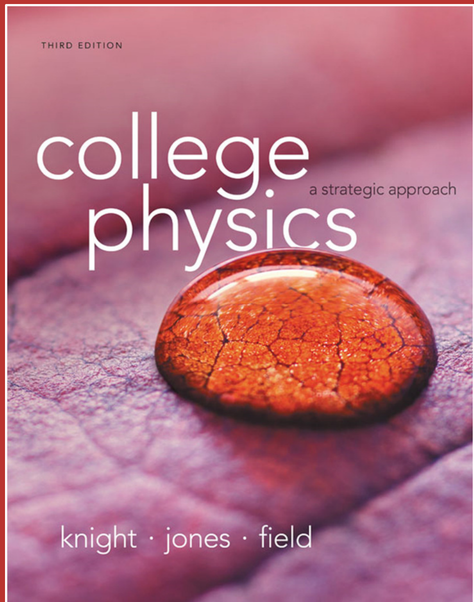


Lecture Presentation

Chapter 6

Circular Motion, Orbits, and Gravity

Chapter 6 Preview
Looking Ahead

Circular Motion

An object moving in a circle has an acceleration toward the center, so there must be a net force toward the center as well.

How much force does it take to swing the girl in a circle? You'll learn how to solve such problems.

Apparent Forces

The riders feel pushed out. This isn't a real force, though it is often called centrifugal force; it's an **apparent force.**

This apparent force makes the riders "feel heavy." You'll learn to calculate their apparent weight.

Gravity and Orbits

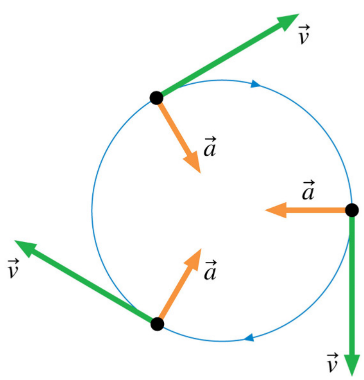
The space station appears to float in space, but gravity is pulling down on it quite forcefully.

You'll learn **Newton's law of gravity**, and you'll see how the force of gravity keeps the station in orbit.

Text: p. 160

Chapter 6 Preview
Looking Back: Centripetal Acceleration

- In Section 3.8, you learned that an object moving in a circle at a constant speed experiences an acceleration directed toward the center of the circle.



- In this chapter, you'll learn how to extend Newton's second law, which relates acceleration to the forces that cause it, to this type of acceleration.

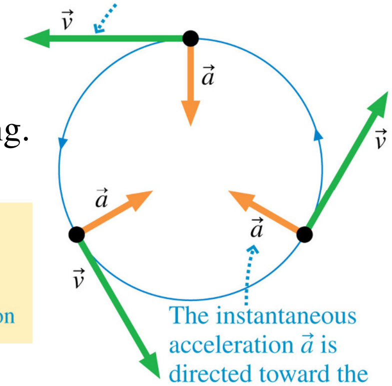
Section 6.1 Uniform Circular Motion

Velocity and Acceleration in Uniform Circular Motion

- Although the *speed* of a particle in uniform circular motion is constant, its *velocity* is not constant because the *direction* of the motion is always changing.

$$a = \frac{v^2}{r}$$

Centripetal acceleration for uniform circular motion



QuickCheck 6.2

A ball at the end of a string is being swung in a horizontal circle. The ball is accelerating because

A. The speed is changing.
B. The direction is changing.
C. The speed and the direction are changing.
D. The ball is not accelerating.

QuickCheck 6.2

A ball at the end of a string is being swung in a horizontal circle. The ball is accelerating because

A. The speed is changing.
✔ B. The direction is changing.
C. The speed and the direction are changing.
D. The ball is not accelerating.

QuickCheck 6.3

A ball at the end of a string is being swung in a horizontal circle. What is the direction of the acceleration of the ball?

A. Tangent to the circle, in the direction of the ball's motion
B. Toward the center of the circle

QuickCheck 6.3

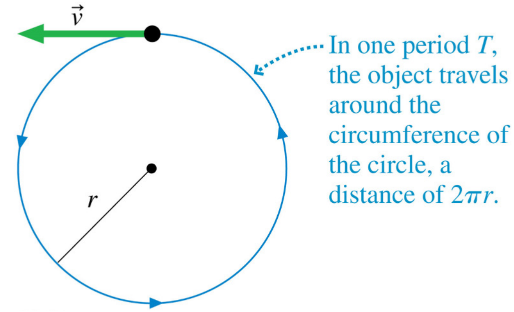
A ball at the end of a string is being swung in a horizontal circle. What is the direction of the acceleration of the ball?

A. Tangent to the circle, in the direction of the ball's motion

✔ B. Toward the center of the circle

Period, Frequency, and Speed

- The time interval it takes an object to go around a circle one time is called the **period** of the motion.



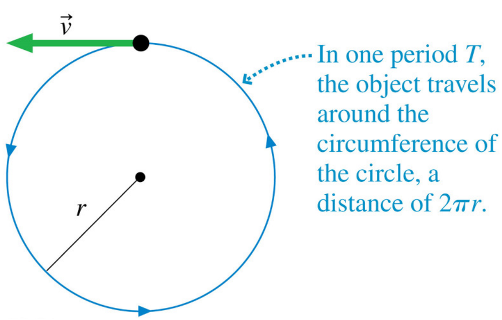
- We can specify circular motion by its frequency, the number of revolutions per second:

$$f = \frac{1}{T}$$

- The SI unit of frequency is inverse seconds, or s^{-1}.

Period, Frequency, and Speed

$$v = \frac{2\pi r}{T}$$



- Given Equation 6.2 relating frequency and period, we can also write this equation as

$$v = 2\pi f r$$

- We can combine this with the expression for centripetal acceleration:

$$a = \frac{v^2}{r} = (2\pi f)^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

Example Problem

A hard drive disk rotates at 7200 rpm. The disk has a diameter of 5.1 in (13 cm). What is the speed of a point 6.0 cm from the center axle? What is the acceleration of this point on the disk?

Section 6.2 Dynamics of Uniform Circular Motion

Dynamics of Uniform Circular Motion

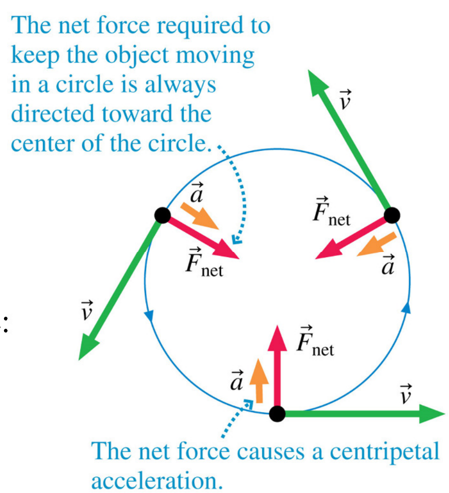
- Riders traveling around on a circular carnival ride are accelerating, as we have seen:

$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$

Net force producing the centripetal acceleration of uniform circular motion

Dynamics of Uniform Circular Motion

- **A particle of mass *m* moving at constant speed *v* around a circle of radius *r* must always have a net force of magnitude mv^2/r pointing toward the center of the circle.**
- This is **not** a new kind of force: The net force is due to one or more of our familiar forces such as tension, friction, or the normal force.



QuickCheck 6.4

A ball at the end of a string is being swung in a horizontal circle. What force is producing the centripetal acceleration of the ball?

A. Gravity
B. Air resistance
C. Normal force
D. Tension in the string

QuickCheck 6.4

A ball at the end of a string is being swung in a horizontal circle. What force is producing the centripetal acceleration of the ball?

A. Gravity
B. Air resistance
C. Normal force
✔ D. Tension in the string

QuickCheck 6.5

A ball at the end of a string is being swung in a horizontal circle. What is the direction of the net force on the ball?

A. Tangent to the circle
B. Toward the center of the circle
C. There is no net force.

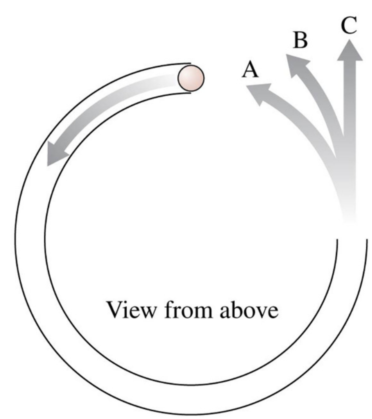
QuickCheck 6.5

A ball at the end of a string is being swung in a horizontal circle. What is the direction of the net force on the ball?

A. Tangent to the circle
✔ B. Toward the center of the circle
C. There is no net force.

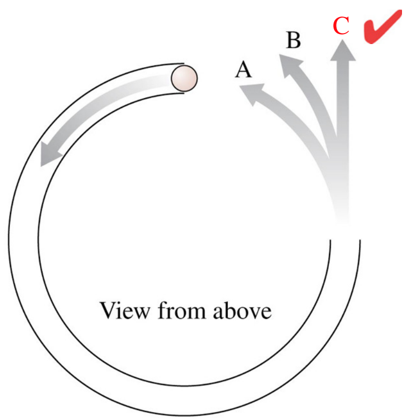
QuickCheck 6.1

A hollow tube lies flat on a table. A ball is shot through the tube. As the ball emerges from the other end, which path does it follow?



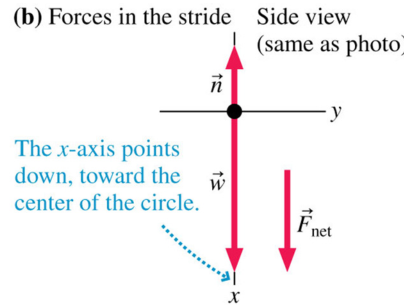
QuickCheck 6.1

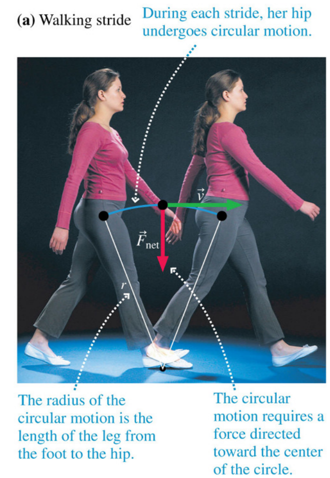
A hollow tube lies flat on a table. A ball is shot through the tube. As the ball emerges from the other end, which path does it follow?



Maximum Walking Speed

In a walking gait, your body is in circular motion as you pivot on your forward foot.





Maximum Walking Speed

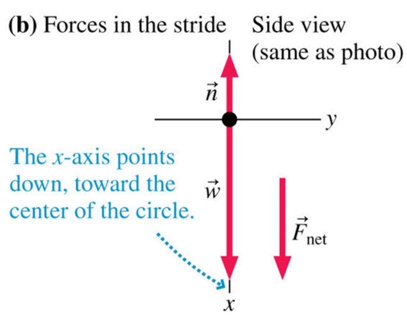
Newton's second law for the x-axis is

$$\sum F_x = w - n = \frac{mv^2}{r}$$

Setting $n = 0$ in Newton's second law gives

$$w = mg = \frac{m v_{max}^2}{r}$$

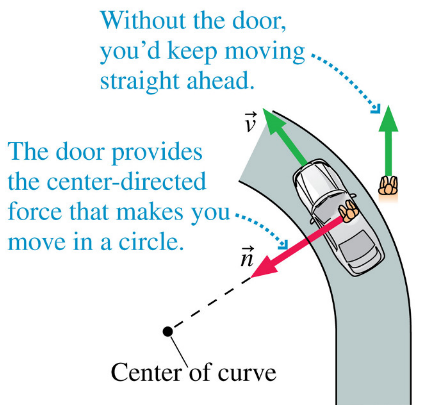
$$v_{max} = \sqrt{gr}$$



Section 6.3 Apparent Forces in Circular Motion

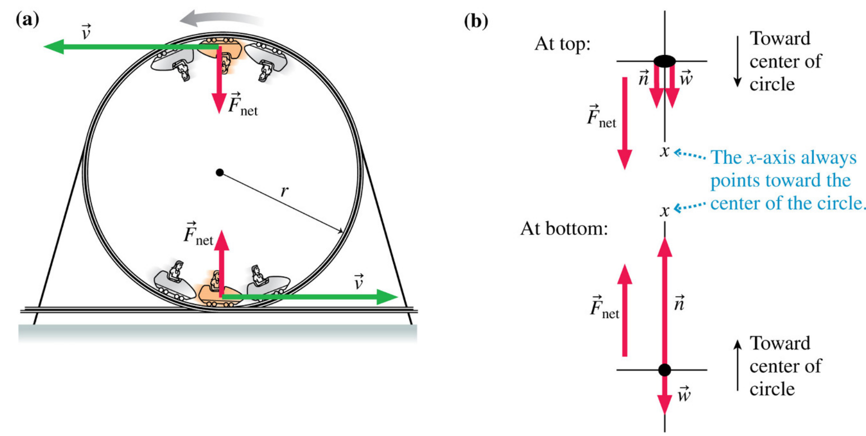
Centrifugal Force?

- If you are a passenger in a car that turns a corner quickly, it is the force of the car door, pushing inward toward the center of the curve, that causes you to turn the corner.
- What you feel is your body trying to move ahead in a straight line as outside forces (the door) act to turn you in a circle.



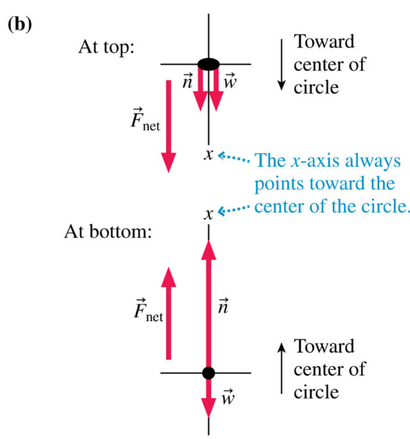
A centrifugal force will never appear on a free-body diagram and never be included in Newton's laws.

Apparent Weight in Circular Motion



Apparent Weight in Circular Motion

- The force you feel, your apparent weight, is the magnitude of the contact force that supports you.
- When the passenger on the roller coaster is at the bottom of the loop:
  - The net force points upward, so $n > w$.
  - Her apparent weight is $w_{app} = n$, so her apparent weight is greater than her true weight.



Apparent Weight in Circular Motion

- Newton's second law for the passenger at the *bottom* of the circle is

$$\sum F_x = n_x + w_x = n - w = \frac{mv^2}{r}$$

- From this equation, the passenger's apparent weight is

$$w_{app} = n = w + \frac{mv^2}{r}$$

- Her apparent weight at the bottom is *greater* than her true weight, w.

Apparent Weight in Circular Motion

- Newton's second law for the passenger at the *top* of the circle is

$$\sum F_x = n_x + w_x = n + w = \frac{mv^2}{r}$$

- Note that w_x is now *positive* because the *x*-axis is directed downward. We can solve for the passenger's apparent weight:

$$w_{app} = n = \frac{mv^2}{r} - w$$

- If v is sufficiently large, her apparent weight can exceed the true weight.

Apparent Weight in Circular Motion

- As the car goes slower there comes a point where n becomes zero:

$$w_{app} = n = \frac{mv^2}{r} - w$$

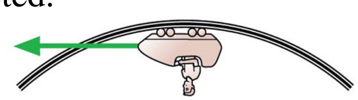
- The speed for which $n = 0$ is called the *critical speed* v_c. Because for n to be zero we must have $mv_c^2/r = w$, the critical speed is

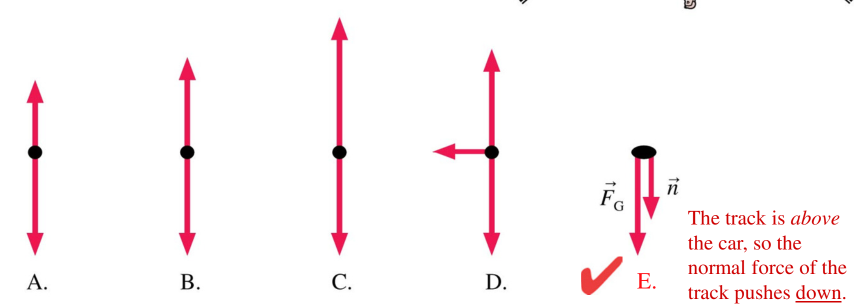
$$v_c = \sqrt{\frac{rw}{m}} = \sqrt{\frac{rmg}{m}} = \sqrt{gr}$$

- **The critical speed is the slowest speed at which the car can complete the circle.**

QuickCheck 6.12

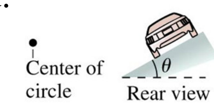
A roller coaster car does a loop-the-loop. Which of the free-body diagrams shows the forces on the car at the top of the loop? Rolling friction can be neglected.

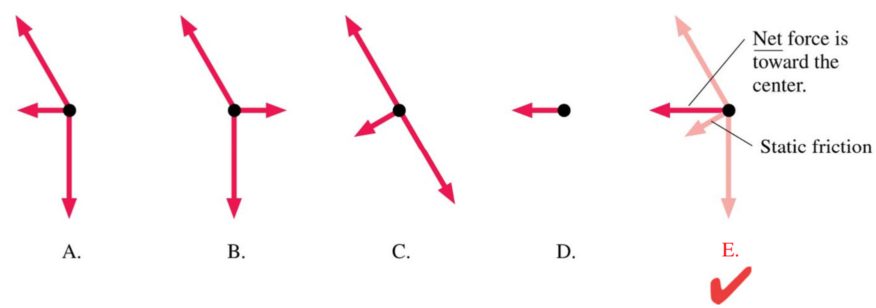




QuickCheck 6.15

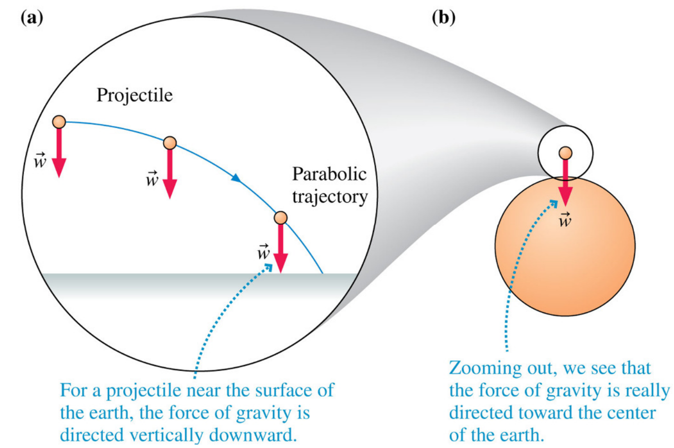
A car turns a corner on a banked road. Which of the diagrams could be the car's free-body diagram?





Section 6.4 Circular Orbits and Weightlessness

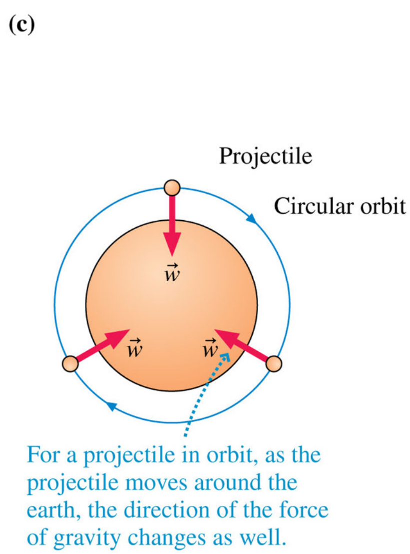
Orbital Motion



The force of gravity on a projectile is directed toward the center of the earth.

Orbital Motion

- If the launch speed of a projectile is sufficiently large, there comes a point at which the curve of the trajectory and the curve of the earth are parallel.
- Such a *closed trajectory* is called an **orbit**.
- **An orbiting projectile is in free fall.**



Orbital Motion

- The force of gravity is the force that causes the centripetal acceleration of an orbiting object:

$$a = \frac{F_{\text{net}}}{m} = \frac{w}{m} = \frac{mg}{m} = g$$

- An object moving in a circle of radius r at speed v_{orbit} will have this centripetal acceleration if

$$a = \frac{(v_{\text{orbit}})^2}{r} = g$$

- That is, if an object moves parallel to the surface with the speed

$$v_{\text{orbit}} = \sqrt{gr}$$

Orbital Motion

- The orbital speed of a projectile just skimming the surface of a smooth, airless earth is

$$v_{\text{orbit}} = \sqrt{gR_e} = \sqrt{(9.80\ \text{m/s}^2)(6.37 \times 10^6\ \text{m})}$$
$$= 7900\ \text{m/s} \approx 18{,}000\ \text{mph}$$

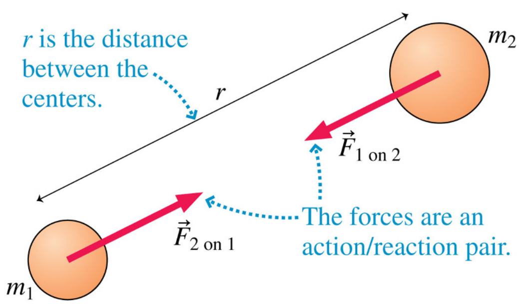
- We can use v_{orbit} to calculate the period of the satellite's orbit:

$$T = \frac{2\pi r}{v_{\text{orbit}}} = 2\pi\sqrt{\frac{r}{g}}$$

Section 6.5 Newton's Law of Gravity

Gravity Obeys an Inverse-Square Law

- Gravity is a universal force that affects all objects in the universe.
- Newton proposed that the force of gravity has the following properties:



1. The force is inversely proportional to the square of the distance between the objects.
2. The force is directly proportional to the product of the masses of the two objects.

Gravity Obeys an Inverse-Square Law

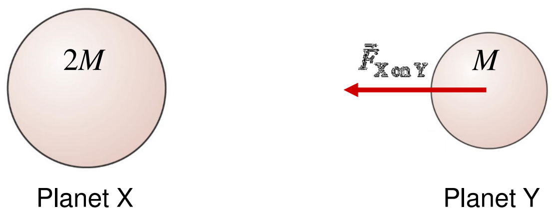
$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1 m_2}{r^2}$$

- Newton's law of gravity is an inverse-square law.
- Doubling the distance between two masses causes the force between them to decrease by a factor of 4.

QuickCheck 6.16

The force of Planet Y on Planet X is ___ the magnitude of $\vec{F}_{\text{X on Y}}$.

A. One quarter
B. One half
C. The same as
D. Twice
E. Four times



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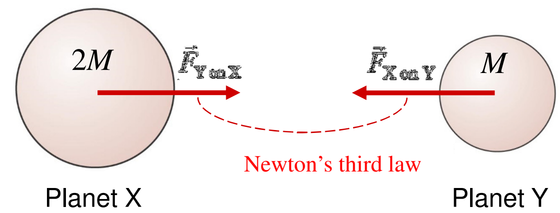
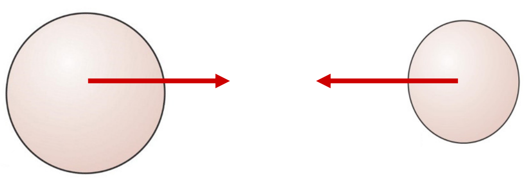


QuickCheck 6.17

The gravitational force between two asteroids is 1,000,000 N. What will the force be if the distance between the asteroids is doubled?

A. 250,000 N
B. 500,000 N
C. 1,000,000 N
D. 2,000,000 N
E. 4,000,000 N



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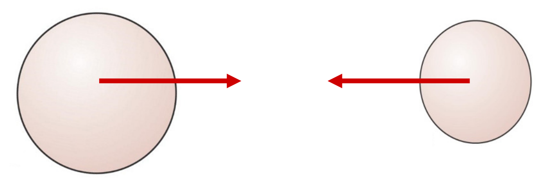


Gravity on Other Worlds

- If you traveled to another planet, your *mass* would be the same but your *weight* would vary. The weight of a mass m on the moon is given by

$$w = mg_{\text{moon}}$$

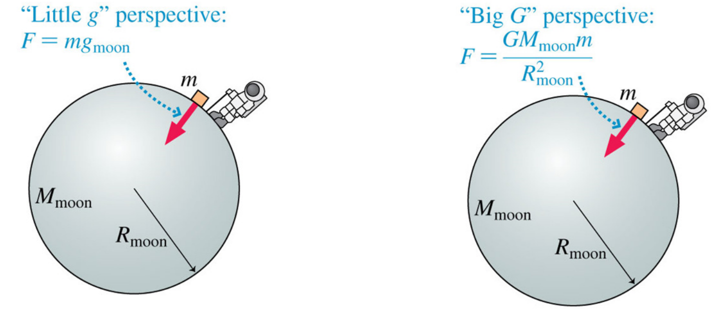
- Using Newton's law of gravity (Eq. (6.15)) the weight is given by

$$F_{\text{moon on } m} = \frac{GM_{\text{moon}}m}{R_{\text{moon}}^2}$$

- Since these are two expressions for the same force, they are equal and

$$\boxed{g_{\text{planet}} = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2}}$$

Gravity on Other Worlds



- If we use values for the mass and the radius of the moon, we compute $g_{\text{moon}} = 1.62 \text{ m/s}^2$.
- A 70-kg astronaut wearing an 80-kg spacesuit would weigh more than 330 lb on the earth but only 54 lb on the moon.

QuickCheck 6.18

Planet X has free-fall acceleration 8 m/s^2 at the surface. Planet Y has twice the mass and twice the radius of planet X. On Planet Y

A. $g = 2 \text{ m/s}^2$
B. $g = 4 \text{ m/s}^2$
C. $g = 8 \text{ m/s}^2$
D. $g = 16 \text{ m/s}^2$
E. $g = 32 \text{ m/s}^2$

QuickCheck 6.18

Planet X has free-fall acceleration 8 m/s^2 at the surface. Planet Y has twice the mass and twice the radius of planet X. On Planet Y

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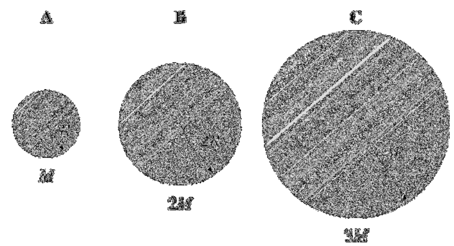
✔ B. $g = 4 \text{ m/s}^2$
C. $g = 8 \text{ m/s}^2$
D. $g = 16 \text{ m/s}^2$
E. $g = 32 \text{ m/s}^2$

QuickCheck 6.22

A 60-kg person stands on each of the following planets. On which planet is his or her weight the greatest?



QuickCheck 6.22

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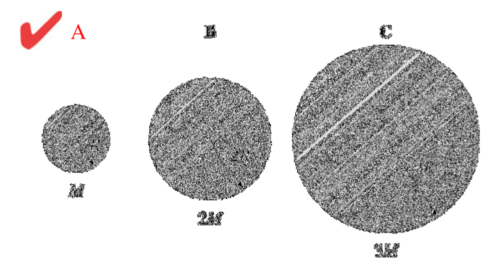
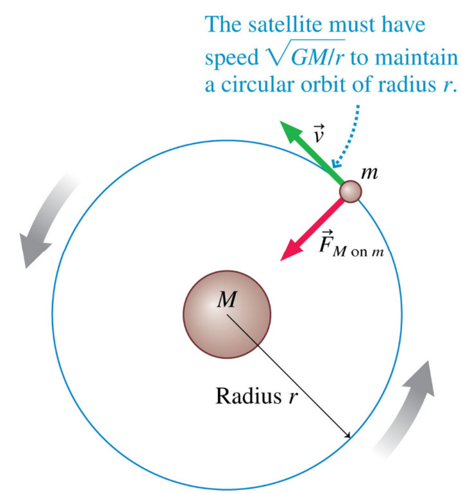


Section 6.6 Gravity and Orbits

Gravity and Orbits

- Newton's second law tells us that $F_{M \text{ on } m} = ma$, where $F_{M \text{ on } m}$ is the gravitational force of the large body on the satellite and a is the satellite's acceleration.
- Because it's moving in a circular orbit, Newton's second law gives

$$F_{M \text{ on } m} = \frac{GMm}{r^2} = ma = \frac{mv^2}{r}$$

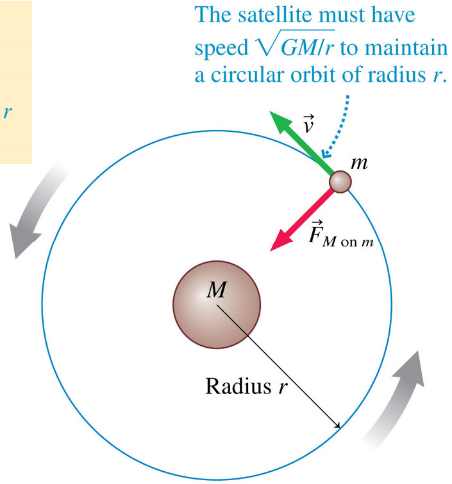


Gravity and Orbits

$$v = \sqrt{\frac{GM}{r}}$$

Speed of a satellite in a circular orbit of radius r about a star or planet of mass M

- A satellite must have this specific speed in order to maintain a circular orbit of radius r about the larger mass M.



Gravity and Orbits

- For a planet orbiting the sun, the period T is the time to complete one full orbit. The relationship among speed, radius, and period is the same as for any circular motion:

$$v = 2\pi r/T$$

- Combining this with the value of v for a circular orbit from Equation 6.21 gives

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

- If we square both sides and rearrange, we find the period of a satellite:

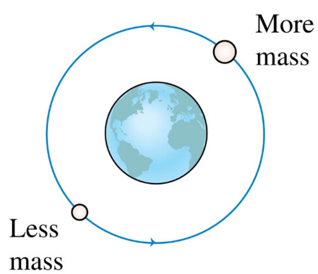
$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Relationship between the orbital period T and radius r for a satellite in a circular orbit around an object of mass M

QuickCheck 6.20

Two satellites have circular orbits with the same radius. Which has a higher speed?

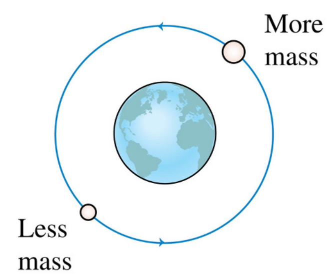
A. The one with more mass.
B. The one with less mass.
C. They have the same speed.



QuickCheck 6.20

Two satellites have circular orbits with the same radius. Which has a higher speed?

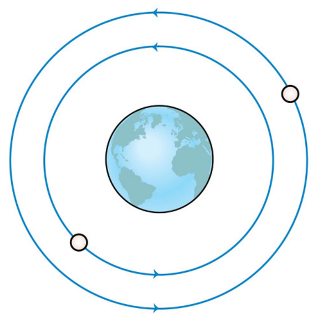
A. The one with more mass.
B. The one with less mass.
✔ C. They have the same speed.



QuickCheck 6.21

Two identical satellites have different circular orbits. Which has a higher speed?

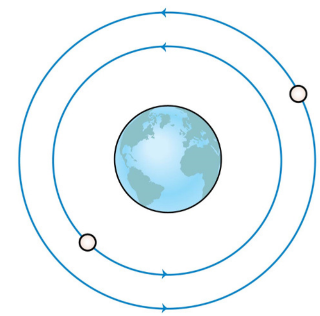
A. The one in the larger orbit
B. The one in the smaller orbit
C. They have the same speed.



QuickCheck 6.21

Two identical satellites have different circular orbits. Which has a higher speed?

A. The one in the larger orbit
✔ B. The one in the smaller orbit
C. They have the same speed.



QuickCheck 6.23

A satellite orbits the earth. A Space Shuttle crew is sent to boost the satellite into a higher orbit. Which of these quantities increases?

A. Speed
B. Angular speed
C. Period
D. Centripetal acceleration
E. Gravitational force of the earth

QuickCheck 6.23

A satellite orbits the earth. A Space Shuttle crew is sent to boost the satellite into a higher orbit. Which of these quantities increases?

A. Speed
B. Angular speed
✔ C. Period
D. Centripetal acceleration
E. Gravitational force of the earth

Example 6.15 Locating a geostationary satellite

Communication satellites appear to "hover" over one point on the earth's equator. A satellite that appears to remain stationary as the earth rotates is said to be in a *geostationary orbit*. What is the radius of the orbit of such a satellite?

PREPARE For the satellite to remain stationary with respect to the earth, the satellite's orbital period must be 24 hours; in seconds this is $T = 8.64 \times 10^4$ s.

Example 6.15 Locating a geostationary satellite (cont.)

SOLVE We solve for the radius of the orbit by rearranging Equation 6.22. The mass at the center of the orbit is the earth:

$$r = \left(\frac{GM_e T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$= \left(\frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(8.64 \times 10^4\ \text{s})^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$= 4.22 \times 10^7\ \text{m}$$

Gravity on a Grand Scale

- No matter how far apart two objects may be, there is a gravitational attraction between them.

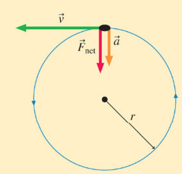
- Galaxies are held together by gravity.
- All of the stars in a galaxy are different distances from the galaxy's center, and so orbit with different periods.

Summary

GENERAL PRINCIPLES

Uniform Circular Motion

An object moving in a circular path is in uniform circular motion if v is constant.



- The speed is constant, but the direction of motion is constantly changing.
- The **centripetal acceleration** is directed toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

- This acceleration requires a net force directed toward the center of the circle. Newton's second law for circular motion is

$$\vec{F}_{net} = m\vec{a} = \left(\frac{mv^2}{r}, \text{toward center of circle}\right)$$

Universal Gravitation

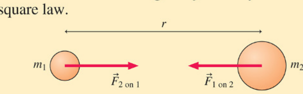
Two objects with masses m_1 and m_2 that are distance r apart exert attractive gravitational forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

where the gravitational constant is

$$G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$$

This is **Newton's law of gravity.** Gravity is an inverse-square law.



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Summary

IMPORTANT CONCEPTS

Describing circular motion

For an object moving in a circle of radius r at a constant speed v:

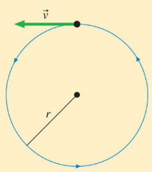
- The **period** T is the time to go once around the circle.

  T = time for one revolution

- The **frequency** f is defined as the number of revolutions per second. It is defined in terms of the period:

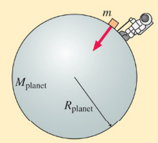
  $$f = \frac{1}{T}$$

- The frequency and period are related to the speed and the radius: $v = 2\pi f r = \frac{2\pi r}{T}$



Planetary gravity

The gravitational attraction between a planet and a mass on the surface depends on the two masses and the distance to the center of the planet.



$$F_{\text{planet on } m} = \frac{GM_{planet}m}{R_{planet}^2}$$

We can use this to define a value of the free-fall acceleration at the surface of a planet: $g_{planet} = \frac{GM_{planet}}{R_{planet}^2}$

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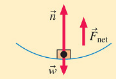
Summary

APPLICATIONS

Apparent weight and weightlessness

Circular motion requires a net force pointing to the center. The apparent weight $w_{app} = n$ is usually not the same as the true weight w. n must be > 0 for the object to be in contact with a surface.



In orbital motion, the net force is provided by gravity. An astronaut and his spacecraft are both in free fall, so he feels weightless.

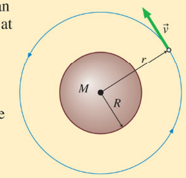
Orbital motion

A **satellite** in a circular orbit of radius r around an object of mass M moves at a speed v given by

$$v = \sqrt{\frac{GM}{r}}$$

The period and radius are related as follows:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$



The speed of a satellite in a low orbit is

$$v = \sqrt{gr}$$

The orbital period is

$$T = 2\pi\sqrt{\frac{r}{g}}$$

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