Electric fields are responsible for the electric currents that flow through your computer and the nerves in your body. Electric fields also line up polymer molecules to form the images in a liquid crystal display (LCD).

**Chapter Goal:** To learn how to calculate and use the electric field.

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**Topics:**
- Electric Field Models
- The Electric Field of Multiple Point Charges
- The Electric Field of a Continuous Charge Distribution
- The Electric Fields of Rings, Disks, Planes, and Spheres
- The Parallel-Plate Capacitor
- Motion of a Charged Particle in an Electric Field
- Motion of a Dipole in an Electric Field

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**Chapter 27. Reading Quizzes**

What device provides a practical way to produce a uniform electric field?

A. A long thin resistor  
B. A Faraday cage  
C. A parallel plate capacitor  
D. A toroidal inductor  
E. An electric field uniformizer
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For charged particles, what is the quantity $q/m$ called?

A. Linear charge density  
B. Charge-to-mass ratio  
C. Charged mass density  
D. Massive electric dipole  
E. Quadrupole moment

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Which of these charge distributions did not have its electric field determined in Chapter 27?

A. A line of charge  
B. A parallel-plate capacitor  
C. A ring of charge  
D. A plane of charge  
E. They were all determined
Which of these charge distributions did not have its electric field determined in Chapter 27?

A. A line of charge
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D. A plane of charge

E. They were all determined

The worked examples of charged-particle motion are relevant to

A. a transistor.
B. a cathode ray tube.
C. magnetic resonance imaging.
D. cosmic rays.
E. lasers.

The worked examples of charged-particle motion are relevant to

A. a transistor.

B. a cathode ray tube.

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E. lasers.
Electric Field Models

The electric field of a point charge $q$ at the origin, $r = 0$, is

$$E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \quad \text{(electric field of a point charge)}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ C$^2$/N m$^2$ is the permittivity constant.

The net electric field due to a group of point charges is

$$\vec{E}_{\text{net}} = \frac{\vec{E}_{\text{1 on q}}}{q} + \frac{\vec{E}_{\text{2 on q}}}{q} + \cdots = \vec{E}_1 + \vec{E}_2 + \cdots = \sum_i \vec{E}_i$$

where $E_i$ is the field from point charge $i$.

Problem-Solving Strategy: The electric field of multiple point charges

**Problem-Solving Strategy 27.1** The electric field of multiple point charges

**Model** Model charged objects as point charges.

**Visualize** For the pictorial representation:
- Establish a coordinate system and show the locations of the charges.
- Identify the point $P$ at which you want to calculate the electric field.
- Draw the electric field of each charge at $P$.
- Use symmetry to determine if any components of $\vec{E}_{\text{net}}$ are zero.
Problem-Solving Strategy: The electric field of multiple point charges

**SOLVE** The mathematical representation is \( \vec{E}_{\text{net}} = \sum \vec{E}_i \).
- For each charge, determine its distance from P and the angle of \( \vec{E}_i \) from the axes.
- Calculate the field strength of each charge’s electric field.
- Write each vector \( \vec{E}_i \) in component form.
- Sum the vector components to determine \( \vec{E}_{\text{net}} \).
- If needed, determine the magnitude and direction of \( \vec{E}_{\text{net}} \).

The Electric Field of a Dipole

We can represent an electric dipole by two opposite charges \( \pm q \) separated by the small distance \( s \).

The dipole moment is defined as the vector

\[ \vec{p} = (qs, \text{from the negative to the positive charge}) \]

The dipole-moment magnitude \( p = qs \) determines the electric field strength. The SI units of the dipole moment are C m.

The Electric Field of a Dipole: Two Equal but Opposite Charges Separated by a Small Distance

A water molecule is a permanent dipole because the negative electrons spend more time with the oxygen atom.

The Electric Field of a Dipole

On the axis of an electric dipole

\[ (E_{\text{dipole}})_y = (E_+)_y + (E_-)_y = \frac{1}{4\pi\varepsilon_0} \left[ \frac{q}{(y - s/2)^2} + \frac{(-q)}{(y + s/2)^2} \right] \]

\[ (E_{\text{dipole}})_y = \frac{-q}{4\pi\varepsilon_0} \left[ \frac{2ys}{(y - 0.5\cdot s)^2} \right] \]

\[ (E_{\text{dipole}})_y \approx \frac{1}{4\pi\varepsilon_0} \frac{2qs}{y^3} \]
The Electric Field of a Dipole

The electric field at a point on the axis of a dipole is

\[
\vec{E}_{\text{dipole}} = \frac{2\vec{p}}{4\pi \varepsilon_0 r^3}
\]

(on the axis of an electric dipole)

where \( r \) is the distance measured from the center of the dipole.

The electric field in the plane that bisects and is perpendicular to the dipole is

\[
\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi \varepsilon_0 r^3} \vec{p}
\]

(perpendicular plane)

This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

EXAMPLE 27.2 The electric field of a water molecule

QUESTION:

The water molecule \( \text{H}_2\text{O} \) has a permanent dipole moment of magnitude \( 6.2 \times 10^{-30} \text{ C.m} \). What is the electric field strength 1.0 nm from a water molecule at a point on the dipole’s axis?

MODEL: The size of a molecule is \( \approx 0.1 \text{ nm} \). Thus \( r \gg s \), and we can use Equation 27.11 for the on-axis electric field of the molecule’s dipole moment.
EXAMPLE 27.2 The electric field of a water molecule

SOLVE The on-axis electric field strength at $r = 1.0 \text{ nm}$ is

$$E \approx \frac{1}{4\pi \varepsilon_0} \frac{2\rho}{r^3} = \frac{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{2(6.2 \times 10^{-30} \text{ C} \cdot \text{m})} \frac{1.0 \times 10^{-9} \text{ m}^3}{(1.0 \times 10^{-9} \text{ m})^3}$$

$$= 1.1 \times 10^8 \text{ N/C}$$

Tactics: Drawing and using electric field lines

Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point. Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.

Continuous Charge Distribution: Charge Density

- **Linear, length $L$**
  - Amount of charge in a small volume $dl$: $dq = \frac{Q}{L} \, dl = \lambda \, dl$ \hspace{1cm} $\lambda = \frac{Q}{L}$
  - Linear charge density

- **Volume $V$**
  - Amount of charge in a small volume $dV$: $dq = \frac{Q}{V} \, dV = \rho \, dV$ \hspace{1cm} $\rho = \frac{Q}{V}$
  - Volume charge density

- **Surface, area $A$**
  - Amount of charge in a small volume $dA$: $dq = \frac{Q}{A} \, dA = \sigma \, dA$ \hspace{1cm} $\sigma = \frac{Q}{A}$
  - Surface charge density

Electric field lines never cross.

Electric field lines start from positive charges and end on negative charges.

Exercises 2–4, 12, 13
**Electric Field: Continuous Charge Distribution**

**Procedure:**
- Divide the charge distribution into small elements, each of which contains $\Delta q$.
- Calculate the electric field due to one of these elements at point $P$.
- Evaluate the total field by summing the contributions of all the charge elements.

**Symmetry:** take advantage of any symmetry to simplify calculations.

Because the charge distribution is continuous,

$$E = k_e \lim_{\Delta q \to 0} \sum_i \frac{\Delta q_i \hat{r}_i}{r_i^2} = k_e \frac{dq_i}{r^2} \hat{r}$$

For the individual charge elements

$$\Delta E = k_e \frac{\Delta q}{r^2} \hat{r}$$

**Electric Field: Symmetry**

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

$q_1 = 10 \mu C$  $q_2 = 10 \mu C$

Because the charge distribution is continuous

$$E = 2E_1 \cos \varphi$$

$E = 2E_{1,z} = 2E_{2,z}$

$E_{1,z} = E_1 \cos \varphi$

$E = 4E_{1,z} = 4E_2 \cos \varphi$

The symmetry gives us the direction of resultant electric field.

**The Electric Field of a Continuous Charge Distribution at a point in the plane that bisects the rode**

$$E = 2E_1 \cos \varphi$$

$E = 2E_{1,z} = 2E_{2,z}$

$E_{1,z} = E_1 \cos \varphi$

$E = 4E_{1,z} = 4E_2 \cos \varphi$

The symmetry gives us the direction of resultant electric field.

**Electric field due to three equal point charges?**

$$\vec{r}_1 = \sqrt{x^2 + d^2}$$

This is the point at which we will calculate the electric field.

The symmetry gives us the direction of resultant electric field.

$E = 2E_1 \cos \varphi$

$E = 2E_{1,z} = 2E_{2,z}$

$E_{1,z} = E_1 \cos \varphi$

$E = 4E_{1,z} = 4E_2 \cos \varphi$

The symmetry gives us the direction of resultant electric field.
The Electric Field of a Continuous Charge Distribution on a Thin Rod

\[(E_i)_x = (E_i)_y \cos \theta_i = \frac{1}{4 \pi \varepsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i\]

\[\Delta Q = \lambda \Delta y = (Q / L) \Delta y\]

\[E_x = \frac{Q / L}{4 \pi \varepsilon_0} \int_{-L/2}^{L/2} \frac{r \, dy}{(y^2 + r^2)^{3/2}}\]

How to evaluate this integral?
- Trigonometric Substitution

\[E_x = \frac{Q / L}{4 \pi \varepsilon_0} \int_{-L/2}^{L/2} \frac{r \, dy}{(y^2 + r^2)^{3/2}}\]

Electric Field due to a Thin Rod

\[E_x = \frac{Q / L}{4 \pi \varepsilon_0} \int_{-L/2}^{L/2} \frac{r \, dy}{(y^2 + r^2)^{3/2}}\]

Use:
\[\int \frac{1}{a^2 + x^2} \, dx = \frac{\tan^{-1} \left( \frac{x}{a} \right)}{\sqrt{a^2 + x^2}}\]

\[E_x = \frac{Q / L}{4 \pi \varepsilon_0} \int_{-L/2}^{L/2} \frac{r \, dy}{(y^2 + r^2)^{3/2}}\]
An infinite Line of Charge

\[ E_x = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r(L/2)^2 + r^2} \]

\[ E_{line} = \lim_{r \to \infty} \frac{1}{4\pi \varepsilon_0} \frac{Q}{rL/2} \approx 0 \]

\[ E_{line} = \frac{1}{4\pi \varepsilon_0} \frac{2\lambda}{r} \]

Unlike a point charge, for which the field decreases as \(1/r^2\), the field of an infinitely long charged wire decreases more slowly – as only \(1/r\).

Parallel-Plate Capacitor

Find electric field

\[ E_x = \frac{\sigma}{2\varepsilon_0} \]

\[ E_x = \frac{\sigma}{2\varepsilon_0} \]

\[ E_x = \frac{\sigma}{2\varepsilon_0} \]

\[ E_x = \frac{\sigma}{2\varepsilon_0} \]

\[ E_+ = E_x \]

\[ E_- = -E_x \]

\[ E_+ + E_- = 0 \]

\[ E = E_x - E_- \]

\[ E = E_x - E_- \]
EXAMPLE 27.7 The electric field inside a capacitor

QUESTIONS:

EXAMPLE 27.7 The electric field inside a capacitor
Two 1.0 cm × 2.0 cm rectangular electrodes are 1.0 mm apart. What charge must be placed on each electrode to create a uniform electric field of strength $2.0 \times 10^6$ N/C? How many electrons must be moved from one electrode to the other to accomplish this?

EXAMPLE 27.7 The electric field inside a capacitor

**MODEL** The electrodes can be modeled as a parallel-plate capacitor because the spacing between them is much smaller than their lateral dimensions.

**SOLVE** The electric field strength inside the capacitor is $E = Q/\varepsilon_0 A$. Thus the charge to produce a field of strength $E$ is

\[ Q = \frac{3.5 \times 10^{-9} \text{ C}}{1.60 \times 10^{-19} \text{ C/atom}} = 2.2 \times 10^{10} \text{ electrons} \]

The positive plate must be charged to $+3.5$ nC and the negative plate to $-3.5$ nC. In practice, the plates are charged by using a battery to move electrons from one plate to the other. The number of electrons in 3.5 nC is

\[ N = \frac{Q}{e} = \frac{3.5 \times 10^{-9} \text{ C}}{1.60 \times 10^{-19} \text{ C/atom}} = 2.2 \times 10^{10} \text{ electrons} \]

Thus $2.2 \times 10^{10}$ electrons are moved from one electrode to the other. Note that the capacitor as a *whole* has no net charge.
EXAMPLE 27.7 The electric field inside a capacitor

**ASSESS** The plate spacing does not enter the result. As long as the spacing is much smaller than the plate dimensions, as is true in this example, the field is independent of the spacing.

What is the electric field at point $P$? $\lambda$ - linear charge density

1. **Symmetry** determines the direction of the electric field.

   
   $\lambda = \text{linear charge density}$

2. Divide the charge distribution into small elements, each of which contains $\Delta q$

   $E = \sum_{\text{all elements}} \Delta q = \sum_{\text{all elements}} \frac{\lambda \Delta l}{r^2} \cos \phi$

   $\Delta E = \Delta E \cos \phi = k_e \frac{\Delta q}{r^2} \cos \phi$

   $E = \sum_{\text{all elements}} k_e \frac{\Delta q}{r^2} \cos \phi$

   $\Delta q = \lambda \Delta l$
What is the electric field at point \( P \)? \( \lambda \) - linear charge density

3. Evaluate the total field by summing the contributions of all the charge elements \( \Delta q \)

\[
E = \sum_{\text{all elements}} k_e \frac{\Delta q}{r^2} \cos \varphi
\]

\[
\Delta l = R \Delta \psi
\]

\[
\Delta q = \lambda \Delta l = \lambda R \Delta \psi
\]

\[
E = \sum_{\text{all elements}} k_e \frac{\lambda R \Delta \psi}{r^2} \cos \varphi
\]

Replace Sum by Integral

\[
E = \int_0^{2\pi} d\psi \, k_e \frac{\lambda R \Delta \psi}{r^2} \cos \varphi
\]

4. Evaluate the integral

\[
E = \frac{2\pi \lambda k_e}{4 \pi \varepsilon_0} \frac{hQ}{(h^2 + R^2)^{3/2}}
\]

A Circular Disk of Charge

Electric field due to a ring:

\[
E_i = \frac{1}{4 \pi \varepsilon_0} \frac{hQ}{(h^2 + r_i^2)^{3/2}}
\]

\[
(E_i)_z = \frac{1}{4 \pi \varepsilon_0} \frac{z \Delta Q_i}{(z^2 + r_i^2)^{3/2}}
\]

\[
(E_{\text{disk}})_z = \sum_{i=1}^{N} (E_i)_z = \frac{z}{4 \pi \varepsilon_0} \sum_{i=1}^{N} \frac{\Delta Q_i}{(z^2 + r_i^2)^{3/2}}
\]

\[
(E_{\text{disk}})_z = \eta z \int_0^R \frac{r dr}{(z^2 + r^2)^{3/2}}
\]

Quiz: If \( z \gg R \), then \( E = ? \)

Motion of a Charged Particle
Motion of Charged Particle

• When a charged particle is placed in an electric field, it experiences an electrical force.
• If this is the only force on the particle, it must be the net force.
• The net force will cause the particle to accelerate according to Newton’s second law.

\[ \vec{F} = q \vec{E} \]  - Coulomb’s law

\[ \vec{F} = m \vec{a} \]  - Newton’s second law

\[ \vec{a} = \frac{q}{m} \vec{E} \]

What is the final velocity?

\[ \vec{F} = q \vec{E} \]  - Coulomb’s law

\[ \vec{F} = m \vec{a} \]  - Newton’s second law

\[ a_y = -\frac{q}{m} E \]

Motion in \( x \) – with constant velocity \( v_0 \)

Motion in \( x \) – with constant acceleration \( a_y = -\frac{q}{m} E \)

\[ t = \frac{l}{v_0} \]  - travel time

After time \( t \) the velocity in \( y \) direction becomes:

\[ v_y = a_y t = -\frac{q}{m} Et \]

then

\[ v_f = \sqrt{v_y^2 + \left(\frac{q}{m} Et\right)^2} \]

Chapter 27. Summary Slides

General Principles

Sources of \( \vec{E} \)

Electric fields are created by charges.

Two major tools for calculating \( \vec{E} \) are:
• The field of a point charge:
  \[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \]
• The principle of superposition

Multiple point charges

Use superposition: \( \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots \)

Continuous distribution of charge

• Divide the charge into segments \( \Delta Q \) for which you already know the field.
• Find the field of each \( \Delta Q \).
• Find \( \vec{E} \) by summing the fields of all \( \Delta Q \).

The summation usually becomes an integral. A critical step is replacing \( \Delta Q \) with an expression involving a charge density \( (\rho \, \text{or} \, \sigma) \) and an integration coordinate.
**General Principles**

**Consequences of \( \vec{E} \)**

The electric field exerts a force on a charged particle:

\[
\vec{F} = q\vec{E}
\]

The force causes acceleration:

\[
\vec{a} = (q/m)\vec{E}
\]

Trajectories of charged particles are calculated with kinematics.

The electric field exerts a torque on a dipole:

\[
\tau = p\vec{E}\sin\theta
\]

The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.

---

**Applications**

**Electric dipole**

The electric dipole moment is \( \vec{p} = (p, q, \text{from negative to positive}) \)

Field on axis: \( \vec{E} = -\frac{1}{4\pi\varepsilon_0} \frac{\vec{p}}{r^2} \)

Field in bisecting plane: \( \vec{E} = -\frac{1}{4\pi\varepsilon_0} \frac{\vec{p}}{r^2} \)

**Infinite plane of charge with surface charge density \( \eta \)**

\[ \vec{E} = \left( \frac{\eta}{\varepsilon_0} \right) \text{perpendicular to plane} \]

**Infinite line of charge with linear charge density \( \lambda \)**

\[ \vec{E} = \left( \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r} \right) \text{perpendicular to line} \]

**Sphere of charge**

Same as a point charge \( Q \) for \( r > R \)

---

**Applications**

**Parallel-plate capacitor**

The electric field inside an ideal capacitor is a uniform electric field:

\[ \vec{E} = \frac{V}{\varepsilon_0}, \text{from positive to negative} \]

A real capacitor has a weak fringe field around it.

---

**Chapter 27. Questions**
At the position of the dot, the electric field points

A. Up.
B. Down.
C. Left.
D. Right.
E. The electric field is zero.

A piece of plastic is uniformly charged with surface charge density \( \eta_1 \). The plastic is then broken into a large piece with surface charge density \( \eta_2 \) and a small piece with surface charge density \( \eta_3 \). Rank in order, from largest to smallest, the surface charge densities \( \eta_1 \) to \( \eta_3 \).

A. \( \eta_2 = \eta_3 > \eta_1 \)
B. \( \eta_1 > \eta_2 > \eta_3 \)
C. \( \eta_1 > \eta_2 = \eta_3 \)
D. \( \eta_3 > \eta_2 > \eta_1 \)
E. \( \eta_1 = \eta_2 = \eta_3 \)
Which of the following actions will increase the electric field strength at the position of the dot?

A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
C. Make the rod shorter without changing the charge.
D. Remove charge from the rod.
E. Make the rod narrower without changing the charge.

Rank in order, from largest to smallest, the electric field strengths $E_a$ to $E_e$ at these five points near a plane of charge.

A. $E_a > E_c > E_b > E_e > E_d$
B. $E_a = E_b = E_c = E_d = E_e$
C. $E_a > E_b = E_c > E_d = E_e$
D. $E_b = E_c = E_d = E_e > E_a$
E. $E_c > E_d > E_e > E_b > E_a$

A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
C. **Make the rod shorter without changing the charge.**
D. Remove charge from the rod.
E. Make the rod narrower without changing the charge.

Rank in order, from largest to smallest, the electric field strengths $E_a$ to $E_e$ at these five points near a plane of charge.

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B. $E_a = E_b = E_c = E_d = E_e$
C. $E_a > E_b = E_c > E_d = E_e$
D. $E_b = E_c = E_d = E_e > E_a$
E. $E_c > E_d > E_e > E_b > E_a$
Rank in order, from largest to smallest, the forces \( F_a \) to \( F_e \) a proton would experience if placed at points a – e in this parallel-plate capacitor.

A. \( F_a = F_b = F_d = F_e > F_c \)
B. \( F_a = F_b > F_c > F_d = F_e \)
C. \( F_a = F_b = F_c = F_d = F_e \)
D. \( F_e > F_d > F_c > F_a = F_b \)
E. \( F_e > F_d > F_c > F_b > F_a \)

Which electric field is responsible for the trajectory of the proton?

A. \( F_a = F_b = F_d = F_e > F_c \)
B. \( F_a = F_b > F_c > F_d = F_e \)
C. \( F_a = F_b = F_c = F_d = F_e \)
D. \( F_e > F_d > F_c > F_a = F_b \)
E. \( F_e > F_d > F_c > F_b > F_a \)