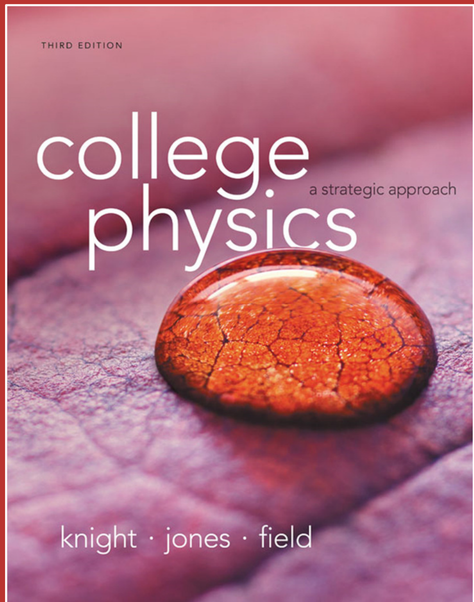

Lecture Presentation

Chapter 2

Motion in One Dimension

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Chapter 2 Motion in One Dimension

Chapter Goal: To describe and analyze linear motion.

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Chapter 2 Preview
Looking Ahead

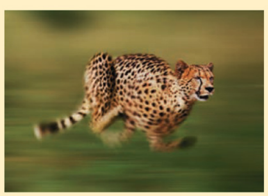
Uniform Motion

Successive images of the rider are the same distance apart, so the velocity is constant. This is **uniform motion.**

You'll learn to describe motion in terms of quantities such as distance and velocity, an important first step in analyzing motion.

Acceleration

A cheetah is capable of very high speeds but, more importantly, it is capable of a rapid *change* in speed—a large **acceleration.**



You'll use the concept of acceleration to solve problems of changing velocity, such as races, or predators chasing prey.

Free Fall

When you toss a coin, the motion—both going up and coming down—is determined by gravity alone. We call this **free fall.**

How long does it take the coin to go up and come back down? This is the type of free-fall problem you'll learn to solve.

Text: p. 28

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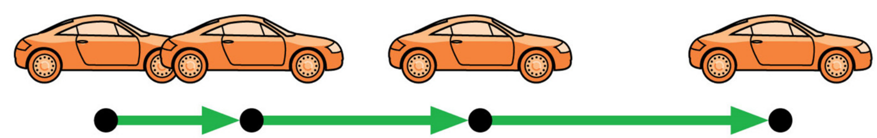
Chapter 2 Preview
Looking Back: Motion Diagrams

- As you saw in Section 1.5, a good first step in analyzing motion is to draw a motion diagram, marking the position of an object in subsequent times.



- In this chapter, you'll learn to create motion diagrams for different types of motion along a line. Drawing pictures like this is a good staring point for solving problems.

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Reading Question 2.1

The slope at a point on a position-versus-time graph of an object is the

A. Object's speed at that point.
B. Object's average velocity at that point.
C. Object's instantaneous velocity at that point.
D. Object's acceleration at that point.
E. Distance traveled by the object to that point.

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Reading Question 2.1

The slope at a point on a position-versus-time graph of an object is the

A. Object's speed at that point.
B. Object's average velocity at that point.
✔ C. Object's instantaneous velocity at that point.
D. Object's acceleration at that point.
E. Distance traveled by the object to that point.

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Reading Question 2.2

Which of the following is an example of uniform motion?

A. A car going around a circular track at a constant speed.
B. A person at rest starts running in a straight line in a fixed direction.
C. A ball dropped from the top of a building.
D. A hockey puck sliding in a straight line at a constant speed.

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Reading Question 2.2

Which of the following is an example of uniform motion?

A. A car going around a circular track at a constant speed.
B. A person at rest starts running in a straight line in a fixed direction.
C. A ball dropped from the top of a building.
✔ D. A hockey puck sliding in a straight line at a constant speed.

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Reading Question 2.3

The area under a velocity-versus-time graph of an object is

A. The object's speed at that point.
B. The object's acceleration at that point.
C. The distance traveled by the object.
D. The displacement of the object.
E. This topic was not covered in this chapter.

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Reading Question 2.3

The area under a velocity-versus-time graph of an object is

A. The object's speed at that point.
B. The object's acceleration at that point.
C. The distance traveled by the object.
✔ D. The displacement of the object.
E. This topic was not covered in this chapter.

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Reading Question 2.4

If an object is speeding up,

A. Its acceleration is positive.
B. Its acceleration is negative.
C. Its acceleration can be positive or negative depending on the direction of motion.

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Reading Question 2.4

If an object is speeding up,

A. Its acceleration is positive.
B. Its acceleration is negative.
✔ C. Its acceleration can be positive or negative depending on the direction of motion.

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Reading Question 2.5

A 1-pound ball and a 100-pound ball are dropped from a height of 10 feet at the same time. In the absence of air resistance

A. The 1-pound ball wins the race.
B. The 100-pound ball wins the race.
C. The two balls end in a tie.
D. There's not enough information to determine which ball wins the race.

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Reading Question 2.5

A 1-pound ball and a 100-pound ball are dropped from a height of 10 feet at the same time. In the absence of air resistance

A. The 1-pound ball wins the race.
B. The 100-pound ball wins the race.
✔ C. The two balls end in a tie.
D. There's not enough information to determine which ball wins the race.

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Section 2.1 Describing Motion

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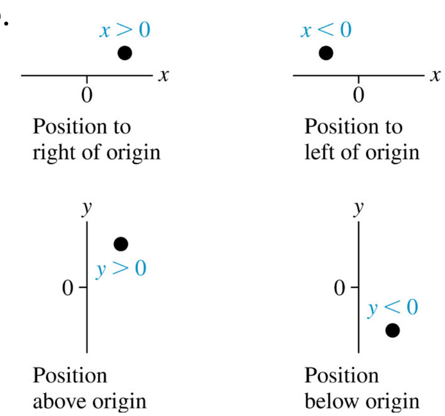
Representing Position

- We will use an ***x*-axis** to analyze horizontal motion and motion on a ramp, with the positive end to the right.
- We will use a ***y*-axis** to analyze vertical motion, with the positive end up.



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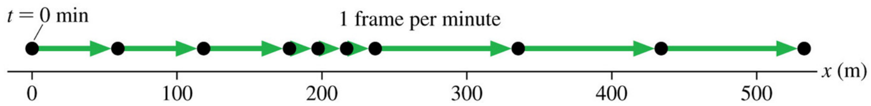
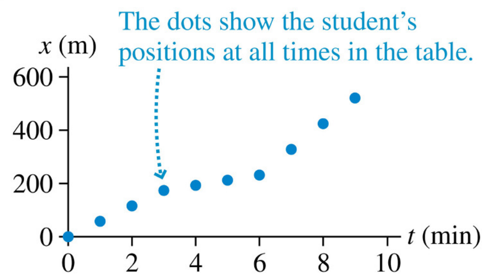
Representing Position



The motion diagram of a student walking to school and a coordinate axis for making measurements

- Every dot in the motion diagram of Figure 2.2 represents the student's position at a particular time.
- Figure 2.3 shows the student's motion shows the student's position as a **graph** of x versus t.



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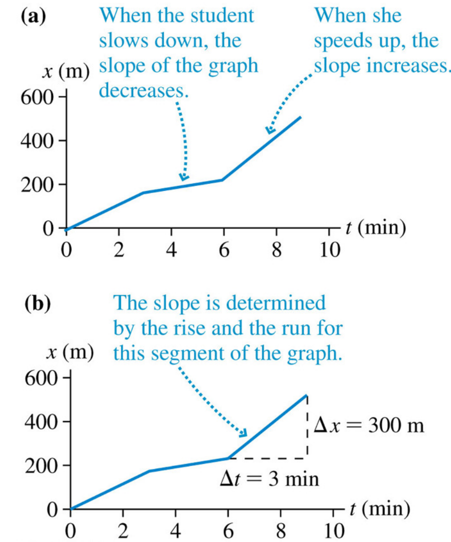
From Position to Velocity

- On a position-versus-time graph, **a faster speed corresponds to a steeper slope**.

$$\text{slope of graph} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$$

- **The slope of an object's position-versus-time graph is the object's velocity at that point in the motion.**



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From Position to Velocity

TACTICS BOX 2.1 Interpreting position-versus-time graphs

Information about motion can be obtained from position-versus-time graphs as follows:

1. Determine an object's *position* at time t by reading the graph at that instant of time.
2. Determine the object's *velocity* at time t by finding the slope of the position graph at that point. Steeper slopes correspond to faster speeds.
3. Determine the *direction of motion* by noting the sign of the slope. Positive slopes correspond to positive velocities and, hence, to motion to the right (or up). Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).

Exercises 2,3

Text: p. 31

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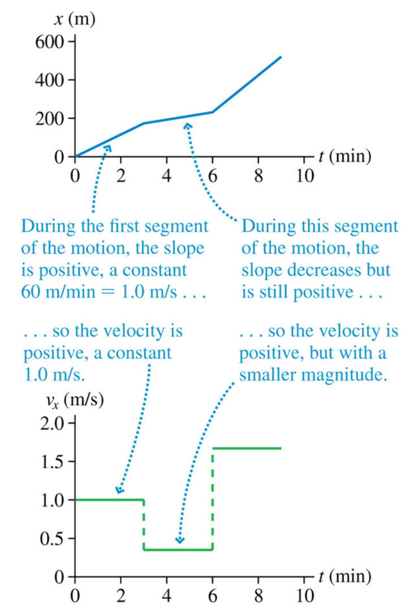
From Position to Velocity

- We can deduce the **velocity-versus-time graph** from the position-versus-time graph.
- The velocity-versus-time graph is yet another way to represent an object's motion.



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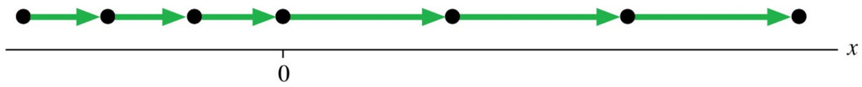
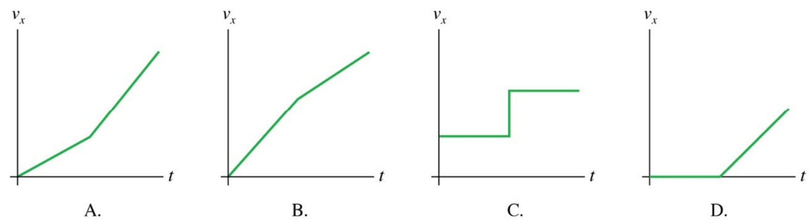
QuickCheck 2.2

Here is a motion diagram of a car moving along a straight road:



Which velocity-versus-time graph matches this motion diagram?



E. None of the above.

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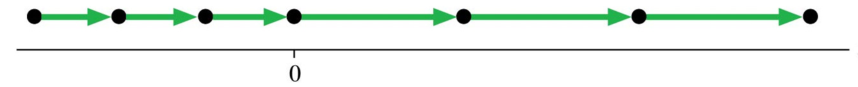
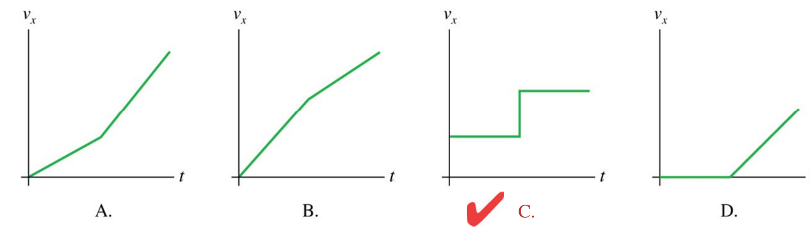
QuickCheck 2.2

Here is a motion diagram of a car moving along a straight road:



Which velocity-versus-time graph matches this motion diagram?



E. None of the above.

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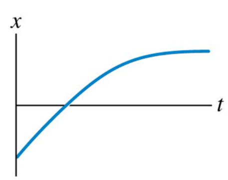
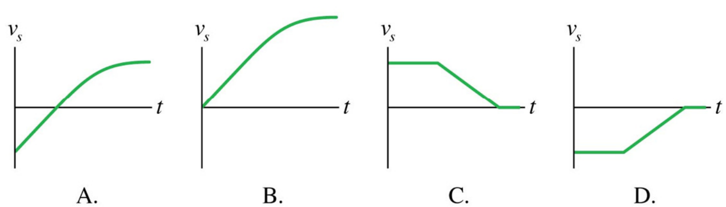
QuickCheck 2.7

Which velocity-versus-time graph goes with this position graph?





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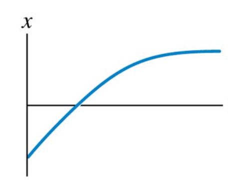
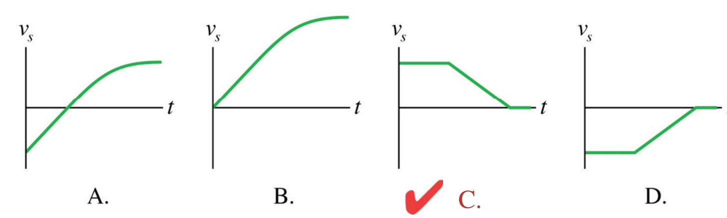
QuickCheck 2.7

Which velocity-versus-time graph goes with this position graph?





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Section 2.2 Uniform Motion

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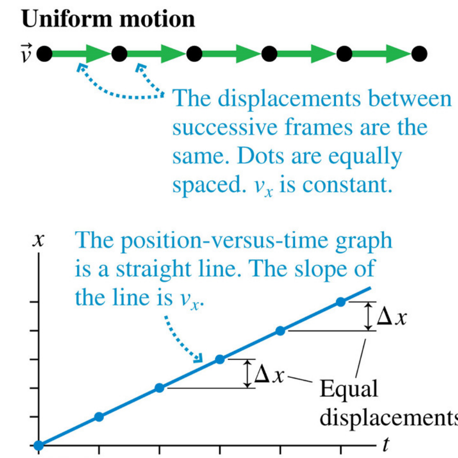
Uniform Motion

- Straight-line motion in which equal displacements occur during any successive equal-time intervals is called **uniform motion** or **constant-velocity motion.**
- **An object's motion is uniform if and only if its position-versus-time graph is a straight line.**



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Equations of Uniform Motion

- The velocity of an object in uniform motion tells us the amount by which its position changes during each second.

$$v_x = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$x_f = x_i + v_x\,\Delta t$$

Position equation for an object in uniform motion (v_x is constant)

$$\Delta x = v_x\,\Delta t$$

- The displacement Δx is proportional to the time interval Δt.

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Equations of Uniform Motion

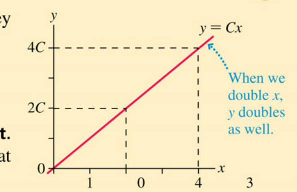
Proportional relationships

We say that y is **proportional** to x if they are related by an equation of this form:

$$y = Cx$$

y is proportional to x

We call C the **proportionality constant.** A graph of y versus x is a straight line that passes through the origin.



SCALING If x has the initial value x_1, then y has the initial value $y_1 = Cx_1$. Changing x from x_1 to x_2 changes y from y_1 to y_2 The ratio of y_2 to y_1 is

$$\frac{y_2}{y_1} = \frac{Cx_2}{Cx_1} = \frac{x_2}{x_1}$$

The ratio of y_2 to y_1 is exactly the same as the ratio of x_2 to x_1. If y is proportional to x, which is often written $y \propto x$, then x and y change by the same factor:

- If you double x, you double y.
- If you decrease x by a factor of 3, you decrease y by a factor of 3.

If two variables have a proportional relationship, we can draw important conclusions from ratios without knowing the value of the proportionality constant C. We can often solve problems in a very straightforward manner by looking at such ratios. This is an important skill called *ratio reasoning*.

Exercise 11

Text: p. 34

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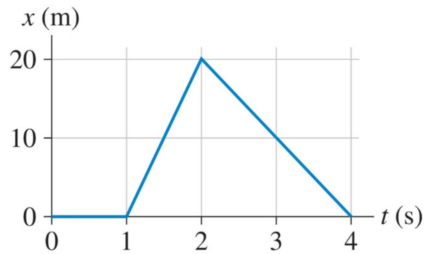
QuickCheck 2.8

Here is a position graph of an object:



At $t = 1.5$ s, the object's velocity is

A. 40 m/s
B. 20 m/s
C. 10 m/s
D. –10 m/s
E. None of the above

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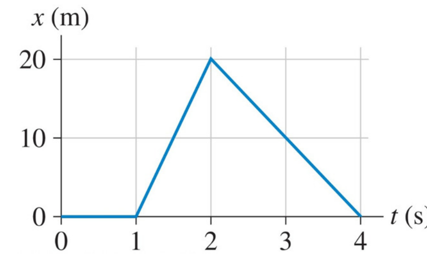
QuickCheck 2.8

Here is a position graph of an object:



At $t = 1.5$ s, the object's velocity is

A. 40 m/s
✔ B. 20 m/s
C. 10 m/s
D. –10 m/s
E. None of the above

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Example 2.3 If a train leaves Cleveland at 2:00…

A train is moving due west at a constant speed. A passenger notes that it takes 10 minutes to travel 12 km. How long will it take the train to travel 60 km?

PREPARE For an object in uniform motion, Equation 2.5 shows that the distance traveled Δx is proportional to the time interval Δt, so this is a good problem to solve using ratio reasoning.

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Example 2.3 If a train leaves Cleveland at 2:00…(cont.)

SOLVE We are comparing two cases: the time to travel 12 km and the time to travel 60 km. Because Δx is proportional to Δt, the ratio of the times will be equal to the ratio of the distances. The ratio of the distances is $\dfrac{\Delta x_2}{\Delta x_1} = \dfrac{60\text{ km}}{12\text{ km}} = 5$

This is equal to the ratio of the times:

$$\frac{\Delta t_2}{\Delta t_1} = 5$$

$$\Delta t_2 = \text{ time to travel 60 km} = 5\Delta t_1 = 5 \times (10\text{ min}) = 50\text{ min}$$

It takes 10 minutes to travel 12 km, so it will take 50 minutes—5 times as long—to travel 60 km.

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Example Problem

A soccer player is 15 m from her opponent's goal. She kicks the ball hard; after 0.50 s, it flies past a defender who stands 5 m away, and continues toward the goal. How much time does the goalie have to move into position to block the kick from the moment the ball leaves the kicker's foot?

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Section 2.3 Instantaneous Velocity

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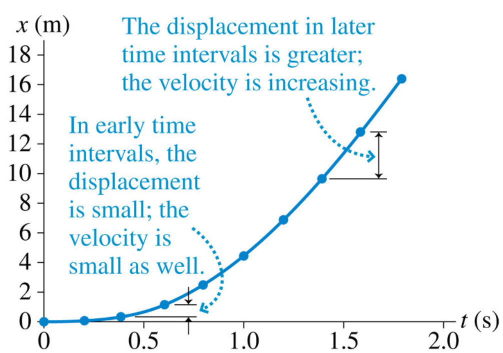
Instantaneous Velocity

- For one-dimensional motion, an object changing its velocity is either speeding up or slowing down.
- An object's velocity—a speed *and* a direction—at a specific *instant* of time *t* is called the object's **instantaneous velocity.**
- **From now on, the word "velocity" will always mean instantaneous velocity.**



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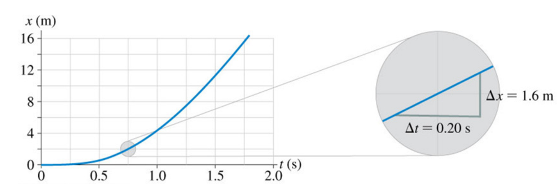
Finding the Instantaneous Velocity



- If the velocity changes, the position graph is a curved line. But we can compute a slope at a point by considering a small segment of the graph. Let's look at the motion in a very small time interval right around $t = 0.75$ s. This is highlighted with a circle, and we show a closeup in the next graph.

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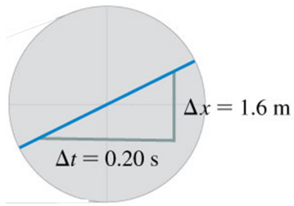
Finding the Instantaneous Velocity



- In this magnified segment of the position graph, the curve isn't apparent. It appears to be a line segment. We can find the slope by calculating the rise over the run, just as before:

$$v_x = (1.6 \text{ m})/(0.20 \text{ s}) = 8.0 \text{ m/s}$$

- This is the slope at $t = 0.75$ s and thus the velocity at this instant of time.

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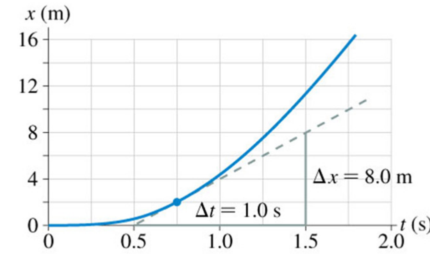
Finding the Instantaneous Velocity

- Graphically, the slope of the curve at a point is the same as the slope of a straight line drawn *tangent* to the curve at that point. Calculating rise over run for the tangent line, we get



$$v_x = (8.0 \text{ m})/(1.0 \text{ s}) = 8.0 \text{ m/s}$$

- This is the same value we obtained from the closeup view. The slope of the tangent line is the instantaneous velocity at that instant of time.

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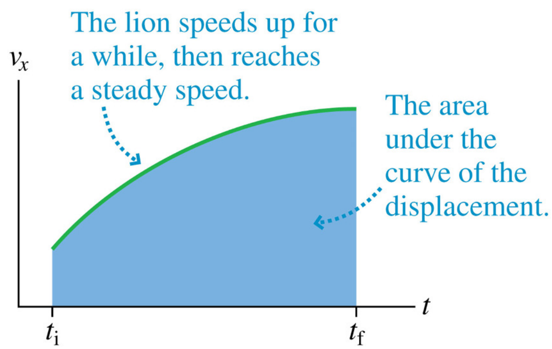
Instantaneous Velocity

- Even when the speed varies we can still use the velocity-versus-time graph to determine displacement.
- The area under the curve in a velocity-versus-time graph equals the displacement even for non-uniform motion.



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QuickCheck 2.5

The slope at a point on a position-versus-time graph of an object is

A. The object's speed at that point.
B. The object's velocity at that point.
C. The object's acceleration at that point.
D. The distance traveled by the object to that point.
E. I am not sure.

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QuickCheck 2.5

The slope at a point on a position-versus-time graph of an object is

A. The object's speed at that point.
B. The object's velocity at that point.
C. The object's acceleration at that point.
D. The distance traveled by the object to that point.
E. I am not sure.

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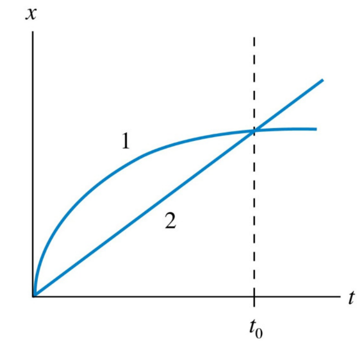
QuickCheck 2.9

When do objects 1 and 2 have the same velocity?

A. At some instant before time t_0
B. At time t_0
C. At some instant after time t_0
D. Both A and B
E. Never



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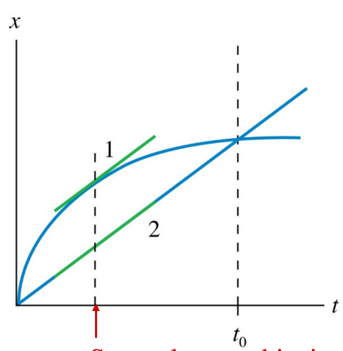
QuickCheck 2.9

When do objects 1 and 2 have the same velocity?

A. At some instant before time t_0
B. At time t_0
C. At some instant after time t_0
D. Both A and B
E. Never



Same slope at this time

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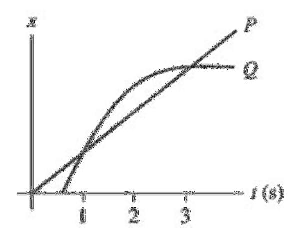
QuickCheck 2.10

Masses *P* and *Q* move with the position graphs shown. Do *P* and *Q* ever have the same velocity? If so, at what time or times?



A. *P* and *Q* have the same velocity at 2 s.
B. *P* and *Q* have the same velocity at 1 s and 3 s.
C. *P* and *Q* have the same velocity at 1 s, 2 s, and 3 s.
D. *P* and *Q* never have the same velocity.

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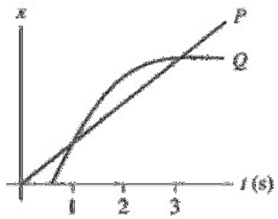
QuickCheck 2.10

Masses P and Q move with the position graphs shown. Do P and Q ever have the same velocity? If so, at what time or times?



✔ A. *P* and *Q* have the same velocity at 2 s.
B. *P* and *Q* have the same velocity at 1 s and 3 s.
C. *P* and *Q* have the same velocity at 1 s, 2 s, and 3 s.
D. *P* and *Q* never have the same velocity.

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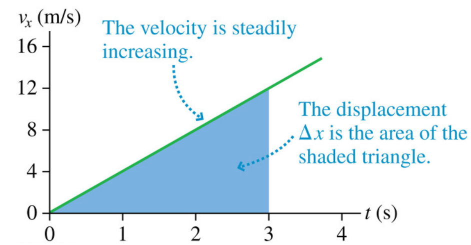
Example 2.5 The displacement during a rapid start

FIGURE 2.21 shows the velocity-versus-time graph of a car pulling away from a stop. How far does the car move during the first 3.0 s?

PREPARE Figure 2.21 is a graphical representation of the motion. The question How far? indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.7, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s.



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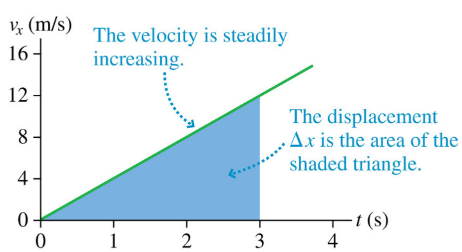
Example 2.5 The displacement during a rapid start (cont.)

SOLVE The curve in this case is an angled line, so the area is that of a triangle:

$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}$$
$$= \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The car moves 18 m during the first 3 seconds as its velocity changes from 0 to 12 m/s.



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Section 2.4 Acceleration

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Acceleration

- We define a new motion concept to describe an object whose velocity is changing.
  - The ratio of $\Delta v_x/\Delta t$ is the *rate of change of velocity.*
  - The ratio of $\Delta v_x/\Delta t$ is the *slope of a velocity-versus-time graph.*

$$a_x = \frac{\Delta v_x}{\Delta t}$$

Definition of acceleration as the rate of change of velocity

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Units of Acceleration

- In our SI unit of velocity, 60 mph = 27 m/s.
- The Corvette speeds up to 27 m/s in Δt = 3.6 s.

$$a_{\text{Corvette }x} = \frac{\Delta v_x}{\Delta t} = \frac{27 \text{ m/s}}{3.6 \text{ s}} = 7.5 \frac{\text{m/s}}{\text{s}}$$

- Every second, the Corvette's velocity changes by 7.5 m/s.
- It is customary to abbreviate the acceleration units (m/s)/s as m/s^2, which we say as "meters per second squared."

TABLE 2.2 Performance data for vehicles

Vehicle	Time to go from 0 to 60 mph
2011 Chevy Corvette	3.6 s
2012 Chevy Sonic	9.0 s

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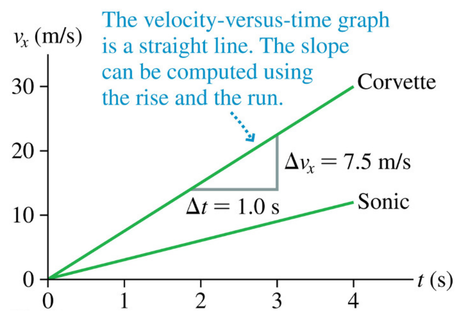
Representing Acceleration

TABLE 2.3 Velocity data for the Sonic and the Corvette

Time (s)	Velocity of Sonic (m/s)	Velocity of Corvette (m/s)
0	0	0
1	3.0	7.5
2	6.0	15.0
3	9.0	22.5
4	12.0	30.0



- **An object's acceleration is the slope of its velocity-versus-time graph.**

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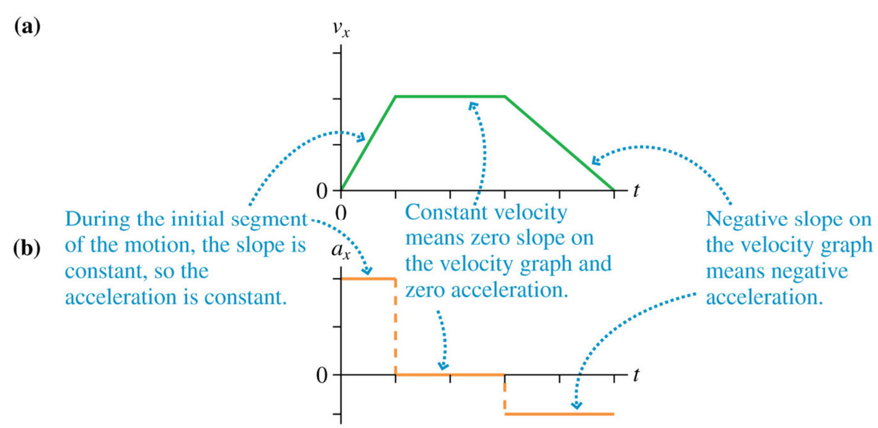
Representing Acceleration

- We can find an acceleration graph from a velocity graph.



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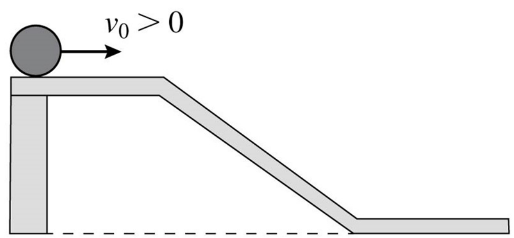
Example Problem

A ball moving to the right traverses the ramp shown below. Sketch a graph of the velocity versus time, and, directly below it, using the same scale for the time axis, sketch a graph of the acceleration versus time.



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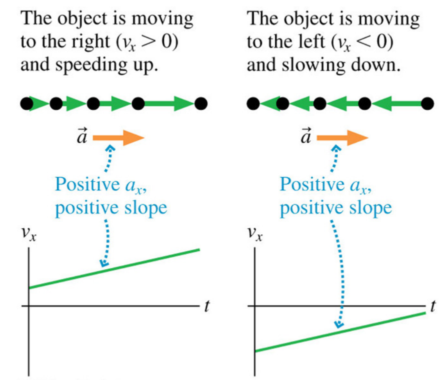
The Sign of the Acceleration

An object can move right or left (or up or down) while either speeding up or slowing down. Whether or not an object that is slowing down has a negative acceleration depends on the direction of motion.



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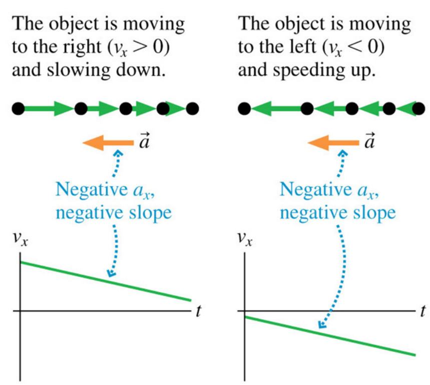
The Sign of the Acceleration (cont.)

An object can move right or left (or up or down) while either speeding up or slowing down. Whether or not an object that is slowing down has a negative acceleration depends on the direction of motion.



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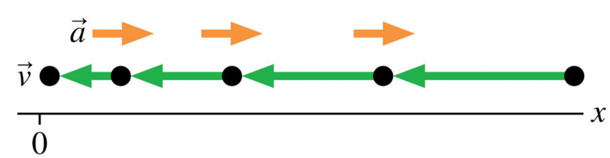
QuickCheck 2.15



The motion diagram shows a particle that is slowing down. The sign of the acceleration a_x is:

A. Acceleration is positive.
B. Acceleration is negative.

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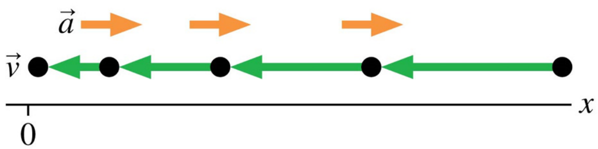
QuickCheck 2.15



The motion diagram shows a particle that is slowing down. The sign of the acceleration a_x is:

✔ A. Acceleration is positive.
B. Acceleration is negative.

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QuickCheck 2.18

Mike jumps out of a tree and lands on a trampoline. The trampoline sags 2 feet before launching Mike back into the air.

At the very bottom, where the sag is the greatest, Mike's acceleration is

A. Upward.
B. Downward.
C. Zero.

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QuickCheck 2.18

Mike jumps out of a tree and lands on a trampoline. The trampoline sags 2 feet before launching Mike back into the air.

At the very bottom, where the sag is the greatest, Mike's acceleration is

✔ A. Upward.
B. Downward.
C. Zero.

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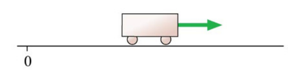
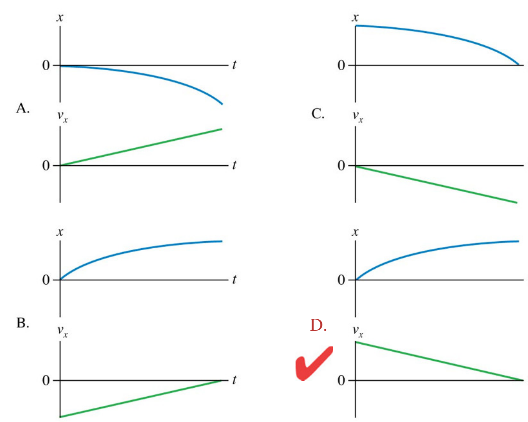
QuickCheck 2.19

A cart slows down while moving away from the origin. What do the position and velocity graphs look like?





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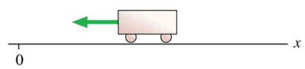
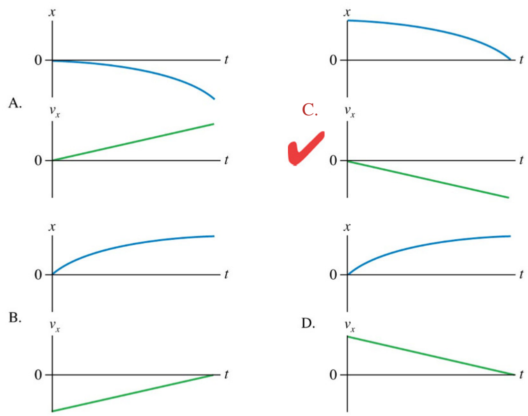
QuickCheck 2.20

A cart speeds up toward the origin. What do the position and velocity graphs look like?





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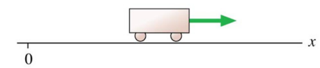
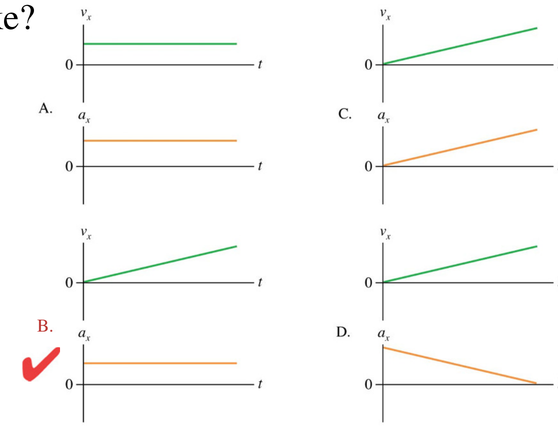
QuickCheck 2.21

A cart *speeds up* while moving away from the origin. What do the velocity and acceleration graphs look like?





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Section 2.5 Motion with Constant Acceleration

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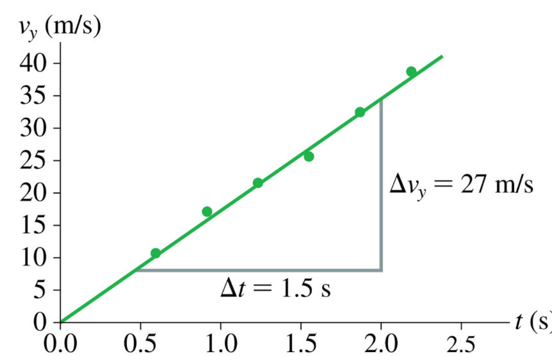
Motion with Constant Acceleration

- We can use the slope of the graph in the velocity graph to determine the acceleration of the rocket.

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{27 \text{ m/s}}{1.5 \text{ s}} = 18 \text{ m/s}^2$$



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Constant Acceleration Equations

- We can use the acceleration to find $(v_x)_f$ at a later time t_f.

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t}$$

$$(v_x)_f = (v_x)_i + a_x \, \Delta t$$

Velocity equation for an object with constant acceleration

- We have expressed this equation for motion along the x-axis, but it is a general result that will apply to any axis.

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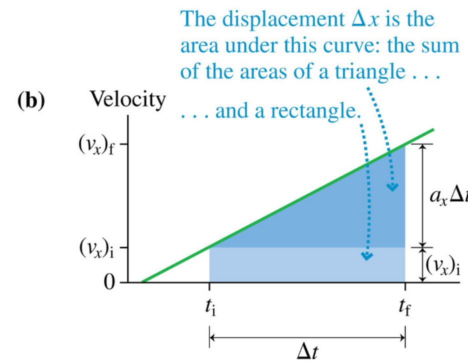
Constant Acceleration Equations

- The velocity-versus-time graph for constant-acceleration motion is a straight line with value $(v_x)_i$ at time t_i and slope a_x.
- The displacement Δx during a time interval Δt is the area under the velocity-versus-time graph shown in the shaded area of the figure.



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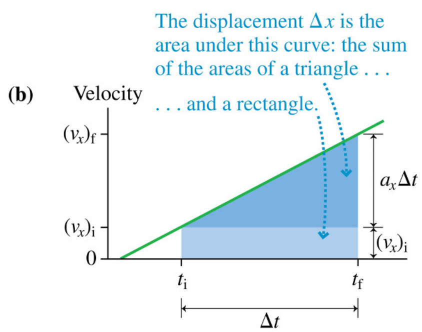
Constant Acceleration Equations



- The shaded area can be subdivided into a rectangle and a triangle. Adding these areas gives

$$x_f = x_i + (v_x)_i \, \Delta t + \tfrac{1}{2} a_x (\Delta t)^2$$

Position equation for an object with constant acceleration

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Constant Acceleration Equations

- Combining Equation 2.11 with Equation 2.12 gives us a relationship between displacement and velocity:

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \, \Delta x$$

Relating velocity and displacement for constant-acceleration motion

- Δx in Equation 2.13 is the displacement (not the distance!).

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Constant Acceleration Equations

For **motion with constant acceleration**:

- Velocity changes steadily:

Final and initial velocity (m/s)

$$(v_x)_f = (v_x)_i + a_x \Delta t \qquad (1)$$

Acceleration (m/s^2) Time interval (s)

- The position changes as the square of the time interval:

Final and initial position (m) Time interval (s)

$$x_f = x_i + (v_x)_i \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \qquad (2)$$

Initial velocity (m/s) Acceleration (m/s^2)

- We can also express the change in velocity in terms of **distance, not time**:

Final and initial velocity (m/s)

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \Delta x \qquad (3)$$

Acceleration (m/s^2) Change in position (m)

Text: p. 43

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Quadratic Relationships

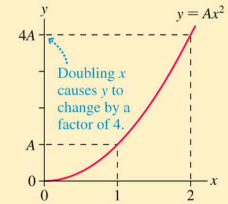
Quadratic relationships

Two quantities are said to have a **quadratic relationship** if y is proportional to the square of x. We write the mathematical relationship as

$$y = Ax^2$$

y is proportional to x^2

The graph of a quadratic relationship is a parabola.



SCALING If x has the initial value x_1, then y has the initial value $y_1 = A(x_1)^2$. Changing x from x_1 to x_2 changes y from y_1 to y_2. The ratio of y_2 to y_1 is

$$\frac{y_2}{y_1} = \frac{A(x_2)^2}{A(x_1)^2} = \left(\frac{x_2}{x_1}\right)^2$$

The ratio of y_2 to y_1 is the square of the ratio of x_2 to x_1. If y is a quadratic function of x, a change in x by some factor changes y by the square of that factor:

- If you increase x by a factor of 2, you increase y by a factor of $2^2 = 4$.
- If you decrease x by a factor of 3, you decrease y by a factor of $3^2 = 9$.

Generally, we can say that:

Changing x by a factor of c changes y by a factor of c^2.

Exercise 19

Text: p. 44

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The Pictorial Representation

TACTICS BOX 2.2 **Drawing a pictorial representation**

1. **Sketch the situation.** Not just any sketch: Show the object at the *beginning* of the motion, at the *end,* and at any point where the character of the motion changes. Very simple drawings are adequate.
2. **Establish a coordinate system.** Select your axes and origin to match the motion.
3. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch.

We will generally combine the pictorial representation with a **list of values,** which will include:

- *Known information*. Make a table of the quantities whose values you can determine from the problem statement or that you can find quickly with simple geometry or unit conversions.
- *Desired unknowns*. What quantity or quantities will allow you to answer the question?

Exercise 21

Text: p. 46

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The Visual Overview

- The **visual overview** will consist of some or all of the following elements:
  - A *motion diagram*. A good strategy for solving a motion problem is to start by drawing a motion diagram.
  - A *pictorial representation,* as defined above.
  - A *graphical representation.* For motion problems, it is often quite useful to include a graph of position and/or velocity.
  - A *list of values.* This list should sum up all of the important values in the problem.

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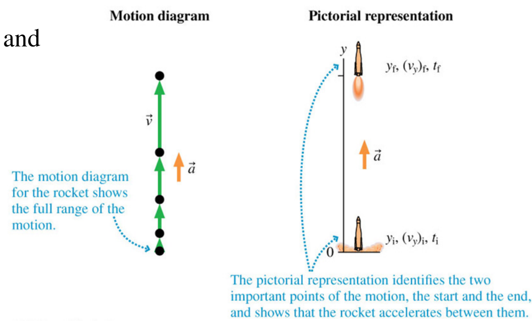
Example 2.11 Kinematics of a rocket launch

A Saturn V rocket is launched straight up with a constant acceleration of 18 m/s^2. After 150 s, how fast is the rocket moving and how far has it traveled?

PREPARE FIGURE 2.32 shows a visual overview of the rocket launch that includes a motion diagram, a pictorial representation, and a list of values. The visual overview shows the whole problem in a nutshell. The motion diagram illustrates the motion of the rocket. The pictorial representation (produced according to Tactics Box 2.2) shows axes, identifies the important points of the motion, and defines variables. Finally, we have included a list of values that gives the known and unknown quantities. In the visual overview we have taken the statement of the problem in words and made it much more precise. The overview contains everything you need to know about the problem.



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Example 2.11 Kinematics of a rocket launch (cont.)

SOLVE Our first task is to find the final velocity. Our list of values includes the initial velocity, the acceleration, and the time interval, so we can use the first kinematic equation of Synthesis 2.1 to find the final velocity:

$$(v_y)_f = (v_y)_i + a_y\,\Delta t = 0\text{ m/s} + (18\text{ m/s}^2)(150\text{ s})$$
$$= 2700\text{ m/s}$$

List of values

Known

$y_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$a_y = 18$ m/s^2
$t_f = 150$ s

Find

$(v_y)_f$ and y_f

The list of values makes everything concrete. We define the start of the problem to be at time 0 s, when the rocket has a position of 0 m and a velocity of 0 m/s. The end of the problem is at time 150 s. We are to find the position and velocity at this time.

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Example 2.11 Kinematics of a rocket launch (cont.)

SOLVE

The distance traveled is found using the second equation in Synthesis 2.1:

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$= 0\text{ m} + (0\text{ m/s})(150\text{ s}) + \tfrac{1}{2}(18\text{ m/s}^2)(150\text{ s})^2$$
$$= 2.0 \times 10^5\text{ m} = 200\text{ km}$$

$y_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$a_y = 18$ m/s^2
$t_f = 150$ s

Find

$(v_y)_f$ and y_f

The list of values makes everything concrete. We define the start of the problem to be at time 0 s, when the rocket has a position of 0 m and a velocity of 0 m/s. The end of the problem is at time 150 s. We are to find the position and velocity at this time.

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Problem-Solving Strategy for Motion with Constant Acceleration

PROBLEM-SOLVING STRATEGY 2.1 **Motion with constant acceleration**

Problems involving constant acceleration—speeding up, slowing down, vertical motion, horizontal motion—can all be treated with the same problem-solving strategy.

PREPARE Draw a visual overview of the problem. This should include a motion diagram, a pictorial representation, and a list of values; a graphical representation may be useful for certain problems.

SOLVE The mathematical solution is based on the three equations in Synthesis 2.1.

- Though the equations are phrased in terms of the variable x, it's customary to use y for motion in the vertical direction.
- Use the equation that best matches what you know and what you need to find. For example, if you know acceleration and time and are looking for a change in velocity, the first equation is the best one to use.
- Uniform motion with constant velocity has $a = 0$.

ASSESS Is your result believable? Does it have proper units? Does it make sense?

Exercise 25

Text: p. 48

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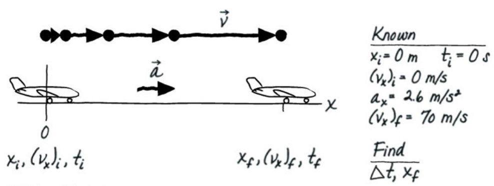
Example 2.12 Calculating the minimum length of a runway

A fully loaded Boeing 747 with all engines at full thrust accelerates at 2.6 m/s². Its minimum takeoff speed is 70 m/s. How much time will the plane take to reach its takeoff speed? What minimum length of runway does the plane require for takeoff?

PREPARE The visual overview of FIGURE 2.33 summarizes the important details of the problem. We set x_i and t_i equal to zero at the starting point of the motion, when the plane is at rest and the acceleration begins. The final point of the motion is when the plane achieves the necessary takeoff speed of 70 m/s. The plane is accelerating to the right, so we will compute the time for the plane to reach a velocity of 70 m/s and the position of the plane at this time, giving us the minimum length of the runway.



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Example 2.12 Calculating the minimum length of a runway (cont.)

SOLVE First we solve for the time required for the plane to reach takeoff speed. We can use the first equation in Synthesis 2.1 to compute this time:

$$(v_x)_f = (v_x)_i + a_x\,\Delta t$$

$$70\text{ m/s} = 0\text{ m/s} + (2.6\text{ m/s}^2)\,\Delta t$$

$$\Delta t = \frac{70\text{ m/s}}{2.6\text{ m/s}^2} = 26.9\text{ s}$$

We keep an extra significant figure here because we will use this result in the next step of the calculation.

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Example 2.12 Calculating the minimum length of a runway (cont.)

SOLVE

Given the time that the plane takes to reach takeoff speed, we can compute the position of the plane when it reaches this speed using the second equation in Synthesis 2.1:

$$x_f = x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$= 0\text{ m} + (0\text{ m/s})(26.9\text{ s}) + \tfrac{1}{2}(2.6\text{ m/s}^2)(26.9\text{ s})^2$$

$$= 940\text{ m}$$

Our final answers are thus that the plane will take 27 s to reach takeoff speed, with a minimum runway length of 940 m.

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Example 2.12 Calculating the minimum length of a runway (cont.)

ASSESS Think about the last time you flew; 27 s seems like a reasonable time for a plane to accelerate on takeoff. Actual runway lengths at major airports are 3000 m or more, a few times greater than the minimum length, because they have to allow for emergency stops during an aborted takeoff. (If we had calculated a distance far greater than 3000 m, we would know we had done something wrong!)

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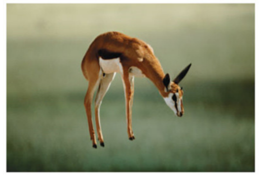
Example Problem: Champion Jumper



The African antelope known as a springbok will occasionally jump straight up into the air, a movement known as a **pronk**. The speed when leaving the ground can be as high as 7.0 m/s.

If a springbok leaves the ground at 7.0 m/s:

A. How much time will it take to reach its highest point?
B. How long will it stay in the air?
C. When it returns to earth, how fast will it be moving?

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Section 2.7 Free Fall

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Free Fall

- If an object moves under the influence of gravity only, and no other forces, we call the resulting motion **free fall**.
- **Any two objects in free fall, regardless of their mass, have the same acceleration.**
- On the earth, air resistance is a factor. For now we will restrict our attention to situations in which air resistance can be ignored.

Apollo 15 lunar astronaut David Scott performed a classic experiment on the moon, simultaneously dropping a hammer and a feather from the same height. Both hit the ground at the exact same time—something that would not happen in the atmosphere of the earth!

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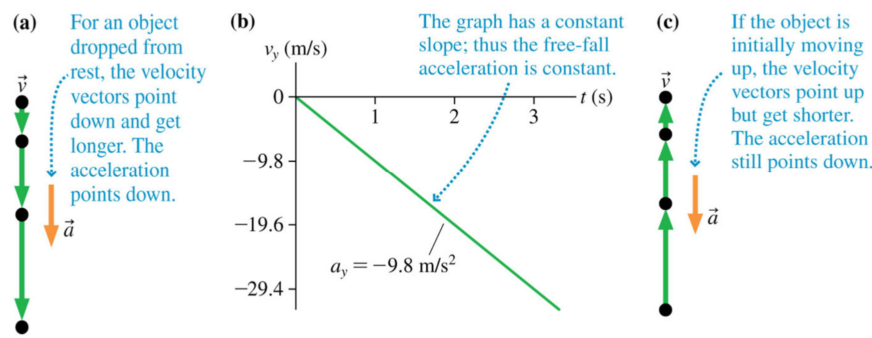
Free Fall



- The figure shows the motion diagram for an object that was released from rest and falls freely. The diagram and the graph would be the same for all falling objects.

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Free Fall

- **The free-fall acceleration always points down**, no matter what direction an object is moving.
- Any object moving under the influence of gravity only, and no other force, is in free fall.

$$\vec{a}_{\text{free fall}} = (9.80\ \text{m/s}^2,\ \text{vertically downward})$$

Standard value for the acceleration of an object in free fall

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Free Fall

- g, by definition, is always positive. **There will never be a problem that uses a negative value for g.**
- Even though a falling object speeds up, it has negative acceleration ($-g$).
- Because free fall is motion with constant acceleration, we can use the kinematic equations for constant acceleration with $a_y = -g$.
- g is not called "gravity." g is the *free-fall acceleration*.
- $g = 9.80\ \text{m/s}^2$ only on earth. Other planets have different values of g.
- We will sometimes compute acceleration in units of g.

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QuickCheck 2.26

A ball is tossed straight up in the air. At its very highest point, the ball's instantaneous acceleration a_y is

A. Positive.
✔ B. Negative.
C. Zero.

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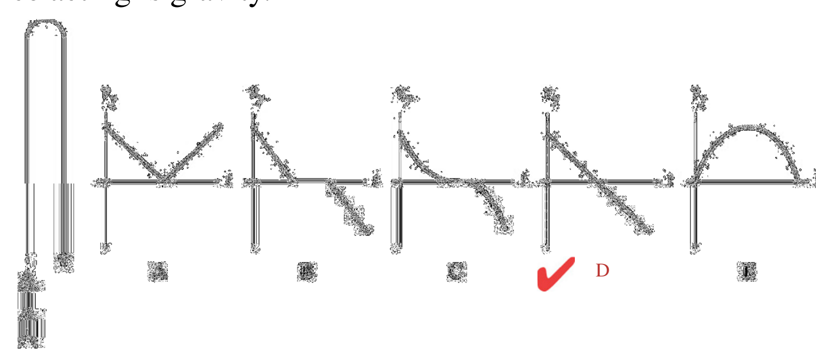
QuickCheck 2.28

An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. The trajectory of the arrow is noted. Which graph best represents the vertical velocity of the arrow as a function of time? Ignore air resistance; the only force acting is gravity.



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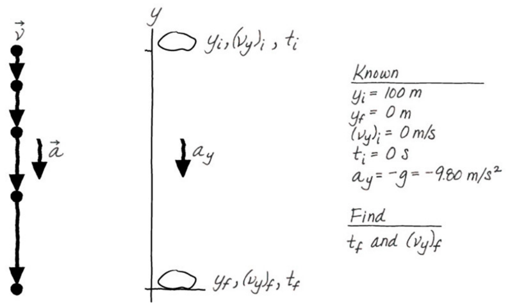
Example 2.14 Analyzing a rock's fall

A heavy rock is dropped from rest at the top of a cliff and falls 100 m before hitting the ground. How long does the rock take to fall to the ground, and what is its velocity when it hits?

PREPARE FIGURE 2.36 shows a visual overview with all necessary data. We have placed the origin at the ground, which makes $y_i = 100$ m.



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Example 2.14 Analyzing a rock's fall (cont.)

SOLVE Free fall is motion with the specific constant acceleration $a_y = -g$. The first question involves a relation between time and distance, a relation expressed by the second equation in Synthesis 2.1. Using $(v_y)_i = 0$ m/s and $t_i = 0$ s, we find

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2 = y_i - \tfrac{1}{2}g(\Delta t)^2 = y_i - \tfrac{1}{2}gt_f^2$$

We can now solve for t_f:

$$t_f = \sqrt{\frac{2(y_i - y_f)}{g}} = \sqrt{\frac{2(100\text{ m} - 0\text{ m})}{9.80\text{ m/s}^2}} = 4.52\text{ s}$$

Now that we know the fall time, we can use the first kinematic equation to find $(v_y)_f$:

$$(v_y)_f = (v_y)_i - g\,\Delta t = -gt_f = -(9.80\text{ m/s}^2)(4.52\text{ s})$$
$$= -44.3\text{ m/s}$$

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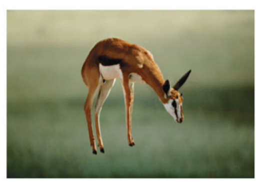
Example 2.16 Finding the height of a leap



A springbok is an antelope found in southern Africa that gets its name from its remarkable jumping ability. When a springbok is startled, it will leap straight up into the air—a maneuver called a "pronk." A springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at 35 m/s^2 for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air.

a. At what speed does the springbok leave the ground?

b. How high does it go?

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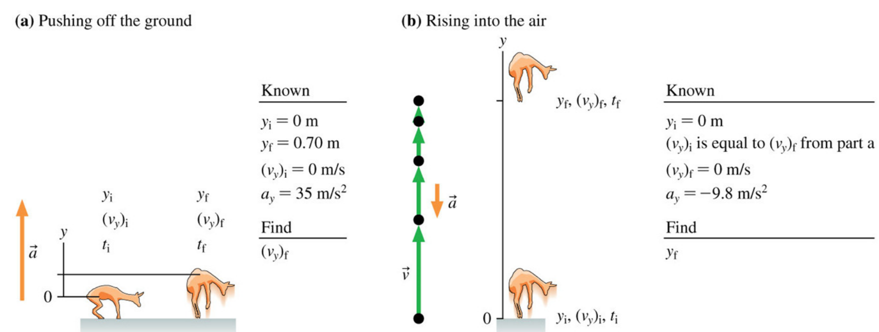
Example 2.16 Finding the height of a leap (cont.)



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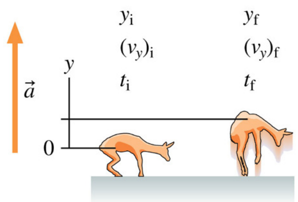
Example 2.16 Finding the height of a leap (cont.)

PREPARE We begin with the visual overview shown in FIGURE 2.38, where we've identified two different phases of the motion: the springbok pushing off the ground and the springbok rising into the air. We'll treat these as two separate problems that we solve in turn. We will "re-use" the variables y_i, y_f, $(v_y)_i$, and $(v_y)_f$ for the two phases of the motion.

For the first part of our solution, in Figure 2.38a we choose the origin of the y-axis at the position of the springbok deep in the crouch. The final position is the top extent of the push, at the instant the springbok leaves the ground. We want to find the velocity at this position because that's how fast the springbok is moving as it leaves the ground.



Known
$y_i = 0$ m
$y_f = 0.70$ m
$(v_y)_i = 0$ m/s
$a_y = 35$ m/s^2

Find
$(v_y)_f$

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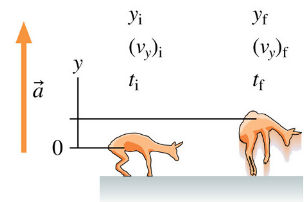
Example 2.16 Finding the height of a leap (cont.)

SOLVE a. For the first phase, pushing off the ground, we have information about displacement, initial velocity, and acceleration, but we don't know anything about the time interval. The third equation in Synthesis 2.1 is perfect for this type of situation. We can rearrange it to solve for the velocity with which the springbok lifts off the ground:

$$(v_y)_f^2 = (v_y)_i^2 + 2a_y\Delta y = (0\text{ m/s})^2 + 2(35\text{ m/s}^2)(0.70\text{ m}) = 49\text{ m}^2/\text{s}^2$$
$$(v_y)_f = \sqrt{49\text{ m}^2/\text{s}^2} = 7.0\text{ m/s}$$

The springbok leaves the ground with a speed of 7.0 m/s.



Known
$y_i = 0$ m
$y_f = 0.70$ m
$(v_y)_i = 0$ m/s
$a_y = 35$ m/s^2

Find
$(v_y)_f$

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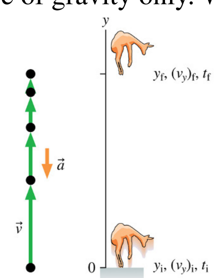
Example 2.16 Finding the height of a leap (cont.)

Figure 2.38b essentially starts over—we have defined a new vertical axis with its origin at the ground, so the highest point of the springbok's motion is a distance above the ground. The table of values shows the key piece of information for this second part of the problem: The initial velocity for part b is the final velocity from part a.

After the springbok leaves the ground, this is a free-fall problem because the springbok is moving under the influence of gravity only. We want to know the height of the leap, so we are looking for the height at the top point of the motion. This is a turning point of the motion, with the instantaneous velocity equal to zero. Thus y_f, the height of the leap, is the springbok's position at the instant $(v_y)_f = 0$.



Known
$y_i = 0$ m
$(v_y)_i$ is equal to $(v_y)_f$ from part a
$(v_y)_f = 0$ m/s
$a_y = -9.8$ m/s^2

Find
y_f

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Example 2.16 Finding the height of a leap (cont.)

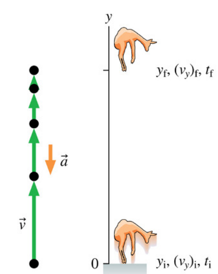
SOLVE b. Now we are ready for the second phase of the motion, the vertical motion after leaving the ground. The third equation in Synthesis 2.1 is again appropriate because again we don't know the time. Because $y_i = 0$, the springbok's displacement is $\Delta y = y_f - y_i = y_f$, the height of the vertical leap. From part a, the initial velocity is $(v_y)_i = 7.0$ m/s, and the final velocity is $(v_y)_f = 0$. This is free-fall motion, with $a_y = -g$; thus

$$(v_y)_f^2 = 0 = (v_y)_i^2 - 2g\Delta y = (v_y)_i^2 - 2gy_f$$

which gives $(v_y)_i^2 = 2gy_f$

Solving for y_f, we get a jump height of

$$y_f = \frac{(7.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 2.5\text{ m}$$



Known
$y_i = 0$ m
$(v_y)_i$ is equal to $(v_y)_f$ from part a
$(v_y)_f = 0$ m/s
$a_y = -9.8$ m/s^2

Find
y_f

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Example 2.16 Finding the height of a leap (cont.)

ASSESS 2.5 m is a remarkable leap—a bit over 8 ft—but these animals are known for their jumping ability, so this seems reasonable.

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Summary

GENERAL STRATEGIES

Problem-Solving Strategy

Our general problem-solving strategy has three parts:

PREPARE Set up the problem:

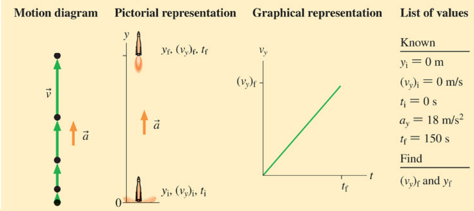
- Draw a picture.
- Collect necessary information.
- Do preliminary calculations.

SOLVE Do the necessary mathematics or reasoning.

ASSESS Check your answer to see if it is complete in all details and makes physical sense.

Visual Overview

A visual overview consists of several pieces that completely specify a problem. This may include any or all of the elements below:



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Summary

IMPORTANT CONCEPTS

Velocity is the rate of change of position:

$$v_x = \frac{\Delta x}{\Delta t}$$

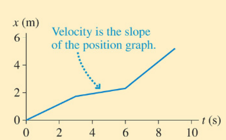
Acceleration is the rate of change of velocity:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

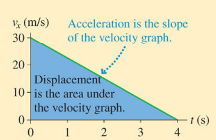
The units of acceleration are m/s^2.

An object is speeding up if v_x and a_x have the same sign, slowing down if they have opposite signs.

A **position-versus-time graph** plots position on the vertical axis against time on the horizontal axis.



A **velocity-versus-time graph** plots velocity on the vertical axis against time on the horizontal axis.



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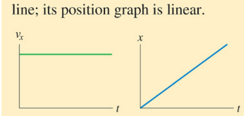
Summary

APPLICATIONS

Uniform motion

An object in uniform motion has a constant velocity. Its velocity graph is a horizontal line; its position graph is linear.



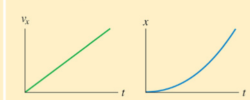
Kinematic equation for uniform motion:

$$x_f = x_i + v_x \Delta t$$

Uniform motion is a special case of constant-acceleration motion, with $a_x = 0$.

Motion with constant acceleration

An object with constant acceleration has a constantly changing velocity. Its velocity graph is linear; its position graph is a parabola.



Kinematic equations for motion with constant acceleration:

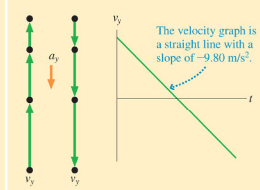
$$(v_x)_f = (v_x)_i + a_x \Delta t$$

$$x_f = x_i + (v_x)_i \Delta t + \tfrac{1}{2} a_x (\Delta t)^2$$

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \Delta x$$

Free fall

Free fall is a special case of constant-acceleration motion. The acceleration has magnitude $g = 9.80$ m/s^2 and is always directed vertically downward whether an object is moving up or down.



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