college

THIRD EDITION

Lecture Presentation

Chapter 7 Rotational Motion

Chapter 7 Preview Looking Ahead

Rotational Kinematics The spinning roulette wheel isn't going anywhere, but it is moving. This is rotational motion.



as this sailor is doing to the wheel.



You'll learn about angular velocity and other quantities we use to describe rotational motion. Torque To start something moving, apply a force. To start something rotating, apply a torque,



You'll see that torque depends on how hard

far from the axle gives a large torque.

you push and also on where you push. A push

Rotational Dynamics

The girl pushes on the outside edge of the merry-go-round, gradually increasing its rotation rate.



You'll learn a version of Newton's second law for rotational motion and use it to solve problems.

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Reading Question 7.1

knight · jones · field

If an object is rotating clockwise, this corresponds to a angular velocity.

A. Positive

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B. Negative

Reading Question 7.1

If an object is rotating clockwise, this corresponds to a angular velocity.

A. Positive **B**. Negative

Reading Question 7.2

The angular displacement of a rotating object is measured in

Reading Question 7.2

The angular displacement of a rotating object is measured in



Reading Question 7.3

Moment of inertia is

- A. The rotational equivalent of mass.
- B. The time at which inertia occurs.
- C. The point at which all forces appear to act.
- D. An alternative term for moment arm.

Reading Question 7.3

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✓ A. The rotational equivalent of mass.

- B. The time at which inertia occurs.
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- D. An alternative term for *moment arm*.

Reading Question 7.4

Which factor does the torque on an object not depend on?

- A. The magnitude of the applied force
- B. The object's angular velocity
- C. The angle at which the force is applied
- D. The distance from the axis to the point at which the force is applied

Reading Question 7.4

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Reading Question 7.5

A net torque applied to an object causes

- A. A linear acceleration of the object.
- B. The object to rotate at a constant rate.
- C. The angular velocity of the object to change.
- D. The moment of inertia of the object to change.

Reading Question 7.5

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Describing Circular and Rotational Motion

• **Rotational motion** is the motion of objects that spin about an axis.



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Angular Position

- We use the angle *θ* from the positive *x*-axis to describe the particle's location.
- Angle θ is the **angular position** of the particle.
- *θ* is positive when measured *counterclockwise* from the positive *x*-axis.



• An angle measured *clockwise* from the positive *x*-axis has a negative value.

Angular Position

- We measure angle θ in the angular unit of *radians*, not degrees.
- The radian is abbreviated "rad."
- The **arc length**, *s*, is the distance that the particle has traveled along its circular path.



Angular Position

• We define the particle's angle θ in terms of arc length and radius of the circle:

$$\theta$$
 (radians) = $\frac{s}{r}$

 $s = r\theta$

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Angular Position

around the circle.

• The angle of the full circle is

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Angular Displacement and Angular Velocity

• For *linear* motion, a particle with a larger velocity undergoes a greater displacement

(a) Uniform linear motion

A particle with a small velocity y undergoes a small displacement each second.



Angular Displacement and Angular Velocity

• One *revolution* (rev) is when a particle travels all the way

 $\theta_{\text{full choice}} = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$

For uniform *circular* motion, a particle with a larger angular velocity will undergo a greater angular displacement Δθ.



• Angular velocity is the angular displacement through which the particle moves each second.

Angular Displacement and Angular Velocity



• The angular velocity $\omega = \Delta \theta / \Delta t$ is constant for a particle moving with uniform circular motion.

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Example 7.1 Comparing angular velocities (cont.)



SOLVE The particle on the left travels one-quarter of a full circle during the 5 s time interval. We learned earlier that a full circle corresponds to an angle of 2π rad, so the angular displacement for this particle is $\Delta\theta = (2\pi \text{ rad})/4 = \pi/2$ rad. Thus its angular velocity is

$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{\pi/2 \text{ rad}}{5 \text{ s}} = 0.314 \text{ rad/s}$$

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Example 7.1 Comparing angular velocities

Find the angular velocities of the two particles in Figure 7.2b.



PREPARE For uniform circular motion, we can use any angular displacement $\Delta\theta$, as long as we use the corresponding time interval Δt . For each particle, we'll choose the angular displacement corresponding to the motion from t = 0 s to t = 5 s.

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Example 7.1 Comparing angular velocities (cont.)



The particle on the right travels halfway around the circle, or π rad, in the 5 s interval. Its angular velocity is

$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{\pi \text{ rad}}{5 \text{ s}} = 0.628 \text{ rad/s}$$

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Angular Displacement and Angular Velocity

• The linear displacement during a time interval is

 $x_{f} - x_{f} = \Delta x = v_{f} \Delta t$

• Similarly, the angular displacement for uniform circular motion is

 $\theta_{\rm f} - \theta_{\rm i} = \Delta \theta = \omega \ \Delta t$ Angular displacement for uniform circular motion

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Angular Displacement and Angular Velocity

- **Angular speed** is the absolute value of the angular velocity.
- The angular speed is related to the period *T*:

$$\omega = \frac{2\pi \operatorname{rad}}{T}$$

• Frequency (in rev/s) f = 1/T:

```
\omega = (2\pi \text{ rad})f
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Example 7.3 Rotations in a car engine

The crankshaft in your car engine is turning at 3000 rpm. What is the shaft's angular speed?

PREPARE We'll need to convert rpm to rev/s and then use Equation 7.6.

SOLVE We convert rpm to rev/s by

$$\left(3000\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 50.0\,\text{rev/s}$$

Thus the crankshaft's angular speed is

 $\omega = (2\pi \text{ rad})f = (2\pi \text{ rad})(50.0 \text{ rev/s}) = 314 \text{ rad/s}$

Angular-Position and Angular-Velocity Graphs

- We construct **angular position-versus-time graphs** using the change in angular *position* for each second.
- Angular velocity-versus-time graphs can be created by finding the slope of the corresponding angular position-versus-time graph.

Relating Speed and Angular Speed

• Speed v and angular speed ω are related by

 $v = \omega r$ Relationship between speed and angular speed

• Angular speed ω must be in units of rad/s.

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Example 7.5 Finding the speed at two points on a CD

The diameter of an audio compact disk is 12.0 cm. When the disk is spinning at its maximum rate of 540 rpm, what is the speed of a point (a) at a distance 3.0 cm from the center and (b) at the outside edge of the disk, 6.0 cm from the center?

PREPARE Consider two points A and B on the rotating compact disk in

FIGURE 7.7. During one period *T*, the disk rotates once, and both points rotate through the same angle, 2π rad. Thus the angular speed, $\omega = 2\pi/T$, is the same for these two points; in fact, it is the same for all points on the disk.

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Slide 7-30

B

Example 7.5 Finding the speed at two points on a CD (cont.)

But as they go around one time, the two points move different *distances*. The outer point B goes around a larger circle. The two points thus have different *speeds*. We can solve this problem by first finding the angular speed of the disk and then computing the speeds at



disk and then computing the speeds at the two points.

Example 7.5 Finding the speed at two points on a CD (cont.)

SOLVE We first convert the frequency of the disk to rev/s:

$$f = \left(540 \frac{\text{rev}}{\text{min}}\right) \times \left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$
$$= 9.00 \text{ rev/s}$$



We then compute the angular speed using Equation 7.6:

 $\omega = (2\pi \text{ rad})(9.00 \text{ rev/s}) = 56.5 \text{ rad/s}$

Example 7.5 Finding the speed at two points on a CD (cont.)

We can now use Equation 7.7 to compute the speeds of points on the disk. At point A, r = 3.0 cm = 0.030 m, so the speed is



 $v_{\rm A} = \omega r = (56.5 \text{ rad/s})(0.030 \text{ m}) = 1.7 \text{ m/s}$

At point B, r = 6.0 cm = 0.060 m, so the speed at the outside edge is

$$v_{\rm B} = \omega r = (56.5 \text{ rad/s})(0.060 \text{ m}) = 3.4 \text{ m/s}$$

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Example 7.5 Finding the speed at two points on a CD (cont.)

ASSESS The speeds are a few meters per second, which seems reasonable. The point farther from the center is moving at a higher speed, as we expected.



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QuickCheck 7.7

This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?



QuickCheck 7.7

This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?





Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time t. Through what angle will it have turned after time 2*t*?

 A. 25 rad B. 50 rad C. 75 rad D. 100 rad E. 200 rad 		A. 25 rad B. 50 rad C. 75 rad \checkmark D. 100 rad $\Delta \theta \propto (\Delta t)^2$ E. 200 rad
© 2015 Pearson Education, Inc.	Slide 7-37	© 2015 Pearson Education, Inc.
Section 7.2 The Rotation of a Ri	gid Body	 The Rotation of a Rig A rigid body is an extend object whose size and sha do not change as it moves The rigid-body model is good approximation for m real objects.

QuickCheck 7.9

Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time t. Through what angle will it have turned after time 2*t*?

Slide 7-38

id Body

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- а nany





Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is _____ that of Rasheed.

- A. Half
- B. The same as
- C. Twice
- D. Four times
- E. We can't say without knowing their radii.



QuickCheck 7.2

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is _____ that of Rasheed.

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Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's speed is _____ that of Rasheed.



QuickCheck 7.3

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's speed is _____ that of Rasheed.



QuickCheck 7.4

Two coins rotate on a turntable. Coin B is twice as far from the axis as coin A.



- A. The angular velocity of A is twice that of B
- B. The angular velocity of A equals that of B
- C. The angular velocity of A is half that of B

QuickCheck 7.4

Two coins rotate on a turntable. Coin B is twice as far from the axis as coin A.



- A. The angular velocity of A is twice that of B
- ✔ B. The angular velocity of A equals that of B
 - C. The angular velocity of A is half that of B

Angular Acceleration

• Angular acceleration is defined as:

 $\alpha = \frac{\text{change in angular velocity}}{\text{time interval}} = \frac{\Delta\omega}{\Delta t}$ Angular acceleration for a particle in nonuniform circular motion

• The units of angular acceleration are rad/s².



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QuickCheck 7.5

The fan blade is slowing down. What are the signs of ω and α ?

- A. ω is positive and α is positive.
- B. ω is positive and α is negative.
- C. ω is negative and α is positive.
- D. ω is negative and α is negative.
- E. ω is positive and α is zero.



Angular Acceleration



QuickCheck 7.5

The fan blade is slowing down. What are the signs of ω and α ?

- A. ω is positive and α is positive.
- B. ω is positive and α is negative.
- \checkmark C. ω is negative and α is positive.
 - D. ω is negative and α is negative.
 - E. ω is positive and α is zero.

"Slowing down" means that ω and α have opposite signs, not that α is negative.



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The fan blade is speeding up. What are the signs of ω and α ?

- A. ω is positive and α is positive.
- B. ω is positive and α is negative.
- C. ω is negative and α is positive.
- D. ω is negative and α is negative.



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QuickCheck 7.6

The fan blade is speeding up. What are the signs of ω and α ?

- A. ω is positive and α is positive.
- B. ω is positive and α is negative.
- C. ω is negative and α is positive.
- **/** D. ω is negative and α is negative.



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Example Problem

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A high-speed drill rotating counterclockwise takes 2.5 s to speed up to 2400 rpm.

- A. What is the drill's angular acceleration?
- B. How many revolutions does it make as it reaches top speed?

Linear and Circular Motion

SYNTHESIS 7.1 Linear and circular motion

The variables and equations for linear motion have analogs for circular motion.



Starting from rest, a wheel with constant angular acceleration spins up to 25 rpm in a time t. What will its angular velocity be after time 2*t*?

Tangantial Appalaration		Tangantial Acceleration	
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E. 200 rpm		E. 200 rpm	
D 100 rpm		D 100 rpm	
B. 50 rpm		$\blacksquare B. 50 \text{ rpm} \qquad \Delta \omega \propto \Delta t$	
A. 25 rpm		A. 25 rpm	

- langential Acceleration
- Tangential acceleration is the component of acceleration directed tangentially to the circle.
- The tangential acceleration measures the rate at which the particle's speed around the circle increases.



always has a centripetal acceleration

langential Acceleration

QuickCheck 7.8

• We can relate tangential acceleration to the angular acceleration by $v = \omega r$.

Starting from rest, a wheel with constant angular

angular velocity be after time 2*t*?

acceleration spins up to 25 rpm in a time t. What will its

$$a_{t} = \frac{\Delta v}{\Delta t} = \frac{\Delta(\omega r)}{\Delta t} = \frac{\Delta \omega}{\Delta t} r$$

 $a_t = \alpha r$ Relationship between tangential and angular acceleration



• Torque units are newton-meters, abbreviated $N \cdot m$.

ϕ is the angle between the radial line and the direction of the force.

measured from the

direction of the force.

radial line to the

Radial

line

Torque

- The **radial line** is the line starting at the pivot and extending through the point where force is applied.
- The angle φ is measured from the radial line to the direction of the force.
- Torque is dependent on the perpendicular component of the force being applied.

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 $F_{\perp} = F \sin \phi$ The component of \vec{F} that is *perpendicular* to the radial line causes a torque. \vec{F}_{\perp}

Torque

- An alternate way to calculate torque is in terms of the moment arm.
- The **moment arm** (or lever arm) is the perpendicular distance from the *line of action* to the pivot.
- The *line of action* is the line that is in the direction of the force and passes through the point at which the force acts.

in the direction of the force r_{1} vector and passes through the point at which the force acts. Pivot r_{1} Line of action r_{1} vector and r_{2} vector r_{2} vector r_{3} vector r_{4} vector r_{4} vector r_{5} vector r_{4} vector r_{5} vector r_{5}

The line of action extends

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Torque

• The equivalent expression for torque is

 $au = r_{\perp} F$ Torque due to a force *F* with moment arm r_{\perp}

• For both methods for calculating torque, the resulting expression is the same:

 $\tau = rF\sin\phi$



QuickCheck 7.10

The four forces shown have the same strength. Which force would be most effective in opening the door?



The four forces shown have the same strength. Which force would be most effective in opening the door?



Example 7.9 Torque in opening a door (cont.)



The torque depends on how hard Ryan pushes, where he pushes, and at what angle. If he wants to exert more torque, he could push at a point a bit farther out from the hinge, or he could push exactly perpendicular to the door. Or he could simply push harder!

ASSESS As you'll see by doing more problems, $170 \text{ N} \cdot \text{m}$ is a significant torque, but this makes sense if you are trying to free a stuck door.

Example 7.9 Torque in opening a door

Ryan is trying to open a stuck door. He pushes it at a point 0.75 m from the hinges with a 240 N force directed 20° away from being perpendicular to the door. There's a natural pivot point, the hinges. What torque does Ryan exert? How could he exert more torque?



PREPARE In FIGURE 7.20 the radial line is shown drawn from the pivot—the hinge—through the point at which the force \vec{F} is applied. We see that the component of \vec{F} that is perpendicular to the radial line is $F_{\perp} = F \cos 20^{\circ} = 226 \text{ N}$. The distance from the hinge to the point at which the force is applied is r = 0.75 m.

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Torque

• A torque that tends to rotate the object in a counterclockwise direction is positive, while a torque that tends to rotate the object in a clockwise direction is negative.



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Net Torque



QuickCheck 7.11

QuickCheck 7.11

Which third force on the wheel, applied at point P, will make the net torque zero?





Section 7.4 Gravitational Torque and the Center of Gravity

Gravitational Torque and the Center of Gravity

- Gravity pulls downward on *every particle* that makes up an object (like the gymnast).
- Each particle experiences a torque due to the force of gravity.
- (a) Gravity exerts a force and a torque on each particle that makes up the gymnast...... Rotation axis



Gravitational Torque and the Center of Gravity

(b)

- The gravitational torque can be calculated by assuming that the net force of gravity (the object's weight) acts as a single point.
- That single point is called the **center of gravity**.

The weight force provides a torque about the rotation axis.....



The gymnast responds *as if* her entire weight acts at her center of gravity.

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Example 7.12 The torque on a flagpole

to the wall. What is the gravitational torque on the flagpole

PREPARE FIGURE 7.26 shows the situation. For the purpose of

calculating torque, we can consider the entire weight of the pole

as acting at the center of gravity. Because the moment arm r_{\perp} is

simple to visualize here, we'll use Equation 7.11 for the torque.

A 3.2 kg flagpole extends from a wall at an angle of 25° from the horizontal. Its center of gravity is 1.6 m from the point where the pole is attached

about the point of attachment?



Example 7.12 The torque on a flagpole (cont.)

SOLVE From Figure 7.26, we see that the moment arm is $r_{\perp} = (1.6 \text{ m}) \cos 25^{\circ} =$ 1.45 m. Thus the gravitational torque on the flagpole, about the point where it attaches to the wall, is



 $r = -r_{\perp}w = -r_{\perp}mg = -(1.45 \text{ m})(3.2 \text{ kg})(9.8 \text{ m/s}^2) = -45 \text{ N} \cdot \text{m}$

We inserted the minus sign because the torque tries to rotate the pole in a clockwise direction.

ASSESS If the pole were attached to the wall by a hinge, the gravitational torque would cause the pole to fall. However, the actual rigid connection provides a counteracting (positive) torque to the pole that prevents this. The net torque is zero.

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Gravitational Torque and the Center of Gravity

- An object that is free to rotate about a pivot will come to rest with the center of gravity below the pivot point.
- If you hold a ruler by one end and allow it to rotate, it will stop rotating when the center of gravity

Pivot When the center of gravity is not below the pivot, w_{\perp} is zero and there is no torque. When the center \vec{w}_{\perp} \vec{w}_{\perp} \vec{w}_{\perp} \vec{w}_{\perp}

is directly above or below the pivot point. There is no torque acting at these positions.

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Which point could be the center of gravity of this L-shaped piece?



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QuickCheck 7.12

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Which point could be the center of gravity of this L-shaped piece?



Calculating the Position of the Center of Gravity

- The torque due to gravity when the pivot is *at* the center of gravity is zero.
- We can use this to find an expression for the position of the center of gravity.

Calculating the Position of the Center of Gravity

- For the dumbbell to balance, the pivot must be at the center of gravity.
- We calculate the torque on either side of the pivot, which is located at the position x_{cg} .



Calculating the Position of the Center of Gravity

• The torque due to the weight on the left side of the pivot is

 $\tau_1 = r_1 w_1 = (x_{cg} - x_1)m_1g$

• The torque due to the weight on the right side of the pivot is

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$$\tau_2 = -r_2 w_2 = -(x_2 - x_{cg}) m_2 g$$



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Calculating the Position of the Center of Gravity

• The total torque is

$$\tau_{\text{net}} = 0 = \tau_1 + \tau_2 = (x_{\text{cg}} - x_1)m_1g - (x_2 - x_{\text{cg}})m_2g$$

• The location of the center of gravity is



Calculating the Position of the Center of Gravity

- Because the center of gravity depends on distance and mass from the pivot point, objects with large masses count more heavily.
- The center of gravity tends to lie closer to the heavier objects or particles that make up the object.



Calculating the Position of the Center of Gravity



Example 7.13 Where should the dumbbell be lifted?

A 1.0-m-long dumbbell has a 10 kg mass on the left and a 5.0 kg mass on the right. Find the position of the center of gravity, the point where the dumbbell should be lifted in order to remain balanced.

PREPARE First we sketch the situation as in FIGURE 7.30.



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Example 7.13 Where should the dumbbell be lifted? (cont.)



Next, we can use the steps from Tactics Box 7.1 to find the center of gravity. Let's choose the origin to be at the position of the 10 kg mass on the left, making $x_1 = 0$ m and $x^2 = 1.0$ m. Because the dumbbell masses lie on the *x*-axis, the *y*-coordinate of the center of gravity must also lie on the *x*-axis. Thus we only need to solve for the *x*-coordinate of the center of gravity.

Example 7.13 Where should the dumbbell be lifted? (cont.)



The center of gravity is 0.33 m from the 10 kg mass or, equivalently, 0.17 m left of the center of the bar.

ASSESS The position of the center of gravity is closer to the larger mass. This agrees with our general statement that the center of gravity tends to lie closer to the heavier particles.



Rotational Dynamics and Moment of Inertia

- A torque causes an angular acceleration.
- The tangential and angular accelerations are





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Rotational Dynamics and Moment of Inertia

• We compare with torque:

 $\tau = rF$

• We find the relationship with angular acceleration:

$$\alpha = \frac{\tau}{mr^2}$$



Newton's Second Law for Rotational Motion

- For a rigid body rotating about a fixed axis, we can think of the object as consisting of multiple particles.
- We can calculate the torque on each particle.
- Because the object rotates together, each particle has the *same* angular acceleration.

Particle 1 is at radius r_1 and has mass m_1 . $\vec{F_1}$ $\vec{F_2}$ $\vec{F_2}$ $\vec{F_2}$ has the $\vec{F_2}$ $\vec{F_2}$

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Newton's Second Law for Rotational Motion



Newton's Second Law for Rotational Motion

• The quantity Σmr^2 in Equation 7.20, which is the proportionality constant between angular acceleration and net torque, is called the object's moment of inertia *I*:

r3 Rotation m axis \vec{F}_2 $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$

Particle 1 is at radius r_1

and has mass m_1 .

These forces exert a net torque about the rotation axis and cause the object to have an angular acceleration.

• The units of moment of inertia are kg \cdot m².

Moment of inertia of a collection of particles

• The moment of inertia *depends on the axis of rotation*.

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Newton's Second Law for Rotational Motion

Newton's second law for rotation An object that experiences a net torque $\tau_{\rm net}$ about the axis of rotation undergoes an angular acceleration

$$\alpha = \frac{\tau_{\rm ne}}{I}$$

where *I* is the moment of inertia of the object *about the rotation axis*.

A net torque is the cause of angular acceleration.

Interpreting the Moment of Inertia

- The moment of inertia is the rotational equivalent of mass.
- An object's moment of inertia depends not only on the object's mass but also on how the mass is distributed around the rotation axis.



inertia, harder to inertia, easier to get rotating get rotating

Interpreting the Moment of Inertia

- The moment of inertia is the rotational equivalent of mass.
- It is more difficult to spin the merry-go-round when people sit far from the center because it has a higher inertia than when people sit close to the center.



Larger moment of inertia, harder to get rotating

Smaller moment of inertia, easier to get rotating

K in

Interpreting the Moment of Inertia



Example 7.15 Calculating the moment of inertia

Your friend is creating an abstract sculpture that consists of three small, heavy spheres attached by very lightweight 10-cm-long rods as shown in FIGURE 7.36. The spheres have masses

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 $m_1 = 1.0$ kg, $m_2 = 1.5$ kg, and $m_3 = 1.0$ kg. What is the object's moment of inertia if it is rotated about axis A? About axis B?

PREPARE We'll use Equation 7.21 for the moment of inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

In this expression, r_1 , r_2 , and r_3 are the distances of each particle from the axis of rotation, so they depend on the axis chosen.

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Example 7.15 Calculating the moment of inertia (cont.)

Particle 1 lies on both axes, so $r_1 = 0$ cm in both cases. Particle 2 lies 10 cm (0.10 m) from both axes. Particle 3 is 10 cm from axis A but farther from axis B. We



can find r_3 for axis B by using the Pythagorean theorem, which gives $r_3 = 14.1$ cm. These distances are indicated in the figure.



SOLVE For each axis, we can prepare a table of the values of r, m, and mr^2 for each particle, then add the values of mr^2 . For axis A we have

Particle

1

2

3

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r

0.10 m

0.10 m

0 m



Example 7.15 Calculating the moment of inertia (cont.)



Example 7.15 Calculating the moment of inertia (cont.)

axis B than from axis A, leading to a higher moment of

ASSESS We've already noted that the moment of inertia of an object is higher when its mass is distributed farther from the axis of rotation. Here, m_3 is farther from

inertia about that axis.



The Moments of Inertia of Common Shapes



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Using Newton's Second Law for Rotation

PROBLEM-SOLVING STRATEGY 7.1 Rotational dynamics problems



We can use a problem-solving strategy for rotational dynamics that is very similar to the strategy for linear dynamics in Chapter 5.

PREPARE Model the object as a simple shape. Draw a pictorial representation to clarify the situation, define coordinates and symbols, and list known information.

- Identify the axis about which the object rotates.
- Identify the forces and determine their distance from the axis.
- Calculate the torques caused by the forces, and find the signs of the torques.

SOLVE The mathematical representation is based on Newton's second law for rotational motion:

$$\tau_{\rm net} = I \alpha$$
 or $\alpha = \frac{\tau_{\rm net}}{I}$

- Find the moment of inertia either by direct calculation using Equation 7.21 or from Table 7.1 for common shapes of objects.
- Use rotational kinematics to find angular positions and velocities.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 31 💋

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Example 7.18 Starting an airplane engine

The engine in a small air-plane is specified to have a torque of $500 \text{ N} \cdot \text{m}$. This engine drives a 2.0-m-long, 40 kg single-blade propeller. On start-up, how long does it take the propeller to reach 2000 rpm?



Example 7.18 Starting an airplane engine (cont.)

PREPARE The propeller can be modeled as a rod that rotates about its center. The engine exerts a torque on the propeller. FIGURE 7.38 shows the propeller and the rotation axis.



Example 7.18 Starting an airplane engine (cont.)



Example 7.18 Starting an airplane engine (cont.)

The time needed to reach $\omega_f = 2000 \text{ rpm} = 33.3 \text{ rev/s} = 209 \text{ rad/s is}$

$$\Delta t = \frac{\Delta \omega}{\alpha} = \frac{\omega_{\rm f} - \omega_{\rm i}}{\alpha}$$

 $=\frac{209 \text{ rad/s} - 0 \text{ rad/s}}{37.5 \text{ rad/s}^2} = 5.6 \text{ s}$



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Example 7.18 Starting an airplane engine (cont.)

ASSESS We've assumed a constant angular acceleration, which is reasonable for the first few seconds while the propeller is still turning slowly. Eventually, air resistance and friction will cause opposing torques and the angular acceleration will decrease. At full speed, the negative torque due to air resistance



and friction cancels the torque of the engine. Then $\tau_{\text{net}} = 0$ and the propeller turns at constant angular velocity with no angular acceleration.

Constraints Due to Ropes and Pulleys

- If the pulley turns *without the rope slipping on it* then the rope's speed must exactly match the speed of the rim of the pulley.
- The attached object must have the same speed and acceleration as the rope.



Motion constraints for an object connected to a pulley of radius *R* by a nonslipping rope

 $v_{\rm obi} = \omega R$

 $a_{\rm obi} = \alpha R$



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Rolling Motion



- The figure above shows exactly one revolution for a wheel or sphere that rolls forward *without slipping*.
- The overall position is measured at the object's center.

Rolling Motion



• In one revolution, the center moves forward by exactly one circumference ($\Delta x = 2\pi R$).

$$v = \frac{\Delta x}{T} = \frac{2\pi R}{T}$$

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Rolling Motion



• This is the **rolling constraint**, the basic link between translation and rotation for objects that roll without slipping.

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Rolling Motion



- The point at the bottom of the wheel has a translational velocity and a rotational velocity in opposite directions, which cancel each other.
- The point on the bottom of a rolling object is instantaneously at rest.
- This is the idea behind "rolling without slipping."

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Example 7.20 Rotating your tires

The diameter of your tires is 0.60 m. You take a 60 mile trip at a speed of 45 mph.

- a. During this trip, what was your tires' angular speed?
- b. How many times did they revolve?

Example 7.20 Rotating your tires (cont.)

PREPARE The angular speed is related to the speed of a wheel's center by Equation 7.25: $v = \omega R$. Because the center of the wheel turns on an axle fixed to the car, the speed *v* of the wheel's center is the same as that of the car. We prepare by converting the car's speed to SI units:

$$v = (45 \text{ mph}) \times \left(0.447 \frac{\text{m/s}}{\text{mph}}\right) = 20 \text{ m/s}$$

Once we know the angular speed, we can find the number of times the tires turned from the rotational-kinematic equation $\Delta \theta = \omega \Delta t$. We'll need to find the time traveled Δt from $v = \Delta x / \Delta t$.

Example 7.20 Rotating your tires (cont.)

SOLVE a. From Equation 7.25 we have

$$\omega = \frac{v}{R} = \frac{20 \text{ m/s}}{0.30 \text{ m}} = 67 \text{ rad/s}$$

b. The time of the trip is

$$\Delta t = \frac{\Delta x}{\nu} = \frac{60 \text{ mi}}{45 \text{ mi/h}} = 1.33 \text{ h} \times \frac{3600 \text{ s}}{1 \text{ h}} = 4800 \text{ s}$$

Example 7.20 Rotating your tires (cont.)

Thus the total angle through which the tires turn is

$$\Delta\theta = \omega \ \Delta t = (67 \text{ rad/s})(4800 \text{ s}) = 3.2 \times 10^5 \text{ rad}$$

Because each turn of the wheel is 2π rad, the number of turns is

 $\frac{3.2\times10^5\,\mathrm{rad}}{2\pi\,\mathrm{rad}}=51,000\,\mathrm{turns}$

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Summary

GENERAL PRINCIPLES

Newton's Second Law for Rotational Motion

If a net torque τ_{net} acts on an object, the object will experience an angular acceleration given by $\alpha = \tau_{net}/I$, where *I* is the object's moment of inertia about the rotation axis.

This law is analogous to Newton's second law for linear motion, $\vec{a} = \vec{F}_{net}/m$.

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Summary

APPLICATIONS		
Moments of inertia of common shapes	Rotation about a fixed axis	Rolling motion
$MR^{2} \qquad \qquad$	When a net torque is applied to an object that rotates about a fixed axis, the object will undergo an angular acceleration given by $\alpha = \frac{\tau_{\text{thet}}}{I}$ If a rope unwinds from a pulley of radius <i>R</i> , the linear motion of an object tied to the rope is related to the angular motion of the pulley by $a_{\text{obj}} = \alpha R \qquad v_{\text{obj}} = \omega R$	For an object that rolls without slipping, $\nu = \omega R$ The velocity of a point at the top of the object is twice that of the center.
		Text: p. 217
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