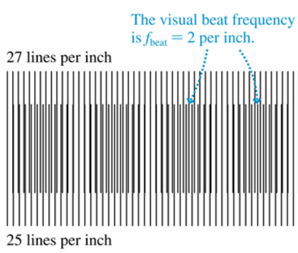
$$f_{beat} = 2f_{mod} = 2\frac{\omega_{mod}}{2\pi} = 2 \cdot \frac{1}{2}\left(\frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi}\right) = |f_1 - f_2|$$

where, to keep f_{beat} from being negative, we will always let f_1 be the larger of the two frequencies.
- The beat frequency is simply the *difference* between the two individual frequencies.

Visual Beats

- Shown is a graphical example of beats.
- Two "fences" of slightly different frequencies are superimposed on each other.
- The center part of the figure has two "beats" per inch:

$f_{beat} = 27 - 25 = 2$



QuickCheck 17.13

You hear 2 beats per second when two sound sources, both at rest, play simultaneously. The beats disappear if source 2 moves toward you while source 1 remains at rest. The frequency of source 1 is 500 Hz. The frequency of source 2 is

A. 496 Hz
B. 498 Hz
C. 500 Hz
D. 502 Hz
E. 504 Hz

QuickCheck 17.13

You hear 2 beats per second when two sound sources, both at rest, play simultaneously. The beats disappear if source 2 moves toward you while source 1 remains at rest. The frequency of source 1 is 500 Hz. The frequency of source 2 is

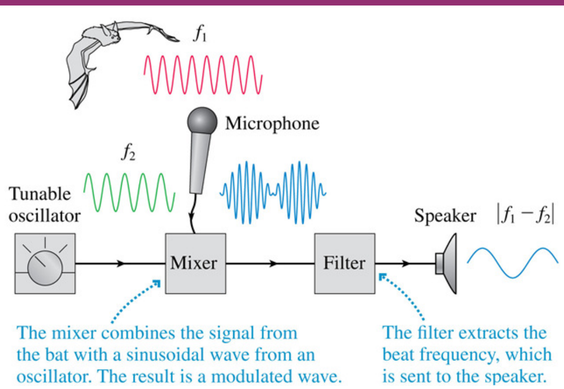
A. 496 Hz
✔**B. 498 Hz**
C. 500 Hz
D. 502 Hz
E. 504 Hz

Example 17.11 Detecting Bats with Beats

EXAMPLE 17.11 Detecting bats with beats

The little brown bat is a common species in North America. It emits echolocation pulses at a frequency of 40 kHz, well above the range of human hearing. To allow researchers to "hear" these bats, the bat detector shown in FIGURE 17.30 combines the bat's sound wave at frequency f_1 with a wave of frequency f_2 from a tunable oscillator. The resulting beat frequency is then amplified and sent to a loudspeaker. To what frequency should the tunable oscillator be set to produce an audible beat frequency of 3 kHz?

Example 17.11 Detecting Bats with Beats



Example 17.11 Detecting Bats with Beats

EXAMPLE 17.11 Detecting bats with beats

SOLVE Combining two waves with different frequencies gives a beat frequency

$$f_{beat} = |f_1 - f_2|$$

A beat frequency will be generated at 3 kHz if the oscillator frequency and the bat frequency *differ* by 3 kHz. An oscillator frequency of either 37 kHz or 43 kHz will work nicely.

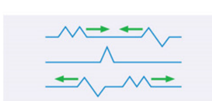
ASSESS The electronic circuitry of radios, televisions, and cell phones makes extensive use of *mixers* to generate difference frequencies.

Chapter 17 Summary Slides

General Principles

Principle of Superposition

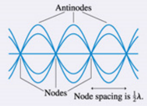
The displacement of a medium when more than one wave is present is the sum at each point of the displacements due to each individual wave.



Important Concepts

Standing Waves

Standing waves are due to the superposition of two traveling waves moving in opposite directions.

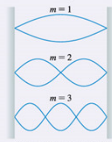


The amplitude at position x is

$$A(x) = 2a \sin kx$$

where a is the amplitude of each wave.

The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are **modes** of the system.



Standing waves on a string

Important Concepts

Solving Interference Problems

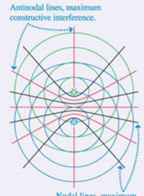
Maximum constructive interference occurs where crests are aligned with crests and troughs with troughs. The waves are in phase.

Maximum destructive interference occurs where crests are aligned with troughs. The waves are out of phase.

MODEL Model the wave as linear, circular, or spherical.

VISUALIZE Find distances to the sources.

SOLVE Interference depends on the **phase difference** $\Delta\phi$ between the waves:



Constructive: $\Delta\phi = 2\pi \dfrac{\Delta r}{\lambda} + \Delta\phi_0 = m \cdot 2\pi$

Destructive: $\Delta\phi = 2\pi \dfrac{\Delta r}{\lambda} + \Delta\phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi$

Δr is the path-length difference of the two waves, and $\Delta\phi_0$ is any phase difference between the sources. For identical (in-phase) sources:

Constructive: $\Delta r = m\lambda$ Destructive: $\Delta r = \left(m + \frac{1}{2}\right)\lambda$

ASSESS Is the result reasonable?

Applications

Boundary conditions

Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends:

$$\lambda_m = \frac{2L}{m} \qquad f_m = m\frac{v}{2L} = mf_1 \qquad m = 1, 2, 3, \ldots$$

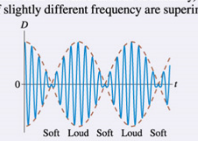
The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.

A sound wave in an open-closed tube must have a node at the closed end but an antinode at the open end. This leads to

$$\lambda_m = \frac{4L}{m} \qquad f_m = m\frac{v}{4L} = mf_1 \qquad m = 1, 3, 5, 7, \ldots$$

Applications

Beats (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.



The beat frequency between waves of frequencies f_1 and f_2 is

$$f_{beat} = |f_1 - f_2|$$