Chapter 22. Wave Optics

Light is an electromagnetic wave. The interference of light waves produces the colors reflected from a CD, the iridescence of bird feathers, and the technology underlying supermarket checkout scanners and optical computers.

**Chapter Goal:** To understand and apply the wave model of light.

**Topics:**
- Light and Optics
- The Interference of Light
- The Diffraction Grating
- Single-Slit Diffraction
- Circular-Aperture Diffraction
- Interferometers

**What was the first experiment to show that light is a wave?**

A. Young’s double slit experiment  
B. Galileo’s observation of Jupiter’s moons  
C. The Michelson–Morley interferometer  
D. The Pound-Rebka experiment  
E. Millikan’s oil drop experiment
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What is a *diffraction grating*?

A. A device used to grate cheese and other materials
B. A musical instrument used to direct sound
C. An opaque screen with a tiny circular aperture
D. An opaque screen with many closely spaced slits
E. Diffraction gratings are not covered in Chapter 22.

When laser light shines on a screen after passing through two closely spaced slits, you see

A. a diffraction pattern.
B. interference fringes.
C. two dim, closely spaced points of light.
D. constructive interference.
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This chapter discussed the

A. acoustical interferometer.
B. Michelson interferometer.
C. Fabry-Perot interferometer.
D. Both A and B.
E. Both B and C.

The spreading of waves behind an aperture is

A. more for long wavelengths, less for short wavelengths.
B. less for long wavelengths, more for short wavelengths.
C. the same for long and short wavelengths.
D. not discussed in this chapter.
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Apertures for which diffraction is studied in this chapter are

A. a single slit.
B. a circle.
C. a square.
D. both A and B.
E. both A and C.

Chapter 22. Basic Content and Examples
Models of Light

• The wave model: under many circumstances, light exhibits the same behavior as sound or water waves. The study of light as a wave is called wave optics.

• The ray model: The properties of prisms, mirrors, and lenses are best understood in terms of light rays. The ray model is the basis of ray optics.

• The photon model: In the quantum world, light behaves like neither a wave nor a particle. Instead, light consists of photons that have both wave-like and particle-like properties. This is the quantum theory of light.
Analyzing Double-Slit Interference

The $m$th bright fringe emerging from the double slit is at an angle

$$\theta_m = \frac{m \lambda}{d} \quad m = 0, 1, 2, 3, \ldots$$

(angles of bright fringes)

where $\theta_m$ is in radians, and we have used the small-angle approximation. The $y$-position on the screen of the $m$th fringe is

$$y_m = \frac{m \lambda L}{d} \quad m = 0, 1, 2, 3, \ldots$$

(positions of bright fringes)

while dark fringes are located at positions

$$y_n = \left( m + \frac{1}{2} \right) \frac{\lambda L}{d} \quad m = 0, 1, 2, \ldots$$

(positions of dark fringes)

EXAMPLE 22.2 Measuring the wavelength of light

QUESTION:

EXAMPLE 22.2 Measuring the wavelength of light

A double-slit interference pattern is observed on a screen 1.0 m behind two slits spaced 0.30 mm apart. Ten bright fringes span a distance of 1.7 cm. What is the wavelength of the light?

MODEL It is not always obvious which fringe is the central maximum. Slight imperfections in the slits can make the interference fringe pattern less than ideal. However, you do not need to identify the $m = 0$ fringe because you can make use of the fact that the fringe spacing $\Delta y$ is uniform. Ten bright fringes have nine spaces between them (not ten—be careful!).
EXAMPLE 22.2 Measuring the wavelength of light

**VISEUZIAL** The interference pattern looks like the photograph of Figure 22.3b.

(a) A plane wave is incident on the double slit.
1. Waves spread out behind each slit.
2. The waves interfere in the region where they overlap.
3. Bright fringes occur where the antinodal lines intersect the viewing screen.

SOLVE The fringe spacing is
\[
\Delta y = \frac{d}{9} = 1.89 \times 10^{-3} \text{ m}
\]
Using this fringe spacing in Equation 22.7, we find that the wavelength is
\[
\lambda = \frac{d}{L} \Delta y = 5.7 \times 10^{-7} \text{ m} = 570 \text{ nm}
\]
It is customary to express the wavelengths of light in nanometers. Be sure to do this as you solve problems.

EXAMPLE 22.2 Measuring the wavelength of light

**ASSESS** Young’s double-slit experiment not only demonstrated that light is a wave, it provided a means for measuring the wavelength. You learned in Chapter 20 that the wavelengths of visible light span the range 400–700 nm. These lengths are smaller than we can easily comprehend. A wavelength of 570 nm, which is in the middle of the visible spectrum, is only about 1% of the diameter of a human hair.

The Diffraction Grating

Suppose we were to replace the double slit with an opaque screen that has \( N \) closely spaced slits. When illuminated from one side, each of these slits becomes the source of a light wave that diffracts, or spreads out, behind the slit. Such a multi-slit device is called a **diffraction grating**. Bright fringes will occur at angles \( \theta_m \), such that
\[
d \sin \theta_m = m \lambda \quad m = 0, 1, 2, 3, \ldots
\]
The \( y \)-positions of these fringes will occur at
\[
y_m = L \tan \theta_m \quad \text{(positions of bright fringes)}
\]
EXAMPLE 22.3 Measuring wavelengths emitted by sodium atoms

QUESTION:

EXAMPLE 22.3 Measuring wavelengths emitted by sodium atoms

Light from a sodium lamp passes through a diffraction grating having 1000 slits per millimeter. The interference pattern is viewed on a screen 1.00 m behind the grating. Two bright yellow fringes are visible 72.88 cm and 73.00 cm from the central maximum. What are the wavelengths of these two fringes?

VISUALIZE This is the situation shown in Figure 22.8b. The two fringes are very close together, so we expect the wavelengths to be only slightly different. No other yellow fringes are mentioned, so we will assume these two fringes are the first-order diffraction ($m = 1$).
EXAMPLE 22.3 Measuring wavelengths emitted by sodium atoms

SOLVE The distance $y_m$ of a bright fringe from the central maximum is related to the diffraction angle by $y_m = L\tan\theta_m$. Thus the diffraction angles of these two fringes are

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{L}\right) = \left\{ \begin{array}{l} 36.08^\circ \text{ fringe at } 72.88 \text{ cm} \\
36.13^\circ \text{ fringe at } 73.00 \text{ cm} \end{array} \right.$$ 

These angles must satisfy the interference condition $d\sin\theta_1 = \lambda$, so the wavelengths are $\lambda = d\sin\theta_1$. What is $d$? If a 1 mm length of the grating has 1000 slits, then the spacing from one slit to the next must be $1/1000$ mm, or $d = 1.000 \times 10^{-3}$ m. Thus the wavelengths creating the two bright fringes are

$$\lambda = d\sin\theta_1 = \left\{ \begin{array}{l} 589.0 \text{ nm fringe at } 72.88 \text{ cm} \\
589.6 \text{ nm fringe at } 73.00 \text{ cm} \end{array} \right.$$ 

ASSESS We had data accurate to four significant figures, and all four were necessary to distinguish the two wavelengths.
Single-Slit Diffraction

Consider light of wavelength $\lambda$ which passes through a slit of width $a$, and is then incident on a viewing screen a distance $L$ behind the slit, where $L >> a$. The light pattern will consist of a central maximum flanked by a series of weaker secondary maxima and dark fringes. The dark fringes occur at angles

$$\theta_p = \frac{p \lambda}{a} \quad p = 1, 2, 3, \ldots \quad (\text{angles of dark fringes})$$
EXAMPLE 22.4 Diffraction of a laser through a slit

QUESTION:

EXAMPLE 22.4 Diffraction of a laser through a slit

Light from a helium-neon laser ($\lambda = 633 \text{ nm}$) passes through a narrow slit and is seen on a screen $2.0 \text{ m}$ behind the slit. The first minimum in the diffraction pattern is $1.2 \text{ cm}$ from the central maximum. How wide is the slit?

MODEL A narrow slit produces a single-slit diffraction pattern. A displacement of only $1.2 \text{ cm}$ in a distance of $200 \text{ cm}$ means that angle $\theta_1$ is certainly a small angle.

VISUALIZE The intensity pattern will look like Figure 22.14.

FIGURE 22.14 A graph of the intensity of a single-slit diffraction pattern.

SOLVE We can use the small-angle approximation to find that the angle to the first minimum is

$$\theta_1 = \frac{1.2 \text{ cm}}{200 \text{ cm}} = 0.00600 \text{ rad} = 0.344^\circ$$

The first minimum is at angle $\theta_1 = \lambda/a$, from which we find that the slit width is

$$a = \frac{\lambda}{\theta_1} = \frac{633 \times 10^{-9} \text{ m}}{6.00 \times 10^{-3} \text{ rad}} = 1.1 \times 10^{-4} \text{ m} = 0.11 \text{ mm}$$
**EXAMPLE 22.4 Diffraction of a laser through a slit**

**ASSESS** This is typical of the slit widths used to observe single-slit diffraction. You can see that the small-angle approximation is well satisfied.

**Circular-Aperture Diffraction**

Light of wavelength $\lambda$ passes through a circular aperture of diameter $D$, and is then incident on a viewing screen a distance $L$ behind the aperture, $L \gg D$. The diffraction pattern has a circular central maximum, surrounded by a series of secondary bright fringes shaped like rings. The angle of the first minimum in the intensity is

$$\theta_1 = \frac{1.22\lambda}{D}$$

The width of the central maximum on the screen is

$$w = 2y_1 = 2L\tan\theta_1 \approx \frac{2.44\lambda L}{D}$$

**Tactics: Choosing a model of light**

- When light passes through openings <1 mm in size, diffraction effects are usually important. Use the wave model of light.
- When light passes through openings >1 mm in size, diffraction effects are usually not important. Use the ray model of light.
Measuring Indices of Refraction

A Michelson interferometer can be used to measure indices of refraction of gases. A cell of thickness $d$ is inserted into one arm of the cell. When the cell contains a vacuum, the number of wavelengths inside the cell is

$$m_1 = \frac{2d}{\lambda_{\text{vac}}}$$

When the cell is filled with a specific gas, the number of wavelengths spanning the distance $d$ is

$$m_2 = \frac{2d}{\lambda} = \frac{2d}{\lambda_{\text{vac}}/n}$$

Filling the cell has increased the lower path by

$$\Delta m = m_2 - m_1 = (n - 1) \frac{2d}{\lambda_{\text{vac}}}$$

wavelengths. By counting fringe shifts as the cell is filled, one can determine $n$.

EXAMPLE 22.9 Measuring the index of refraction

QUESTION:

A Michelson interferometer uses a helium-neon laser with wavelength $\lambda_{\text{vac}} = 633$ nm. As a 4.00-cm-thick cell is slowly filled with a gas, 43 bright-dark-bright fringe shifts are seen and counted. What is the index of refraction of the gas at this wavelength?

MODEL: The gas increases the number of wavelengths in one arm of the interferometer. Each additional wavelength causes one bright-dark-bright fringe shift.
EXAMPLE 22.9 Measuring the index of refraction

SOLVE  We can rearrange Equation 22.36 to find that the index of refraction is

\[ n = 1 + \frac{\lambda_{\text{vac}} \Delta m}{2d} = 1 + \frac{(6.33 \times 10^{-7} \text{ m})(43)}{2(0.0400 \text{ m})} = 1.00034 \]

ASSESS  This may seem like a six-significant-figure result, but it's really only two. What we're measuring is not \( n \) but \( n - 1 \). We know the fringe count to two significant figures, and that has allowed us to compute \( n - 1 = \frac{\lambda_{\text{vac}} \Delta m}{2d} = 3.4 \times 10^{-4} \).

Chapter 22. Summary Slides

General Principles

Huygens' principle says that each point on a wave front is the source of a spherical wavelet. The wave front at a later time is tangent to all the wavelets.
**General Principles**

**Diffraction** is the spreading of a wave after it passes through an opening.

Constructive and destructive *interference* are due to the overlap of two or more waves as they spread behind openings.

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**Important Concepts**

The *wave model* of light considers light to be a wave propagating through space. Diffraction and interference are important. The *ray model* of light considers light to travel in straight lines like little particles. Diffraction and interference are not important.

Diffraction is important when the width of the diffraction pattern of an aperture equals or exceeds the size of the aperture. For a circular aperture, the crossover between the ray and wave models occurs for an opening of diameter \( D_i \approx \sqrt{2} \lambda a \).

In practice, \( D_i \approx 1 \) mm. Thus:

- Use the wave model when light passes through openings < 1 mm in size. Diffraction effects are usually important.
- Use the ray model when light passes through openings > 1 mm in size. Diffraction is usually not important.

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**Applications**

**Single slit** of width \( a \).

A bright central maximum of width

\[ w = \frac{2\lambda L}{a} \]

is flanked by weaker secondary maxima. Dark fringes are located at angles such that

\[ a \sin \theta_p = p\lambda \quad p = 1, 2, 3, \ldots \]

If \( \lambda/a \ll 1 \), then from the small-angle approximation

\[ \theta_p = \frac{p\lambda}{a} \quad y_p = \frac{p\lambda L}{a} \]

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**Applications**

**Interference due to wave-front division**

Waves overlap as they spread out behind slits. Constructive interference occurs along antinodal lines. Bright fringes are seen where the antinodal lines intersect the viewing screen.

**Double slit** with separation \( d \).

Equally spaced bright fringes are located at

\[ \theta_m = \frac{m\lambda}{d} \quad y_m = \frac{m\lambda L}{d} \quad m = 0, 1, 2, \ldots \]

The fringe spacing is \( \Delta y = \frac{\lambda L}{d} \).

**Diffraction grating** with slit spacing \( d \).

Very bright and narrow fringes are located at angles and positions

\[ d \sin \theta_m = m\lambda \quad y_m = L \tan \theta_m \]
Circular aperture of diameter $D$.
A bright central maximum of diameter
$$w = \frac{2.44\lambda L}{D}$$
is surrounded by circular secondary maxima.
The first dark fringe is located at
$$\theta_1 = \frac{1.22\lambda}{D}, \quad y_1 = \frac{1.22\lambda L}{D}$$
For an aperture of any shape, a smaller opening
causes a more rapid spreading of the wave behind the
opening.

Applications

Interference due to amplitude division
An interferometer divides a wave, lets the two waves travel different
paths, then recombines them. Interference is constructive if one wave
travels an integer number of wavelengths more or less than the other
wave. The difference can be due to an actual path-length difference
or to a different index of refraction.

Michelson interferometer
The number of bright-dark-bright fringe shifts as mirror $M_2$ moves
distance $\Delta L_2$ is
$$\Delta m = \frac{\Delta L_2}{\lambda/2}$$

Chapter 22. Clicker Questions

Suppose the viewing screen in the figure is moved closer to the
double slit. What happens to the interference fringes?
A. They fade out and disappear.
B. They get out of focus.
C. They get brighter and closer together.
D. They get brighter and farther apart.
E. They get brighter but otherwise do not change.
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C. **They get brighter and closer together.**  
D. They get brighter and farther apart.  
E. They get brighter but otherwise do not change.

Light of wavelength $\lambda_1$ illuminates a double slit, and interference fringes are observed on a screen behind the slits. When the wavelength is changed to $\lambda_2$, the fringes get closer together. How large is $\lambda_2$ relative to $\lambda_1$?

A. $\lambda_2$ is smaller than $\lambda_1$.  
B. $\lambda_2$ is larger than $\lambda_1$.  
C. Cannot be determined from this information.

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**A. $\lambda_2$ is smaller than $\lambda_1$.**  
B. $\lambda_2$ is larger than $\lambda_1$.  
C. Cannot be determined from this information.

White light passes through a diffraction grating and forms rainbow patterns on a screen behind the grating. For each rainbow,

A. the red side is farthest from the center of the screen, the violet side is closest to the center.  
B. the red side is closest to the center of the screen, the violet side is farthest from the center.  
C. the red side is on the left, the violet side on the right.  
D. the red side is on the right, the violet side on the left.
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A Michelson interferometer using light of wavelength $\lambda$ has been adjusted to produce a bright spot at the center of the interference pattern. Mirror $M_1$ is then moved distance $\lambda$ toward the beam splitter while $M_2$ is moved distance $\lambda$ away from the beam splitter. How many bright-dark-bright fringe shifts are seen?

A. 4
B. 3
C. 2
D. 1
E. 0
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