Experiment 7

The Pendulum

Preparation

Prepare for this week's quiz by reviewing last week's experiment and by reading the section of your textbook that covers pendulums and harmonic motion.

Principles

A simple pendulum consists of a weight, often called a bob, suspended from a string or rod that is assumed not to stretch. The bob is so heavy compared to the string that the weight of the string can be disregarded. When the string is tied to a support, the bob will hang straight down. If the bob is pulled out so that the string makes a small angle (10° or less) with the vertical and then released the pendulum will swing back and forth, executing simple harmonic motion. The bob will retrace its path at regular intervals and the time it takes the pendulum to go through one complete cycle, or swing, is called its period.

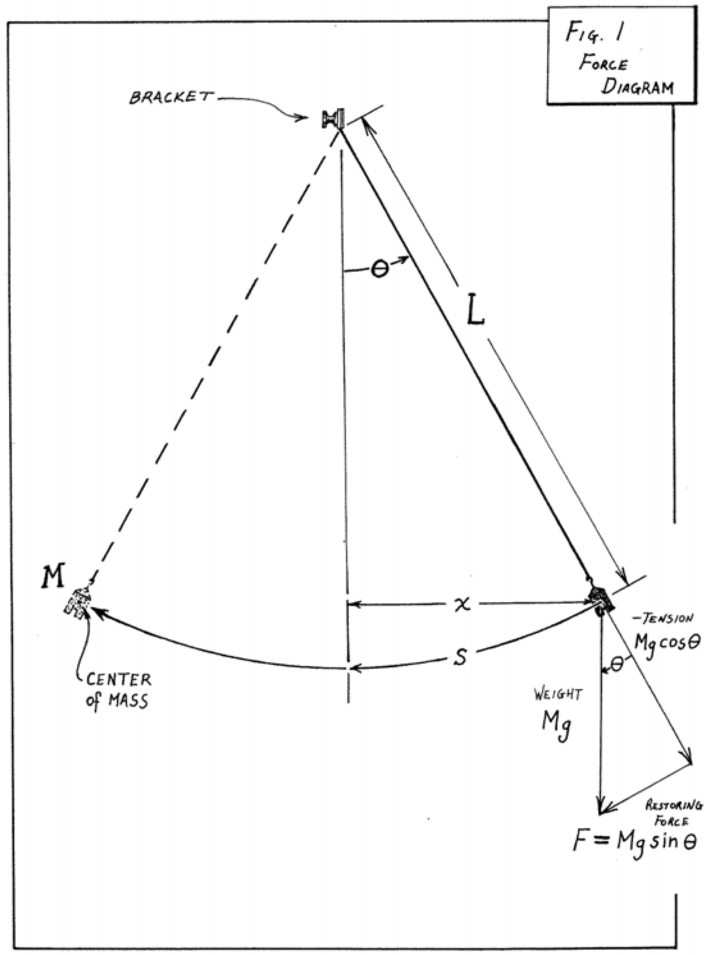
There are a number of variables associated with the period of a pendulum. They are: (1) the mass *(M)* of the bob, (2) the length *(L)* of the string from the point of suspension to the center of mass of the bob, (3) the maximum angle (θ_0) that the string makes with the vertical, (4) the acceleration due to gravity *(g)* acting on the bob, and (5) the period *(T)* of the pendulum. The period is called the dependent variable because it depends on one or more of the other variables. The variables which the experimenter changes are called the independent variables. The acceleration due to gravity varies from place to place but is constant at any one place. This leaves three variables, M, L, and θ_0 which can be controlled in the laboratory. In this experiment you will time the period of a pendulum over a range of these variables and determine the effect, if any, that each has on the period.

From your reading you should know that for all simple harmonic motion the period is given by the formula:

$$T = 2\pi\sqrt{\frac{-\ displacement}{acceleration}}$$

We can use this to find a theoretical formula for the period of a pendulum in terms of the independent variables M, L, and θ_0. Start by considering the force acting on the bob. The weight of the bob, Mg, acts downward. This force can be resolved into two components, one parallel to the string, and one perpendicular to the string. The perpendicular component, F, is the force that acts to return the bob to its rest position. For this reason it is called the restoring force. The restoring force is always in the direction of the rest position of the bob and its sign is always opposite that of the displacement. Study the diagram. You can see that the triangle formed by the vectors, $\boldsymbol{F}$, and $M\boldsymbol{g}$, is similar to the triangle formed in space by L, the vertical reference line h, and the horizontal displacement, x. From the geometric law of similar triangles:

$$\frac{-F}{Mg} = \frac{x}{L}$$



This can be rewritten:

$$\frac{F}{M} = \frac{-gx}{L}$$

But F/m is a, the acceleration of the bob.

When the angle θ_0 is small, x will be small. When x is small it is very nearly the same as the arc length s, which is the actual displacement of the bob. Therefore the equation for a becomes:

$$a = \frac{-gs}{L}$$

Now s is the displacement referred to in the first equation. When these are substituted back into the first equation the minus signs and the s's cancel out leaving:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

This is the theoretical formula for the period of a simple pendulum. Notice that L and g are the only two variables left in the equation and that it can only be considered valid in the case where θ_0 is small. Therefore, the equation predicts that only the length of the string will have any effect on the period. Also, if you know the length and the period, you can calculate a value for g.

Equipment

1 100 gm mass
1 200 gm mass
1 table clamp
1 threaded rod
1 pendulum clamp
1 two meter stick
1 stopwatch
3 meters of string

Procedure

Measure lengths to the nearest 0.001 meter (the nearest mm). Measure time to the nearest 0.1 second. Note that the stopwatches give elapsed time in minutes, seconds, and hundredths of seconds; the minutes must be converted to seconds.

1. Set up the equipment. Note that there are three metal brackets attached to the pendulum holder by screws. The two brackets closest to the supporting rod are used to hold the two ends of the string. Your string should be a little longer than 3 meters. The string is attached in two places so that the pendulum will move at right angles to the holder and will not precess, or swing around in circles. Pass each end of the string under the piece of metal and tighten the screw.

2. Adjust the length of the string by loosening the screw of the bracket nearest the pole. The excess string can then be kept out of the way of the pendulum. The length of the pendulum is measured from the center of mass of the bob to the bottom of the pendulum holder, between the two brackets holding the ends of the string.

3. Keep the angle small (under 5°) and constant. Use two values for the mass of the bob. Keep L the same for each mass. Remember that the bobs are not the same length from the top to the center of mass. This means that you will have to adjust the string when you change bobs. Time 20 cycles each time.

4 Set the string to some convenient, relatively short length. Use any mass for the bob. Measure L and record it in the table. Set θ_o to a very small angle-under 5°. Record the time it takes the bob to go through 20 complete cycles. Next, set the starting angle to around 45°. Time 20 cycles and record. In each case calculate and record the time for one period.

5. Keep the mass of the bob constant. Keep the angle small and constant. Pick eight lengths for the string that cover as wide a range as possible (0.2- 1.5 m or more). Remember to measure the length from the point of suspension to the center of mass of the bob. For each length, time 20 periods, find the time for one period and then square the period.

6. Using the data from procedure 5, graph T^2 as a function of L. Draw the line that best fits your data and find the slope.

7. If you square both sides of the pendulum equation you can see that:

$$\frac{T^2}{L} = \frac{4\pi^2}{g}$$

   The left side of the equation is the slope you found in procedure 6. Use that slope to compute the value for g. Show all work. Find your percent error.