Phys1111: Static Friction

Learning Objectives:

I. Applying Newton's second law to investigate the model of static friction.
II. Understanding the factors that do and don't affect the coefficient of static friction.

Apparatus: Wooden box, aluminum track, wooden board, meter stick, and protractor, 500g mass

Part A: Predict and Test

1. Predict what will happen if the block is placed on the aluminum track and the angle of the track is slowly increased little by little.

2. Now place the block on the track and slowly increase the angle. What do you observe?

3. Why did it behave that way? Why did it slide at a large angle but not at a small angle?
Part B: Forces on a Block at Rest on an Inclined Surface

4. Identify and label the system in this situation (the system is whatever object for which we are examining the motion and identifying the forces that are acting upon). Then select your coordinate system. Then draw free body diagrams for the case where the block is not moving. Label the forces with symbols such as W for weight, n for Normal force, T for tension, f_s for static friction, f_k for kinetic friction, D for drag, etc. depending on which forces are acting in this situation.

   **When the block is at rest**

   Identify the system _____________________________

   Draw the free-body diagram below

   For inclined surfaces we use a tilted coordinate system. Indicate the directions of your x and y axes on the drawing below:

   ![Diagram of tilted coordinate system]

   5. Is your free body diagram consistent with the block being in equilibrium? Explain.

   6. What is the magnitude of the acceleration of the block in the case where the block is not moving?

      \[ a = \text{___________} \]

   7. What are the x- and y-components of the acceleration in this case?

      \[ a_x = \text{___________} \]

      \[ a_y = \text{___________} \]

   8. Write the x-components of each of the forces on your free-body diagram symbolically below. For example, is the x-component of the weight force equal to +mg or –mg or +mg \( \sin \theta \) or –mg \( \cos \theta \) or something else? Notice that the x-components are not the same as the horizontal components since our coordinate system is tilted. Also notice that some forces may have an x-component of zero if they are only in the y-direction.

      Check with your TA to see if the components are correct before going on. TA initials ____________
9. Now we will apply Newton’s second law in the \( x \)-direction to the situation with \textit{the block not sliding}. To do this, use the \( x \)-components for each force from Question 8 and write them on the left side of the equation. The sigma in the equation tells us we are making a sum, so we need to add up all of the \( F_x \) contributions to get the sum or net force in the \( x \)-direction. Also use the acceleration in the \( x \) direction from Question 7 and substitute in on the right side of the equation.

\[
\Sigma F_x = ma_x
\]

\[
\text{_____________________________________} = \text{___________________________________________}
\]

10. Rearrange your equation in Question 9 to solve for the static frictional force, \( f_s \), needed to keep the block from sliding.

\[
f_s = \text{__________________________}
\]

11. The equations you wrote in Questions 9 and 10 should be true even when the angle is changed as long as the block does not slide. Use your equations to determine what happens to the static frictional force on the block as the angle is increased but the block is still not moving. Is \( f_s \) increasing, decreasing, or staying the same as the angle is increased (but still small enough that the block doesn’t slide)?

12. Write the \( y \)-components of each of the forces on your free-body diagram symbolically below. For example, is the \( y \)-component of the normal force equal to +\( n \) or –\( n \) or +\( n \sin \theta \) or –\( n \cos \theta \) or something else? Notice that the \( y \)-components are not the same as the vertical components since our coordinate system is tilted. Also notice that some forces may have a \( y \)-component of zero if they are only in the \( x \)-direction.

Check with your TA to see if the components are correct before going on. TA initials __________

13. Now apply Newton’s second law in the \( y \)-direction just as you did for the \( x \)-direction in Question 9. Use the \( y \)-components for each force from Question 12 and write them on the left side of the equation. Use the acceleration in the \( y \) direction from Question 7 and substitute in on the right side of the equation.

\[
\Sigma F_y = ma_y
\]

\[
\text{_____________________________________} = \text{___________________________________________}
\]

14. Rearrange your equation in Question 13 to solve for the normal force, \( n \).

\[
n = \text{__________________________}
\]
15. The equations you wrote in Questions 13 and 14 should be true even when the angle is changed since the block never moves in the y-direction. Use your equations to determine what happens to the normal force on the block as the angle is increased but the block is still not moving. Is $n$ increasing, decreasing, or staying the same as the angle is increased (but still small enough that the block doesn’t slide)?

16. You learned in class or in the textbook that the static frictional force has a maximum value, $f_{s,max}$, which is equal to the product of the coefficient of static friction ($\mu_s$) and the normal force, that is, $f_{s,max} = \mu_s n$. The coefficient of static friction is a constant in any given situation and characterizes the interaction of the two surfaces involved, in this case the interaction of the block surface with the track surface. So the coefficient of static friction was fixed by our choice of the two materials (wooden block and aluminum track).

Another way to say this is to say that the actual static friction force can never exceed this maximum value so that

$$f_s \leq \mu_s n.$$  

Look back at Question 11: As the angle is increased, is $f_s$ increasing, decreasing, or staying the same?

Look back at Question 15: As the angle is increased, is $n$ increasing, decreasing, or staying the same?

17. At an angle of zero, the static friction force is zero since no friction force is needed to keep the block from sliding (check your equation in Question 10 to see if that makes sense). At an angle of zero, the normal force is at its largest (again check your equation in Question 14). So the inequality above is certainly true at an angle of zero. But as the angle is increased, the static friction force needed increases ($f_s$) and the normal force ($n$) decreases. At some angle, the two sides of the equation will be equal (we'll call that the critical angle, $\theta_{max}$). If we increase the angle to be larger than the critical angle, the equation is no longer true! Something has to change. What happens to the static friction force when we go above the critical angle?

18. At the critical angle the actual static friction force equals the maximum static friction force.

$$f_s = f_{s,max} = \mu_s n$$

At that critical angle, we could put our expressions for $f_s$ and $n$ found in Questions 10 and 14 into the equality above. This is done in your textbook in Example 5.13. When we do that, there is an $mg$ on both sides that cancels out and the expression gets much simpler. We can then rearrange to find out that

$$\mu_s = \tan \theta$$

What are the units of $\mu_s$? ____________________________
Part C: Measuring the coefficient of static friction

19. We are going to examine following effects on the coefficient of static friction.
   a. Effect of mass of the wooden box
   b. Effect of the type of material pair in contact
      • wood on aluminum: wooden box on aluminum track
      • wood on wood: wooden box on wooden board

   We use two different inclines, one made from aluminum and the other made from wood. Design an experiment to measure the static frictional coefficient between the incline (either for the aluminum track or the wooden board) and the block. Use the wooden side of the box. After testing out your procedure describe it below. You can increase the mass of the block by putting an extra mass on the wooden block.

20. Data and calculation: For each incline and with and without added mass to the wooden box, repeat your procedure five different times placing the block at different starting positions on the incline. Then calculate the average of the coefficient for each of the two materials. Note: If you use the tangent function in Excel you must change the angles from degrees to radians first.

<table>
<thead>
<tr>
<th>Select five different positions on the track from the bottom</th>
<th>Wooden box on aluminum track</th>
<th>Wooden box on wooden board</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Angle</td>
<td>Calculated coefficient of static friction</td>
</tr>
<tr>
<td></td>
<td>Box only</td>
<td>Box+ mass</td>
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</tr>
<tr>
<td>1</td>
<td>Avg. $\mu_s =$</td>
<td>Avg. $\mu_s =$</td>
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</tbody>
</table>

21. Compare the values of $\mu_s$ calculated for material pair “wood on aluminum” for with different masses of the box. Do they agree? Explain.
22. Compare the values of $\mu_s$ calculated for material pair “wood on wood” for with different masses of the box. Do they agree within our experimental uncertainties? That is to say, do the ranges overlap? Explain.

23. Compare the values of $\mu_s$ calculated for material pair “wood on wood” and “wood on aluminum.” Are the values $\mu_s$ same or different? Explain how you reached the conclusion.

Part D: Conclusions

24. What does the coefficient of static friction NOT depend on? That is, what can we change about the situation that makes the coefficient stay the same?

25. What determines the coefficient of static friction in a particular situation? That is, what can we change about the situation that would make the coefficient change? Give an example.

26. How is the static frictional force related to the coefficient of static friction?

27. Can the static frictional force change even though the coefficient of static friction stays the same? If so, give an example from this experiment.