Mechanical Equilibrium

Name:

Group Members:

Date:

TA's Name:

Learning Objective:

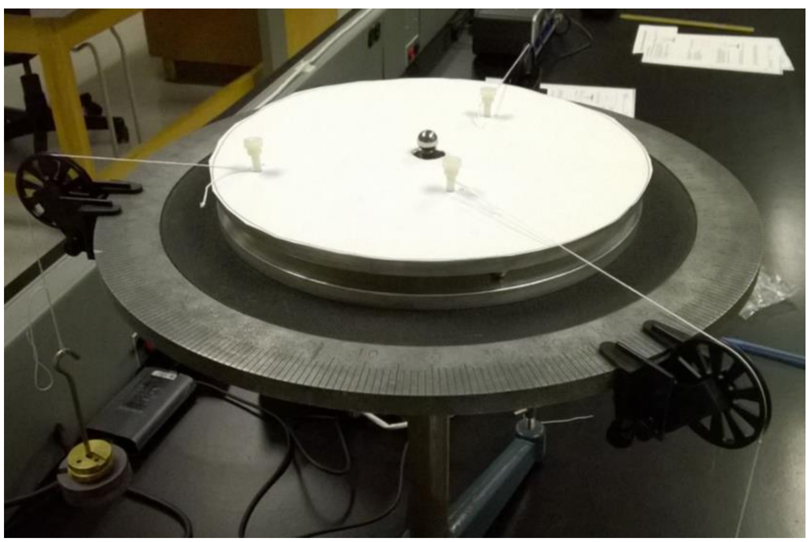
To understand the conditions necessary for mechanical static equilibrium

To be able to determine when there is translational equilibrium using force vectors

To be able to determine when there is rotational equilibrium using torque vectors

Apparatus:

Force table with aluminum torque plate accessory, assorted masses, mass hangers, light string, plastic pegs, ruler, 3 spherical ball bearings, 3 pulleys, paper disk, and protractor.



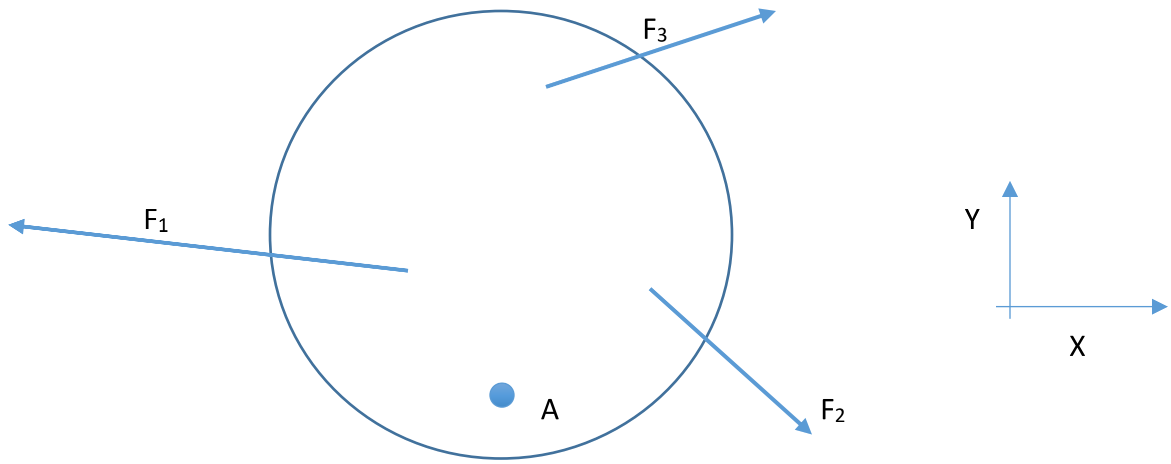
You learned that for a rigid body to be in mechanical equilibrium, the following two conditions must be satisfied.

- The vector sum of the forces acting on the object is zero. (Translational Equilibrium)
- The net torque around any axis is zero. (Rotational Equilibrium)

In this experiment we investigate these conditions applied to a rigid body. The figure shows the apparatus we will use. The rigid body in this experiment is a circular aluminum plate supported by three ball bearings so that it can roll around freely on another aluminum plate. There are off-center holes in the top plate and a set of pegs to fit the holes. Forces are applied to the pegs with strings and mass hangers. Both plates are secured by the center pin so that they do not fall.

In general, a force applied to a rigid body results in the application of a torque as well. To calculate the torque produced by a force, you need to know: the magnitude of the force, the direction of the force, the point of application of the force, and the location of the point where the axis of rotation intersects the plane of rotation.

Now assume that you have following three forces acting on the top plate of the above apparatus. The plate is in **equilibrium**.



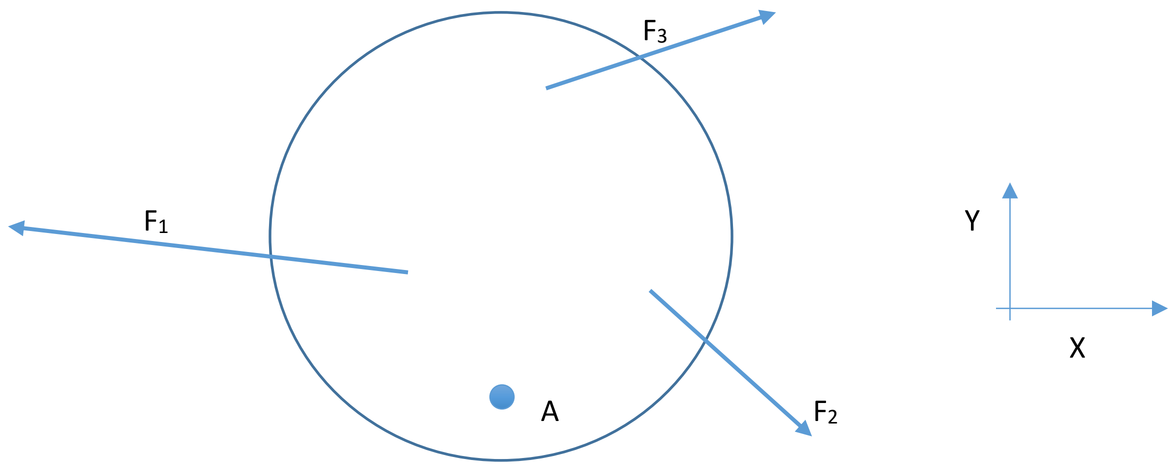
1. On the drawing above, using the plastic ruler and pencil, draw X and Y components of each of the three force vectors.

2. Should the net force in the x direction be zero? Why?

3. Write down Newton's second law ($\sum F_x = ma_x$) as applied to the plate. Use symbols F_{1x}, F_{2x} and F_{3x} to represent the x-components of the three force vectors.

4. Should the net force in the Y direction be zero? Why?

5. Write down Newton's second law ($\sum F_y = ma_y$) applied to plate in the y direction. Use symbols F_{1y}, F_{2y} and F_{3y} to represent the x-components of the three force vectors.



6. For each force on the diagram above, draw the line of action. You can do this by using your ruler to extend the force vector in both directions.
7. Now on the drawing, draw lever arm for each force about the axis point A. That is, draw the shortest line from point A to each line of action. The lever arm should be perpendicular to the line of action if it really is the shortest line. Label the lever arm lengths as r_1, r_2 and r_3 corresponding to the associated force vector.
8. Should the net torque be zero? Why?

9. The torque produced by each force is the product of the lever arm and the magnitude of the force. For example, the torque produced by F_1 about the axis point A is equal to r_1F_1. The torque has a sign since it will produce either a clock-wise or an anti-clockwise rotation about the axis point. We'll call anti-clockwise as positive and clockwise as negative. So the sign of the torque produced by F_1 is positive since it would produce an anti-clockwise rotation about axis point A. Complete the table with the torques from the three forces and indicate their signs.

Force	**Torque**	**Sign**
F_1	r_1F_1	+
F_2		
F_3		

10. Use the torques and the signs to write down Newton's second law for rotation ($\sum \tau = I\alpha$) to the plate. (No need to know I since angular acceleration α is zero in equilibrium)

11. Now we are ready to do the experiment. If there are pins on the top plate remove and keep them away. Level the apparatus. Make sure that the force table is level and firmly seated in the foot before you start doing the experiment. Adjust leveling screws on the feet as necessary so that the top plate is centered on the centering pin. You should be able to pull out the center pin and not have the top plate accelerate. Once the system is leveled, do not move it. Why is levelling the apparatus so important? Explain.

12. Pull the pin out and place the pre-cut circular paper on the top plate. Trace the center hole onto the paper and use scissors to cut this hole out of the paper. Push the center pin through the hole in the paper back in its place. Push three pegs through the paper into any three of the little holes on the top plate and label them **1**, **2**, and **3** on the paper. Loop a string over each peg and run the strings over the pulleys. The position of the pulley on the force table isn't important, so you may turn the pulleys as needed to let the string run without binding. Choose holes for the pegs that don't all have the same radii and arrange the pulleys so that the strings don't all pull directly from the center.

    Add some weights to the 50-g mass hangers. Start around 200 grams so that the frictional effects are overwhelmed. By trial and error, find some combination of weights that produces a state of complete translational and rotational equilibrium. When you are confident that you have achieved equilibrium, the top plate should not be touching the center pin and you should be able to pull the center pin out of the hole. (If the plate is in equilibrium, it will not accelerate in any direction. However, if it is not in equilibrium, it will, so do this carefully!) Record the weights on the **Force Data Table** on the next page.

13. Use the ruler to trace the lines of the strings from peg to pulley. These are the lines of action of the forces. (Try not to move the upper plate while you do this.) Label each line with an arrowhead indicating direction. Also label the forces $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$.

14. Add a little weight to one hanger until the plate goes out of equilibrium causing the plate to move. Record these as a measure of uncertainty in the measurement, $\Delta \boldsymbol{F}$.

15. Remove the paper and draw a coordinate system on it. Now determine the directions, θ, in that coordinate system. Use angles between 0° and 360°.

16. Now from the magnitudes and directions of the forces, find the components of each force, $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$.

Force Data Table

Force	Mass (kg)	$F = mg$ (N)	ΔF (N)	θ (^{0})	F_x (N)	F_y (N)
F_1						
F_2						
F_3						
Net Force						

17. Add the three x-components to get the net force in the x-direction. Do the same for the y-direction.

18. Determine the magnitude of the net force.

    Magnitude of the net force =

19. Does this result in Question 18 agree with what you expected? If not, does the result you expected fall within the uncertainty of what you measured?

20. On your paper showing the lines of action for the three forces, select any point, draw a dot and mark it as **A** to use as the axis point. Don't pick a point that is on any of the lines of force. The measurement of the moment arm is easier if it is longer; pick a point well off the set of the lines of force. Draw the lever arms of the three forces about this axis point as you did back in Question 7. Measure the lengths of the lever arms and put them into the **Torque Data Table** on the next page.

21. Calculate the torque from each force about axis point A and add that to your table. Also determine the sign of each torque.

Torque Date Table

Force	$F = mg$ (N)	Lever arm, r (m)	Torque: τ (Nm)	Sign
F_1				
F_2				
F_3				

22. Using the magnitudes and signs of the three torque vectors, calculate the net torque.

    Net torque =

23. Does the result agree with what you expected for a system in rotational equilibrium? If not, explain why not.

24. If you were to select another arbitrary point Q and find the sum of the torques about this point too what would be the expected result?