

Experiment 8

Harmonic Motion of a Spring

Preparation

Prepare for this week's quiz by reviewing the pendulum experiment and by reading the sections of your book that cover harmonic motion of a spring and Hooke's Law.

Principles

Hooke's law for a spring states that when a spring with a mass attached to its end is displaced from its equilibrium position it will experience a force of magnitude:

$$f = -kx$$

where f is the force on the spring, x is the displacement of the end of the spring from its equilibrium position, and k is the **spring constant**, a property of the spring. A stiff spring will have a large value for k ; a softer spring will have a smaller value. The unit for the spring constant is **newtons/meter**. The spring constant tells you how much force the end of the spring exerts on the mass for per unit length of displacement.

F is sometimes called the **restoring force** since it always points in the direction that would "restore" the mass to its equilibrium position. In other words, it is always in the opposite direction from the displacement. Note that the force is not constant; it is a function of the displacement of the end of the spring. According to Newton's second law:

$$f = ma$$

The forces are the same. You can rearrange the equation:

$$a + \frac{k}{m} x = 0$$

This is called a differential equation. We are looking for some function, x , that will give us the displacement of the end of the spring as a function of time. X must satisfy the above equation, that is the acceleration must equal the negative of the displacement times k/m . Two possible solutions for this equation are:

$$x = A \sin(\omega t)$$

$$x = B \cos(\omega t)$$

where A and B are the **amplitudes** of the spring's motion. If you let a spring oscillate and plot the displacement of the end as a function of time you will get a sine or cosine wave. The wave will repeat itself with a **period**, T . The **angular frequency**, ω , is:

$$\omega = \frac{2\pi}{T}$$

The period of the spring is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Notice that the form of this equation is very much like the form of the equation for the period of a pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The difference is that the relevant variables are l and g for the pendulum and m and k for the spring.

If you put a mass on a spring of known spring constant, time its oscillations and then solve for m the two mass values will not agree. This is because the spring is not massless and part of the spring's mass helps to drive the oscillation. The spring equation is more properly written:

$$T = 2\pi\sqrt{\frac{m_{eff}}{k}}$$

where m_{eff} is called the effective mass of the spring. M_{eff} is the sum of the mass put on the spring and some fraction of the spring's mass. Theoretically, this fraction is one third. One goal of this experiment is to see if this is true.

Consider a spring hanging straight down. Its end will be at some position x_0 . If you hang a mass on the end the spring will stretch to a position x_i . There are two forces acting on the spring. The weight, mg , pulls the spring downward, while the Hooke's Law force, $-k(x - x_0)$ pulls the spring up. If the spring is at rest, the net force on the spring is zero, so the two forces must be equal in magnitude. If you hang a series of masses, m_i , on the spring and measure the new equilibrium lengths, x_i , you will be able to calculate the spring constant from the slope of x_i vs. m_i :

$$\frac{\Delta x_i}{\Delta m_i} = \frac{g}{k}$$

The intercept of the line should equal the unweighted length of the spring.

Equipment

table clamp
threaded rod
90° clamp
small rod
spring clamp
stopwatch
two meter stick
balance

- 1 .100 kg mass
- 2 .200 kg masses

Procedure

Use SI units for all measurements. Hang the spring on the hanger with the wide end down. Assume masses are correct to three significant digits. Measure lengths to the nearest mm; measure time to the tenth of a second.

1. Mass the spring. Hang the spring on the hanger and measure the distance from the **bottom of the support rod to the bottom of the spring (the curved part)** when there is no mass on the spring.
2. Hang the .100 kg mass on the spring and bring it to rest. Measure the distance from the bottom of the support rod to the same point on the spring you measured in the procedure above. **Do not measure to the bottom of the mass!**
3. Displace the mass slightly so that it starts oscillating. Measure the time it takes to go through twenty oscillations.
4. Add another .100 kg to the spring and repeat the procedure. Keep increasing the mass by .100 kg until you reach .500 kg. Measure the displacement at rest and the time for twenty cycles in each case.
5. For one of the masses look at the effect of the amplitude of the displacement on the period. Displace the mass about 2 cm and time twenty cycles. Then displace it about 4 cm and then 8 or 9 cm.

Analysis

1. Multiply each mass you added to the stationary spring by g (use 9.80 m/sec^2) to find the force each mass exerted on the end of the spring. Make a graph with the weights you added to the spring on the x axis and the length of the spring on the y axis. Draw the line that best fits the data and find the slope. Look at your graph. The y intercept should equal the value for the length of the spring when there is no weight on it. Does it?

2. Find the slope of your graph. The slope is the rate of change of displacement of the end of the spring with respect to the applied force. The inverse of the slope will be the spring constant, k , for your particular spring..
3. You measured the time for twenty periods of the spring. Divide each time by 20 to find the time for one period for each mass.
4. You know that $T = 2\pi\sqrt{\frac{m_{eff}}{k}}$. If you square both sides of the equation and rearrange it you can solve for the effective mass. Do the algebra, then use the values for k and T that you found to calculate the effective mass for each period.
5. The effective mass will be bigger than the mass you hung on the spring. This excess mass comes from the spring itself. Subtract the mass you hung on the spring from the effective mass to find each excess mass.
6. Divide each excess mass by the mass of the spring. The theory predicts that this fraction should be about one third. Did your values come close to this?
7. Calculate the time for one period for the part of the experiment where you kept the mass constant and changed the amplitude of the oscillations.
8. Remember to show sample calculations. Be sure to put your equipment away properly