

Experiment 6

Capacitive Reactance

Preparation

Prepare for this week's experiment by reviewing past material and by reading about capacitors in AC circuits.

Principles

Last week you looked at the behavior of a capacitor in a DC circuit. Briefly, at the end, you looked at how the voltage across the capacitor changed as a function of the frequency in an AC circuit. This week you will look at AC circuits in more detail.

Consider what happens in a series circuit consisting of a capacitor and a resistor that are connected to an AC signal generator. When the voltage is turned on current will flow through the wires in the circuit. The current will change direction as the signal generator changes polarity. The voltage across the resistor will vary the same way and its magnitude will be:

$$V_R = I_o R.$$

Since the charge carriers cannot go through the capacitor they pile up on one plate and induce an opposite charge on the other plate. This causes a voltage drop across the capacitor. When the current changes direction, the charge carriers flow in the opposite direction until a voltage drop appears across the capacitor opposite in polarity from the first voltage. The net result is that the voltage across the capacitor will vary sinusoidally. The magnitude of the voltage across the capacitor will be:

$$V_C = I_o X_C$$

where X_C is the **capacitive reactance** of the capacitor of the circuit. It is equal to:

$$X_C = \frac{1}{2\pi f C}.$$

X_C is measured in ohms.

Kirchhoff's Law still applies to AC circuits. The sum of the voltage drops across the individual components will still be equal to the voltage across the voltage source. However, because the resistor and capacitor and power supply voltages do not reach their maxima at the same time you cannot simply add the magnitudes. Instead, they have to be added like vectors. The equation for the magnitude of the power supply voltage is:

$$V_o = \sqrt{V_C^2 + V_R^2}.$$

In a circuit with a resistor and a capacitor the amount of current is determined by the voltage and the **impedance, Z**, of the circuit. The impedance takes the place of the net resistance in a resistor circuit. Z is the vector sum of the resistance and the capacitive reactance in the circuit:

$$Z = \sqrt{X_C^2 + R^2}.$$

The magnitude of the current will be:

$$I_o = \frac{V_o}{Z} = \frac{V_o}{\sqrt{\left(\frac{1}{2\pi fC}\right)^2 + R^2}}.$$

Consider a circuit where V_o stays constant. As the frequency increases X_C and Z will get smaller and the current, I_o , will increase. The magnitude of the voltage drop across the resistor, $V_R = I_o R$, will increase and the voltage across the capacitor, $V_C = I_o X_C$, will decrease.

Equipment

- 1 *1K Ω resistor (approximately)*
- 1 *.05 μ F capacitor (approximately)*
- 1 *signal generator with power cord*
- 1 *oscilloscope*
- 2 *3' banana wires, one red, one black*
- 2 *2' banana wires, one red, one black*
- 1 *4" banana wire*
- 2 *ground isolators*
- 2 *BNC connectors*
- 1 *multimeter (This is only needed for one measurement, so several groups can share one meter.)*

Procedure

1. Record the value of your resistor in Ω , not $k\Omega$.
2. Record the value for the capacitance of the capacitor. Remember that the unit is micro Farads (μ F) or 10^{-6} Farads. This will be important for your calculations.
3. Plug in the oscilloscope and the signal generator. Be sure to use the two prong adapters (the gray things) on the plugs for the 'scope and the signal generator. Plug a BNC connector into the 'scope input and the signal generator output.
4. Use the 4" wire to connect the resistor and capacitor.

5. Use 18" or 2' banana wires to connect the resistor and capacitor to the signal generator so that your circuit looks like the diagram.
6. Have your instructor check the circuit and then turn everything on.
7. Connect the red lead from the scope to the top of the circuit (point 1 on the schematic) and the black lead to the tail of the circuit (point 3 on the schematic). The leads are now connected to measure V_o , the voltage across the entire circuit.

8. Push the button on the signal generator that says "x1K". Turn the dial marked "frequency" until the counter reads some number close to 500 Hz. Try to get close to 500 Hz, but don't waste much time trying to get exactly 500 Hz.

9. Use the "amplitude" knob to set your signal generator voltage to $6 V_{pp}$. This is V_o . Record this value.

10. You are now ready to take your data. It will be very easy if you follow these steps exactly:

1. Set the leads to measure V_o (Connect the red lead to point 1, the black lead to point 3.).

2. Set the frequency.

3. Check the calibration knobs and make sure they are turned all the way in the direction of the arrows.

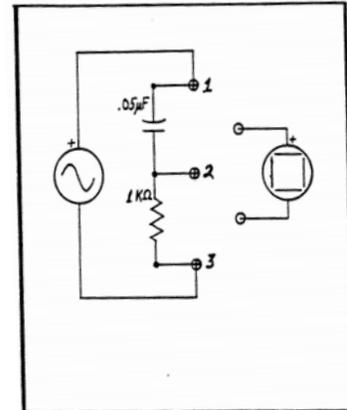
4. Set the generator voltage to $6 V_{pp}$.

5. Measure and record the voltage across the capacitor. (Connect the red lead to point 1, the black lead to point 2.)

6. Measure and record the voltage across the resistor. (Connect the red lead to point 2, the black lead to point 3.)

7. Go back to step 1 until you run out of frequencies.

11. Turn everything off, disconnect everything and put the equipment away carefully.



Analysis

1. Calculate your theoretical value for the reactance using $X_C = \frac{1}{2\pi fC}$.

2. Calculate the experimental value for the reactance. We can do this by dividing the capacitor voltage by the current:

$$X_C = \frac{V_C}{I_o} .$$

The current is just the resistor voltage divided by the resistance:

$$I_o = \frac{V_R}{R} .$$

A little algebra shows that:

$$X_C = \frac{V_C R}{V_R} .$$

3. Pick a couple of values and calculate V_o using $V_o = \sqrt{V_C^2 + V_R^2}$. Are your numbers in pretty good agreement?

4. Make a graph of X_C vs. frequency.