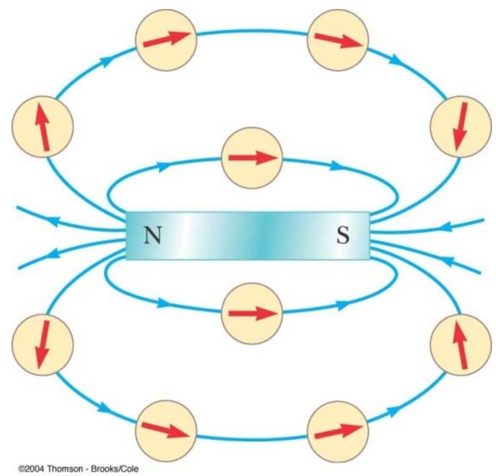
Chapter 32

Magnetic Fields

Magnetic Poles

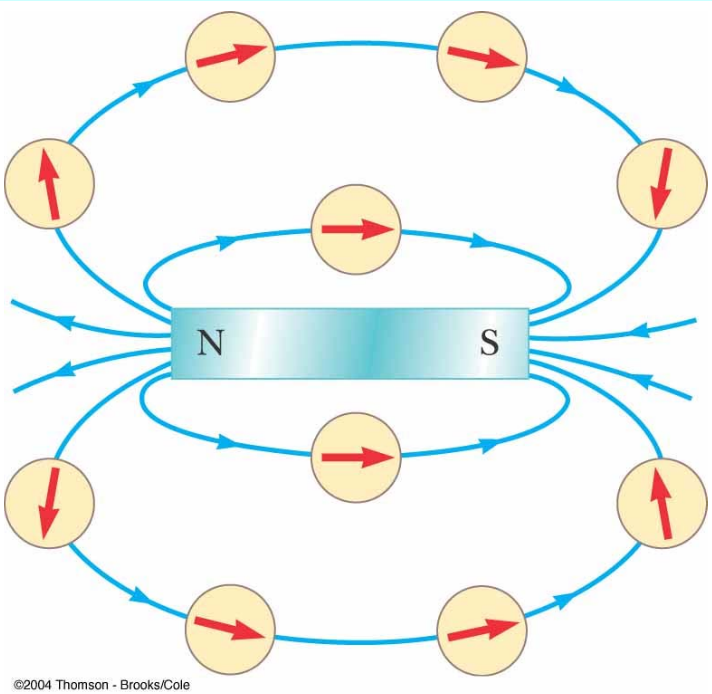
- Every magnet, regardless of its shape, has two poles
  - Called ***north*** and ***south*** poles
  - Poles exert forces on one another
    - **Similar to the way electric charges exert forces on each other**
    - ***Like poles repel each other***
      - N-N or S-S
    - ***Unlike poles attract each other***
      - N-S



- The force between two poles varies as the **inverse square** of the distance between them
- A single magnetic pole has never been isolated

Magnetic Fields

- Magnetic Field is Created by the Magnets
- A vector quantity, Symbolized by **B**
- Direction is given by the direction a *north pole* of a compass needle points in that location
- ***Magnetic field lines*** can be used to show how the field lines, as traced out by a compass, would look



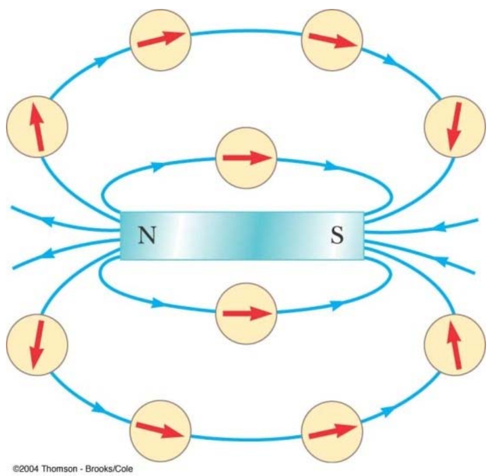
Chapter 32

Sources of the Magnetic Field

Sources of Magnetic Field

Real source of Magnetic Field –

- moving electric charges or
- electric current

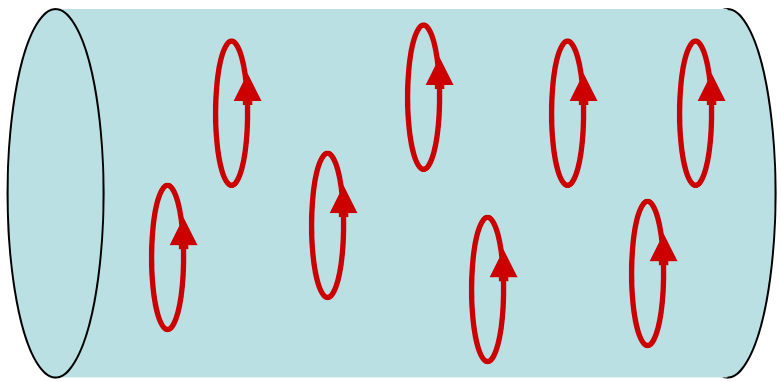


Inside every magnet – electric currents

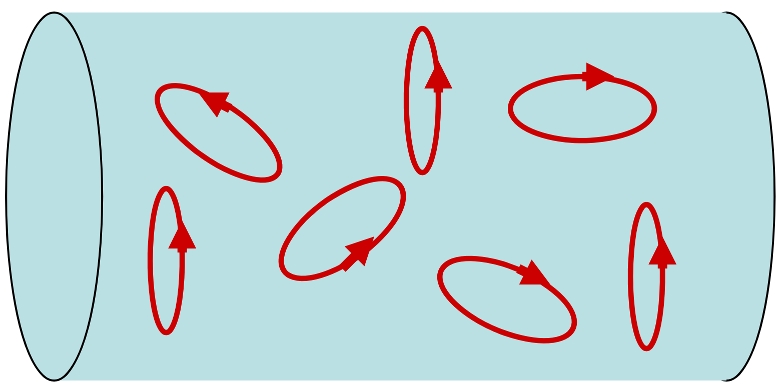
Sources of Magnetic Field

Inside every magnet – electric currents

S



N

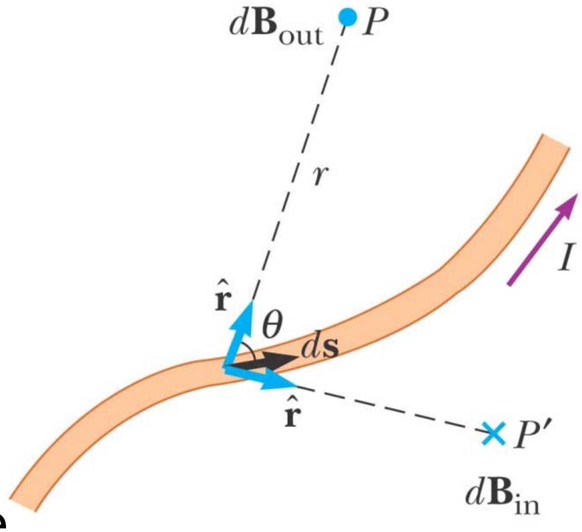


no magnetic field

Biot-Savart Law

- The magnetic field is $d\mathbf{B}$ at some point P
- The length element is $d\mathbf{s}$
- The wire is carrying a steady current of I

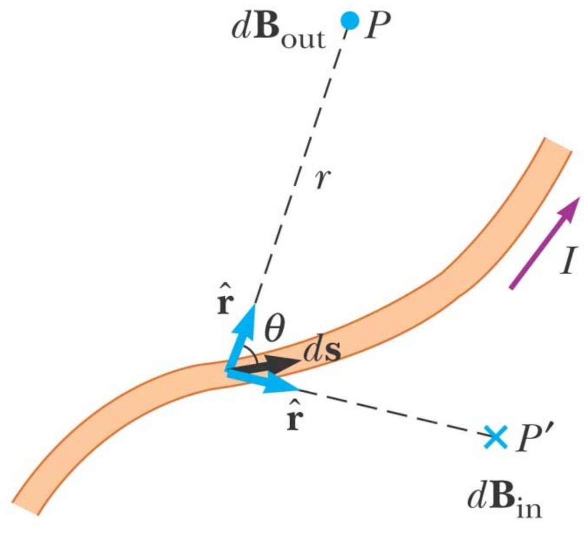
$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{r}}{r^2}$$



- The magnitude of dB is proportional to the current and to the magnitude ds of the length element ds
- The magnitude of dB is proportional to sin θ, where θ is the angle between the vectors ds and $\hat{r}$
- The vector dB is perpendicular to both ds and to $\hat{r}$ the unit vector directed from ds toward P
- The magnitude of dB is inversely proportional to r^2, where r is the distance from ds to P

Biot-Savart Law

$$d\mathbf{B} = \frac{\mu_o}{4\pi} \frac{I\, d\mathbf{s} \times \hat{r}}{r^2}$$



- The constant μ_o is called the **permeability of free space**

➢ $\mu_o = 4\pi \times 10^{-7}$ **T· m / A**

Magnetic Field

- The SI unit of magnetic field is **tesla** (T)

$$T = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

Table 29.1

Some Approximate Magnetic Field Magnitudes	
Source of Field	**Field Magnitude (T)**
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain (due to nerve impulses)	10^{-13}

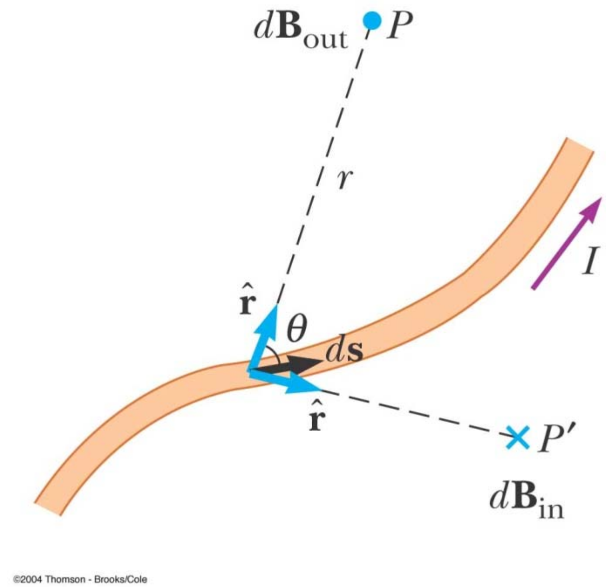
Biot-Savart Law: Total Magnetic Field

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{r}}{r^2}$$

- $d\mathbf{B}$ is the field created by the current in the length segment $d\mathbf{s}$
- To find the total field, sum up the contributions from all the current elements $I d\mathbf{s}$

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{s} \times \hat{r}}{r^2}$$

The integral is over the entire current distribution



Magnetic Field compared to Electric Field

$$d\mathbf{E} = k_e \frac{dq}{r^2}\hat{r} \qquad\qquad d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{r}}{r^2}$$

Distance

- The magnitude of the magnetic field varies as the inverse square of the distance from the source
- The electric field due to a point charge also varies as the inverse square of the distance from the charge

Direction

- The electric field created by a point charge is radial in direction
- The magnetic field created by a current element is perpendicular to both the length element $d\mathbf{s}$ and the unit vector $\hat{r}$

Source

- An electric field is established by an isolated electric charge
- The current element that produces a magnetic field must be part of an extended current distribution

Magnetic Field of a Long Straight Conductor

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{r}}{r^2}$$

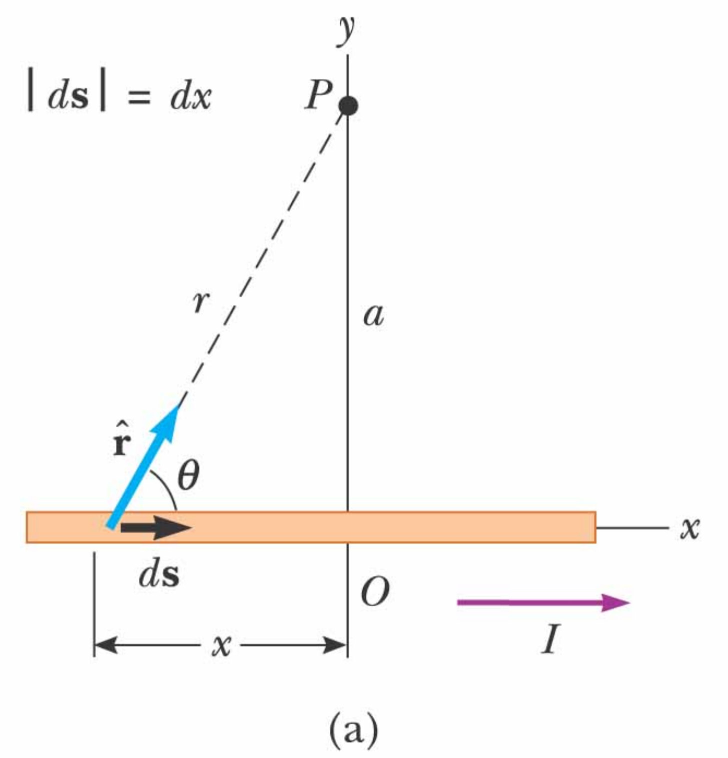
- The thin, straight wire is carrying a constant current

$$d\mathbf{s}\times\hat{r} = (dx\ \sin\ \theta)\hat{k}$$

- Integrating over all the current elements gives

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\sin\ \theta\, d\,\theta$$

$$= \frac{\mu_0 I}{4\pi a}(\cos\ \theta_1 - \cos\ \theta_2)$$



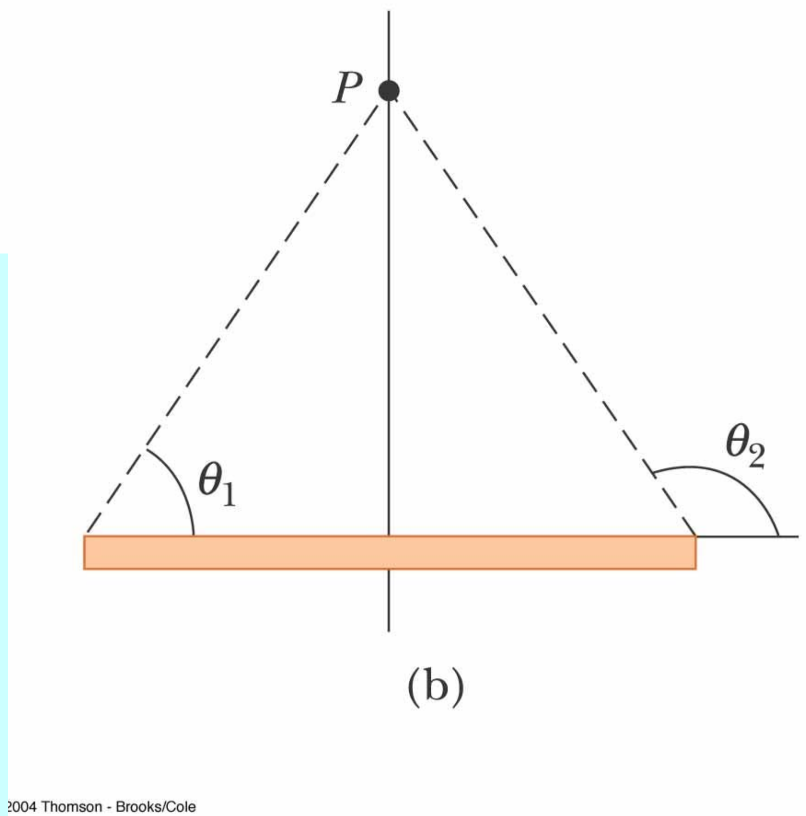
$$x = \frac{a}{\tan\theta} \qquad r = \frac{a}{\sin\theta}$$

Magnetic Field of a Long Straight Conductor

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

- If the conductor is an infinitely long, straight wire, $\theta_1 = 0$ and $\theta_2 = \pi$
- The field becomes

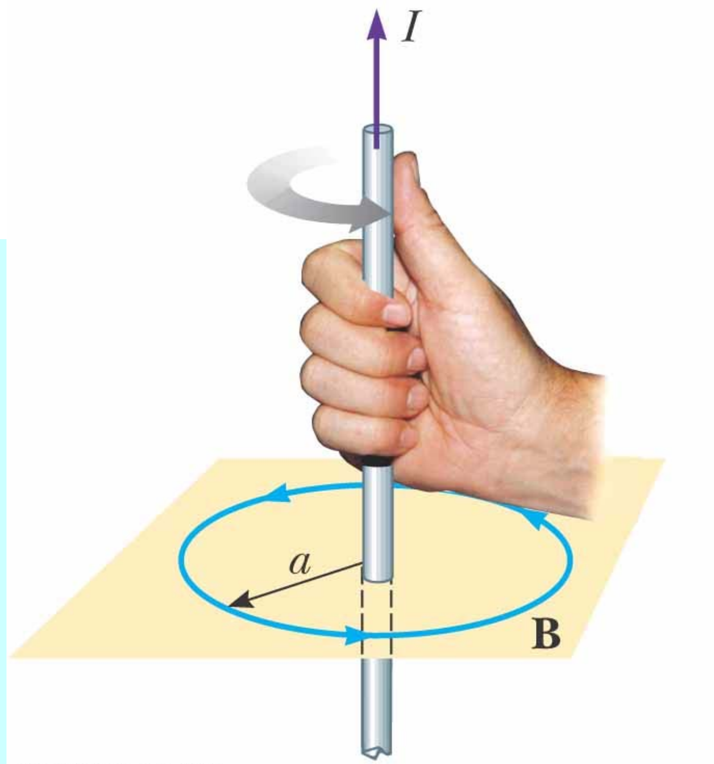
$$B = \frac{\mu_0 I}{2\pi a}$$



Magnetic Field of a Long Straight Conductor

$$B = \frac{\mu_0 I}{2\pi a}$$

- The magnetic field lines are circles concentric with the wire
- The field lines lie in planes perpendicular to to wire
- The magnitude of **B** is constant on any circle of radius ***a***



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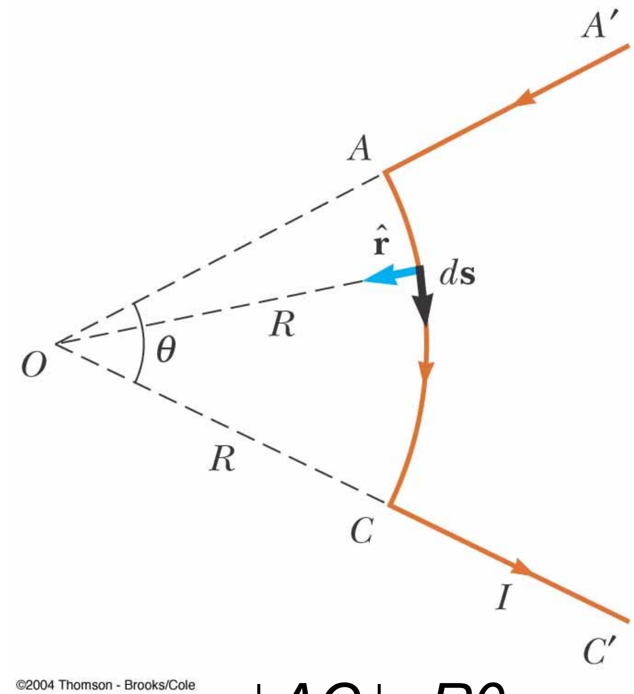
Magnetic Field for a Curved Wire Segment

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{r}}{R^2}$$



$$|AC| = R\theta$$

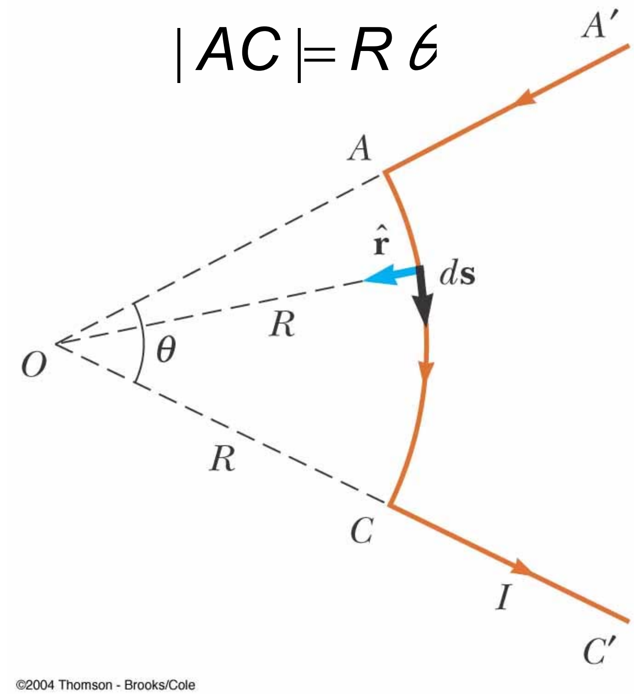
- Find the field at point *O* due to the wire segment
- ***I*** and ***R*** are constants

$$B = \frac{\mu_0 I}{4\pi R^2} |AC| = \frac{\mu_0 I}{4\pi R} \theta$$

  - ***θ*** will be in radians

Magnetic Field for a Curved Wire Segment

$$|AC| = R\theta$$

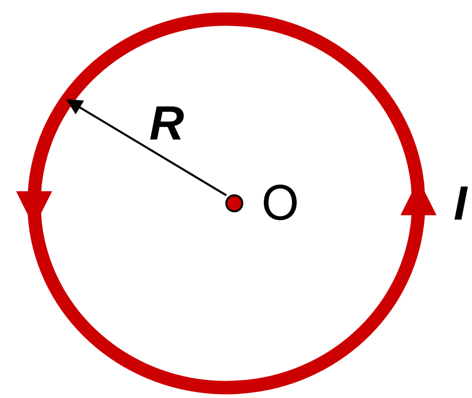


$$B = \frac{\mu_0 I}{4\pi R^2}|AC| = \frac{\mu_0 I}{4\pi R}\theta$$

- Consider the previous result, with $\theta = 2\pi$

$$B = \frac{\mu_0 I}{4\pi R}\theta = \frac{\mu_0 I}{4\pi R}2\pi = \frac{\mu_0 I}{2R}$$

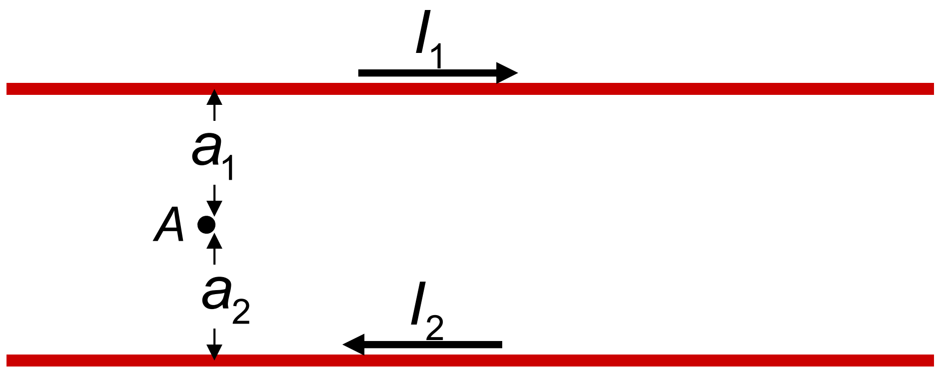
- This is the field at the *center* of the loop



Example 1

Determine the magnetic field at point *A*.

$$B_1 = \frac{\mu_0 I_1}{2\pi a_1}$$



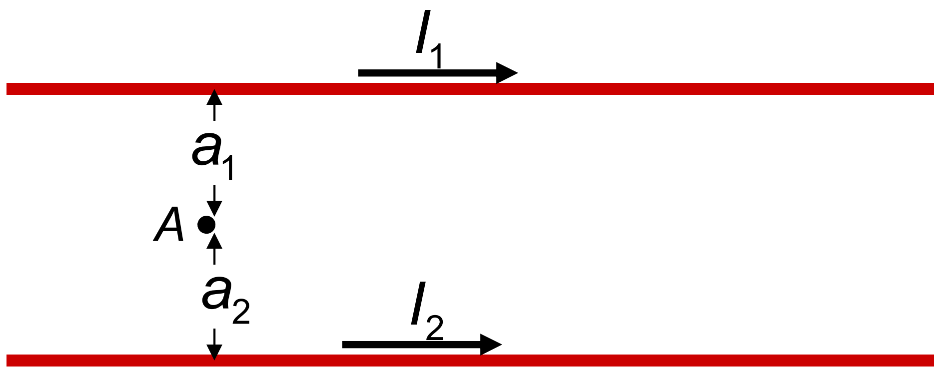
$$B_2 = \frac{\mu_0 I_2}{2\pi a_2}$$

$$B = B_1 + B_2 = \frac{\mu_0 I_1}{2\pi a_1} + \frac{\mu_0 I_2}{2\pi a_2}$$

Example 2

Determine the magnetic field at point *A*.

$$B_1 = \frac{\mu_0 I_1}{2\pi a_1}$$



$$B_2 = \frac{\mu_0 I_2}{2\pi a_2}$$

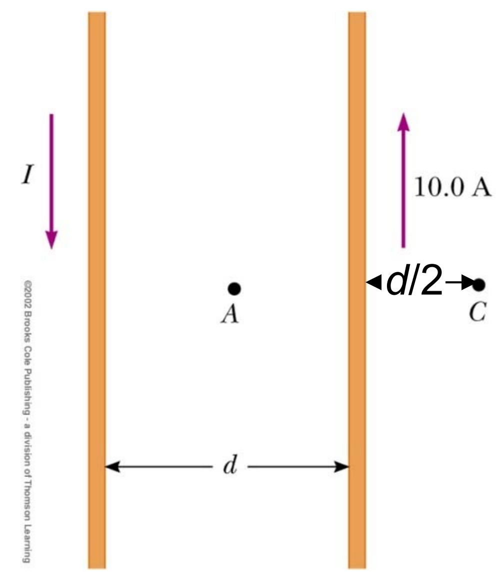
$$B = B_1 - B_2 = \frac{\mu_0 I_1}{2\pi a_1} - \frac{\mu_0 I_2}{2\pi a_2}$$

Example 3

Two parallel conductors carry current in opposite directions. One conductor carries a current of 10.0 A. Point *A* is at the midpoint between the wires, and point *C* is a distance *d*/2 to the right of the 10.0-A current. If *d* = 18.0 cm and *I* is adjusted so that the magnetic field at *C* is zero, find (a) the value of the current *I* and (b) the value of the magnetic field at *A*.

$$B_A = B_{1,A} + B_{2,A} = \frac{\mu_0 I_1}{2\pi a_1} + \frac{\mu_0 I_2}{2\pi a_2} = \frac{\mu_0 I}{\pi d} + \frac{\mu_0 I_0}{\pi d}$$

$$a_1 = d/2 \qquad a_2 = d/2$$



$$B_C = B_{1,C} - B_{2,C} = \frac{\mu_0 I_1}{2\pi a_1} - \frac{\mu_0 I_2}{2\pi a_2} = \frac{\mu_0 I}{3\pi d} - \frac{\mu_0 I_0}{\pi d}$$

$$a_1 = 3d/2 \qquad a_2 = d/2$$

$$B_C = 0 \qquad \frac{\mu_0 I}{3\pi d} = \frac{\mu_0 I_0}{\pi d} \qquad I = 3 I_0 = 30A$$

Example 3

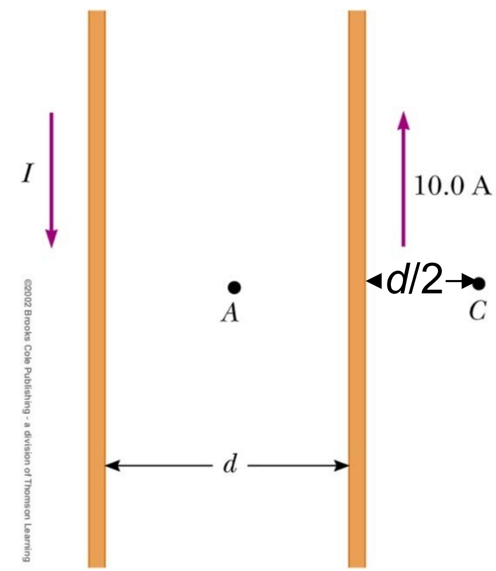
Two parallel conductors carry current in opposite directions. One conductor carries a current of 10.0 A. Point *A* is at the midpoint between the wires, and point *C* is a distance *d*/2 to the right of the 10.0-A current. If *d* = 18.0 cm and *I* is adjusted so that the magnetic field at *C* is zero, find (a) the value of the current *I* and (b) the value of the magnetic field at *A*.

$$I = 3I_0 = 30A$$

$$B_A = B_{1,A} + B_{2,A} = \frac{\mu_0 I_1}{2\pi a_1} + \frac{\mu_0 I_2}{2\pi a_2} = \frac{\mu_0 I}{\pi d} + \frac{\mu_0 I_0}{\pi d}$$

$$a_1 = d/2 \qquad a_2 = d/2$$

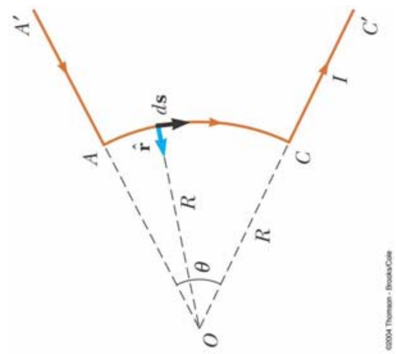
$$B_A = \frac{\mu_0 3I_0}{\pi d} + \frac{\mu_0 I_0}{\pi d} = 4\frac{\mu_0 I_0}{\pi d} = 88.9\ \mu T$$

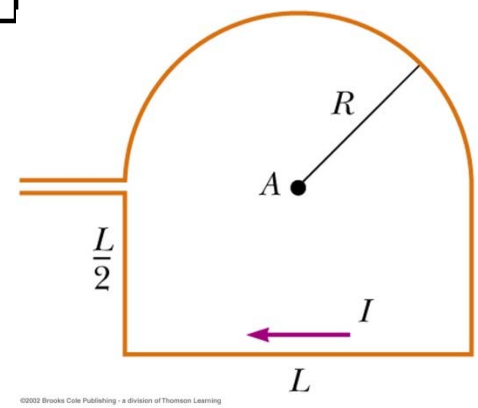


Example 4

The loop carries a current *I*. Determine the magnetic field at point *A* in terms of *I*, *R*, and *L*.

$$B_1 = \frac{\mu_0 I}{4\pi R}\,\theta$$

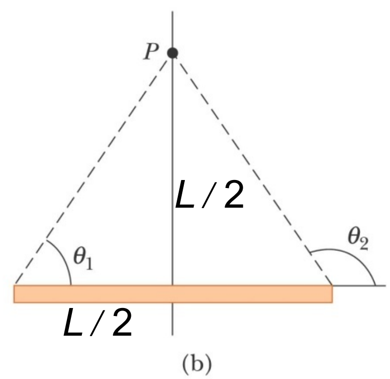




$$\theta = \pi$$

$$B_1 = \frac{\mu_0 I}{4R}$$

$$B_2 = \frac{\mu_0 I}{2\pi L}\left(\cos\theta_1 - \cos\theta_2\right)$$



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$$\theta_1 = \pi/4 \qquad \theta_2 = \pi - \pi/4$$

$$B_2 = \frac{\mu_0 I}{2\pi L}\sqrt{2}$$

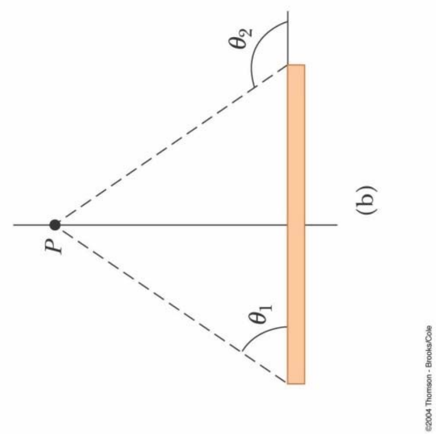
Example 4

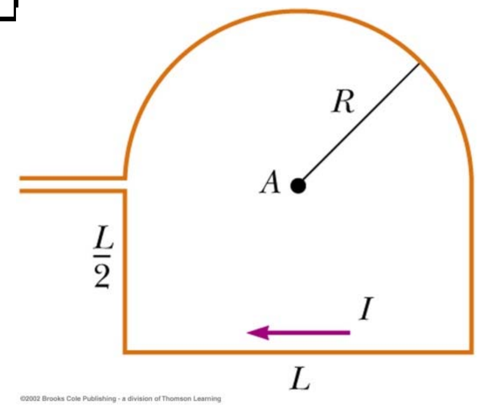
The loop carries a current *I*. Determine the magnetic field at point *A* in terms of *I*, *R*, and *L*.

$$B_3 = \frac{\mu_0 I}{2\pi L}\left(\cos\theta_1 - \cos\theta_2\right)$$

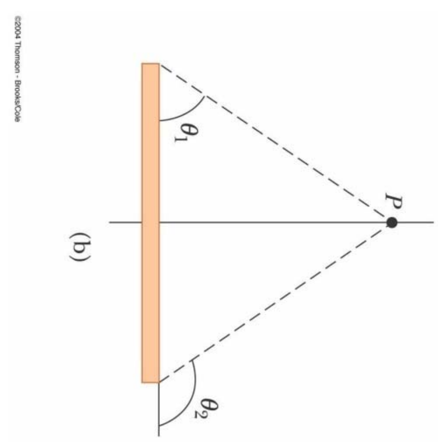
$$\theta_1 = \pi/4 \qquad \theta_2 = \pi/2$$

$$B_3 = \frac{\mu_0 I}{2\pi L}\frac{1}{\sqrt{2}}$$



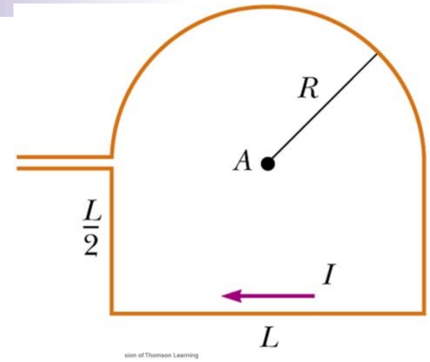


$$B_4 = \frac{\mu_0 I}{2\pi L}\frac{1}{\sqrt{2}}$$

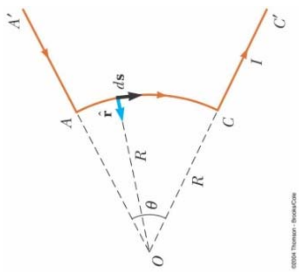


Example 4

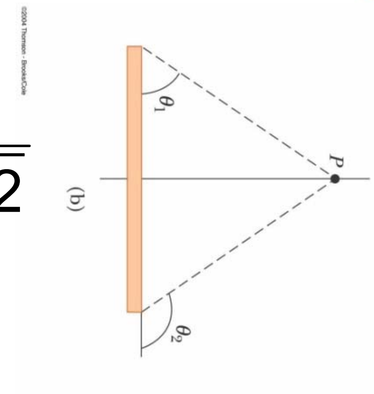
The loop carries a current *I*. Determine the magnetic field at point *A* in terms of *I*, *R*, and *L*.



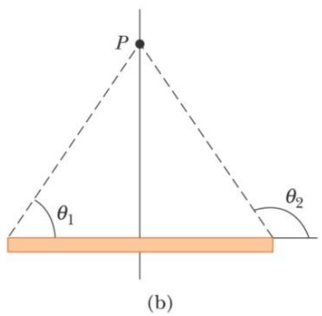
$$B_1 = \frac{\mu_0 I}{4R} = \frac{\mu_0 I}{2L}$$



$$B_4 = \frac{\mu_0 I}{2\pi L}\frac{1}{\sqrt{2}}$$



$$B_2 = \frac{\mu_0 I}{2\pi L}\sqrt{2}$$

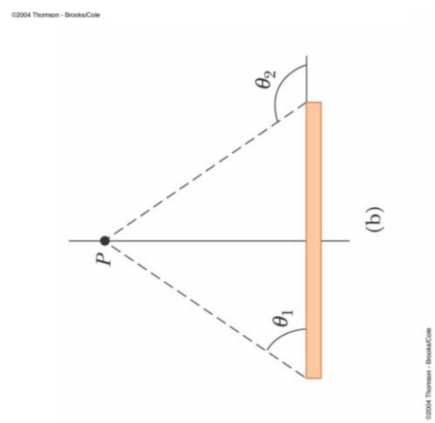


$$B = B_1 + B_2 + B_3 + B_4 =$$

$$= \frac{\mu_0 I}{2L} + \frac{\mu_0 I}{2\pi L}\sqrt{2} + \frac{\mu_0 I}{2\pi L}\frac{1}{\sqrt{2}} + \frac{\mu_0 I}{2\pi L}\frac{1}{\sqrt{2}} =$$

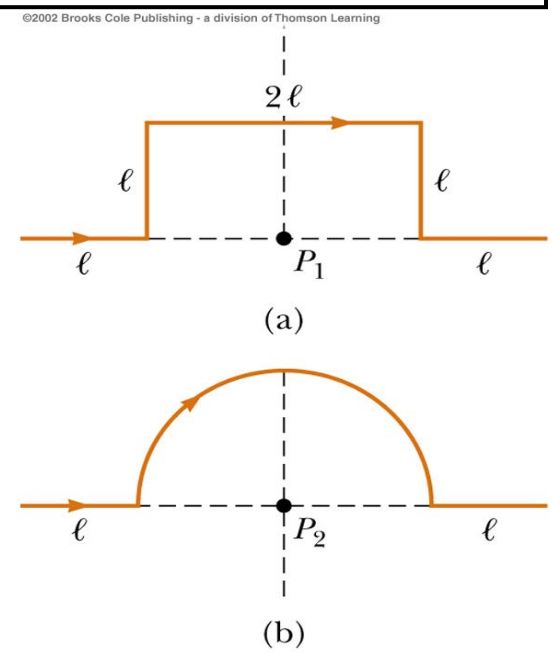
$$= \frac{\mu_0 I}{2L}\left(1 + 2\sqrt{2}\right)$$

$$B_3 = \frac{\mu_0 I}{2\pi L}\frac{1}{\sqrt{2}}$$

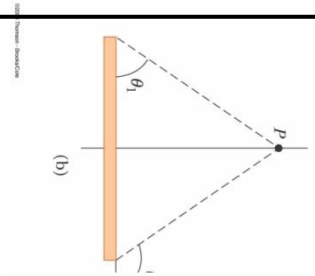


Example 5

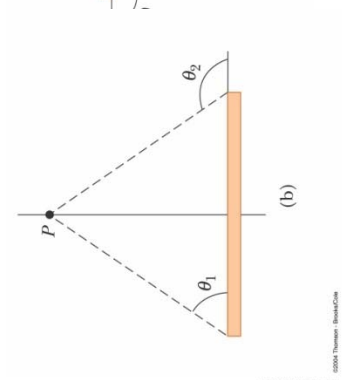
A wire is bent into the shape shown in Fig.(a), and the magnetic field is measured at P_1 when the current in the wire is I. The same wire is then formed into the shape shown in Fig. (b), and the magnetic field is measured at point P_2 when the current is again I. If the total length of wire is the same in each case, what is the ratio of B_1/B_2?



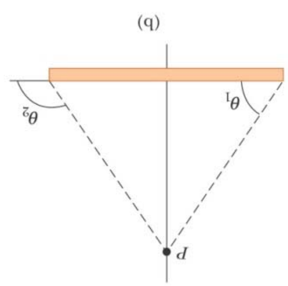
$$B_{1,1} = \frac{\mu_0 I}{4\pi l}\frac{1}{\sqrt{2}}$$



$$B_{1,2} = \frac{\mu_0 I}{4\pi l}\frac{1}{\sqrt{2}}$$



$$B_{1,3} = \frac{\mu_0 I}{4\pi l}\sqrt{2}$$

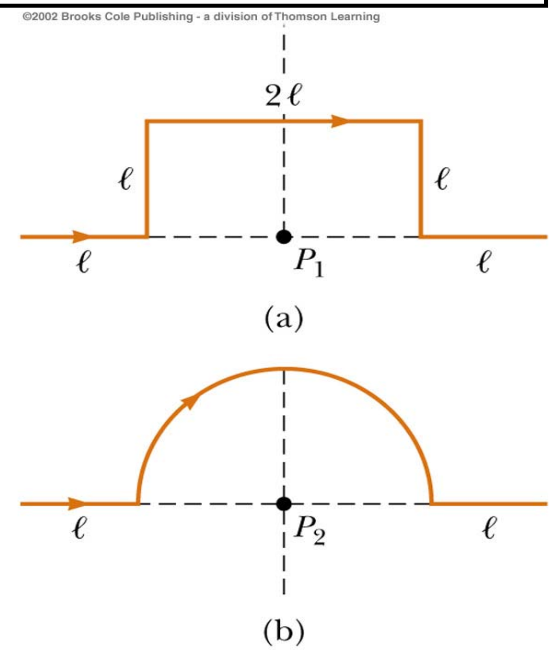


$$B_1 = B_{1,1} + B_{1,2} + B_{1,3} =$$

$$= \frac{\mu_0 I}{4\pi l}\frac{2}{\sqrt{2}} + \frac{\mu_0 I}{4\pi l}\sqrt{2} = \frac{\mu_0 I}{2\pi l}\sqrt{2}$$

Example 5

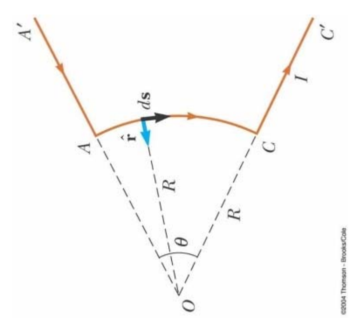
A wire is bent into the shape shown in Fig.(a), and the magnetic field is measured at P_1 when the current in the wire is I. The same wire is then formed into the shape shown in Fig. (b), and the magnetic field is measured at point P_2 when the current is again I. If the total length of wire is the same in each case, what is the ratio of B_1/B_2?



$$B_1 = B_{1,1} + B_{1,2} + B_{1,3} =$$

$$= \frac{\mu_0 I}{4\pi l}\frac{2}{\sqrt{2}} + \frac{\mu_0 I}{4\pi l}\sqrt{2} = \frac{\mu_0 I}{2\pi l}\sqrt{2}$$

$$B_2 = \frac{\mu_0 I}{4R}$$



$$\pi R = 4l$$

$$B_2 = \frac{\mu_0 I}{16 l}\pi$$

$$\frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2\pi l}\sqrt{2}}{\frac{\mu_0 I}{16 l}\pi} = \frac{8\sqrt{2}}{\pi^2}$$

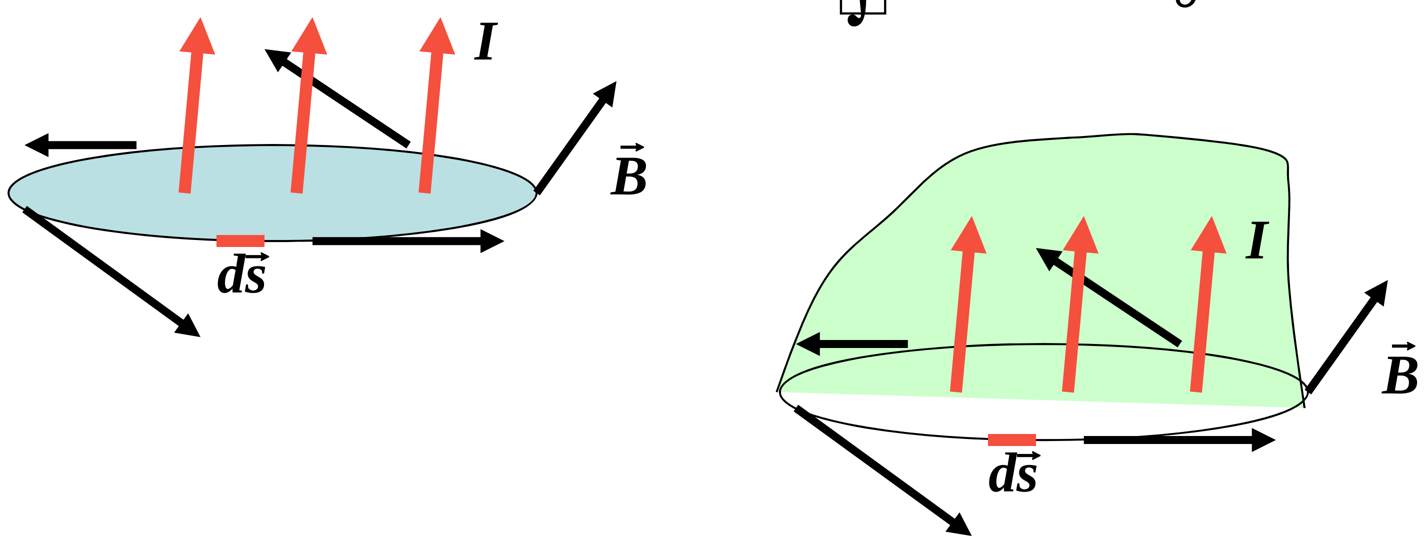
Chapter 32

Ampere’s Law

Ampere's law

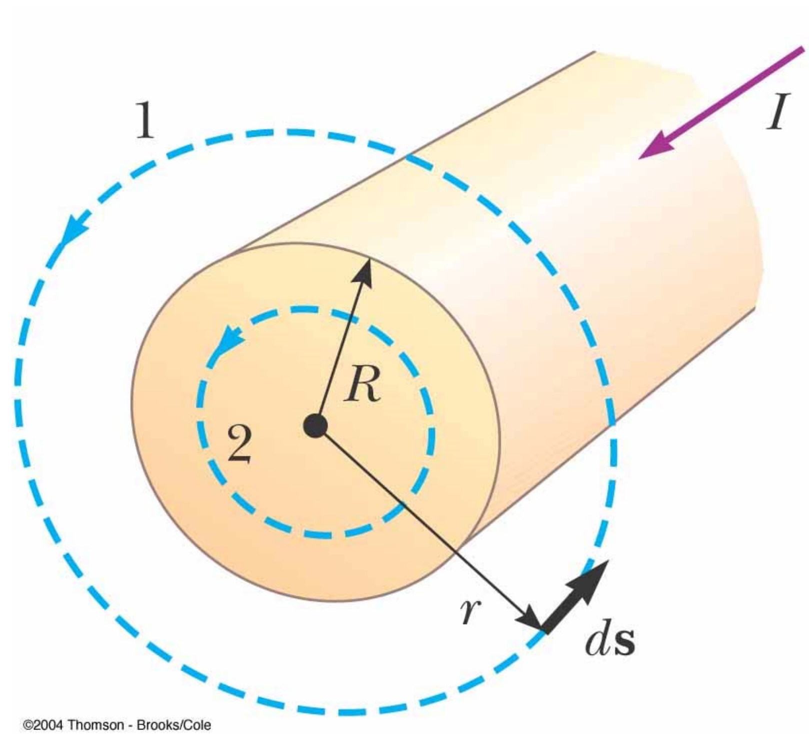
- **Ampere's law** states that the line integral of **B**d**s** around any closed path equals $\mu_0 I$ where ***I*** is the total steady current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$



Field due to a long Straight Wire

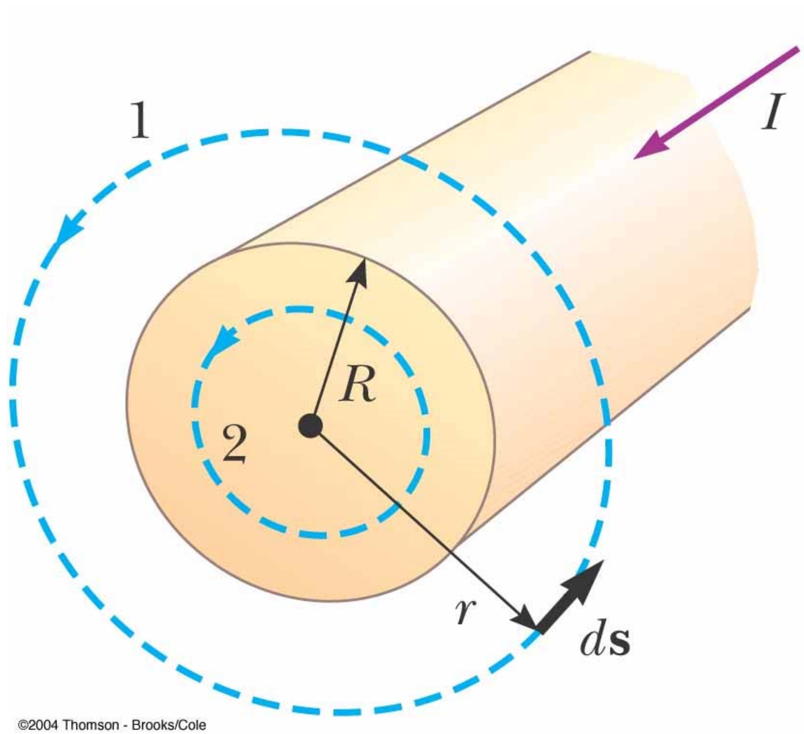
- Need to calculate the magnetic field at a distance *r* from the center of a wire carrying a steady current I
- The current is uniformly distributed through the cross section of the wire



$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

Field due to a long Straight Wire

- The magnitude of magnetic field depends only on distance ***r*** from the center of a wire.



- Outside of the wire, ***r > R***

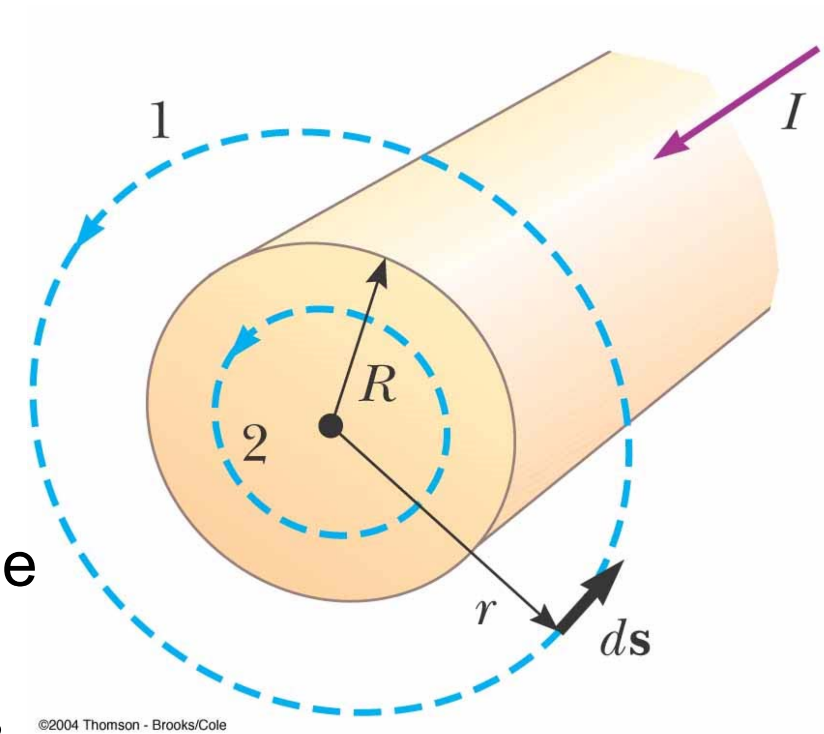
$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

Field due to a long Straight Wire

- The magnitude of magnetic field depends only on distance ***r*** from the center of a wire.



- Inside the wire, we need I', the current inside the amperian circle

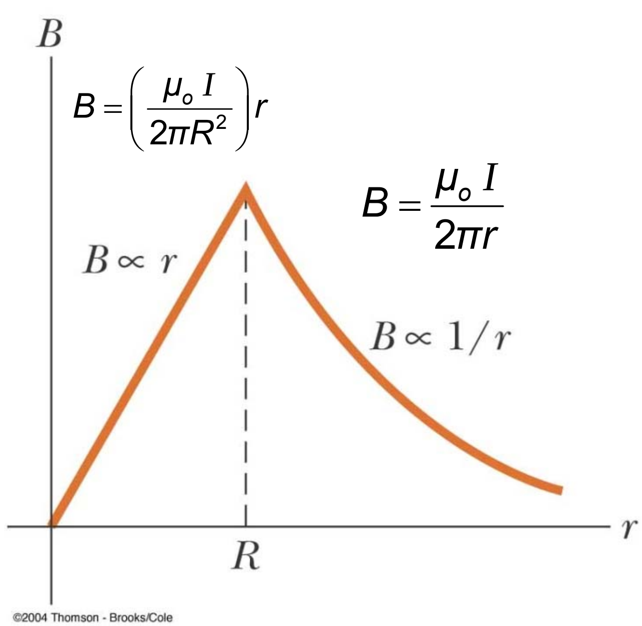
$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I' \quad \rightarrow \quad I' = \frac{r^2}{R^2} I$$

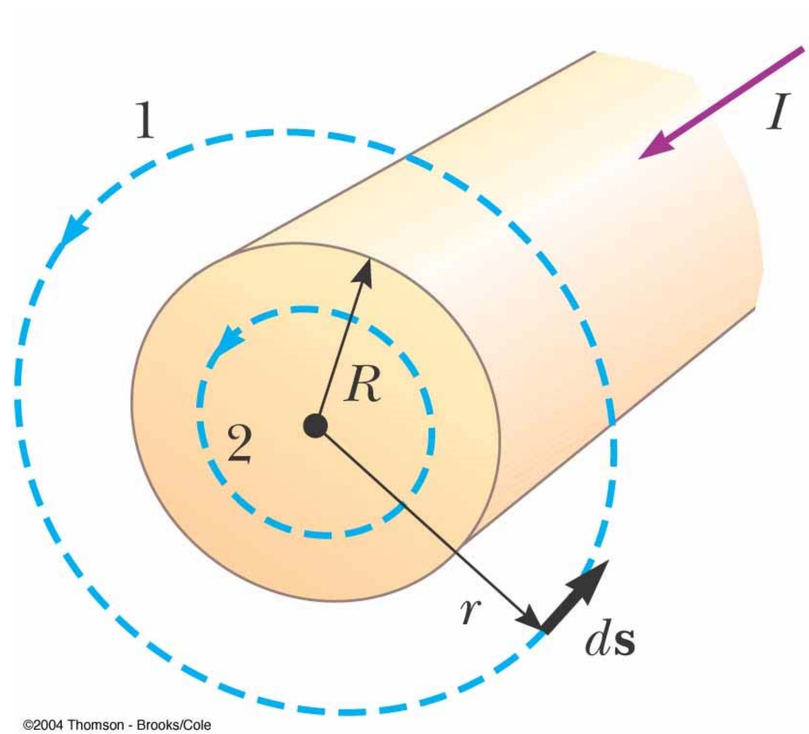
$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r$$

$$\oint \mathbf{B} \cdot ds = \mu_0 I$$

Field due to a long Straight Wire

- The field is proportional to ***r*** inside the wire
- The field varies as **1/*r*** outside the wire
- Both equations are equal at ***r* = *R***

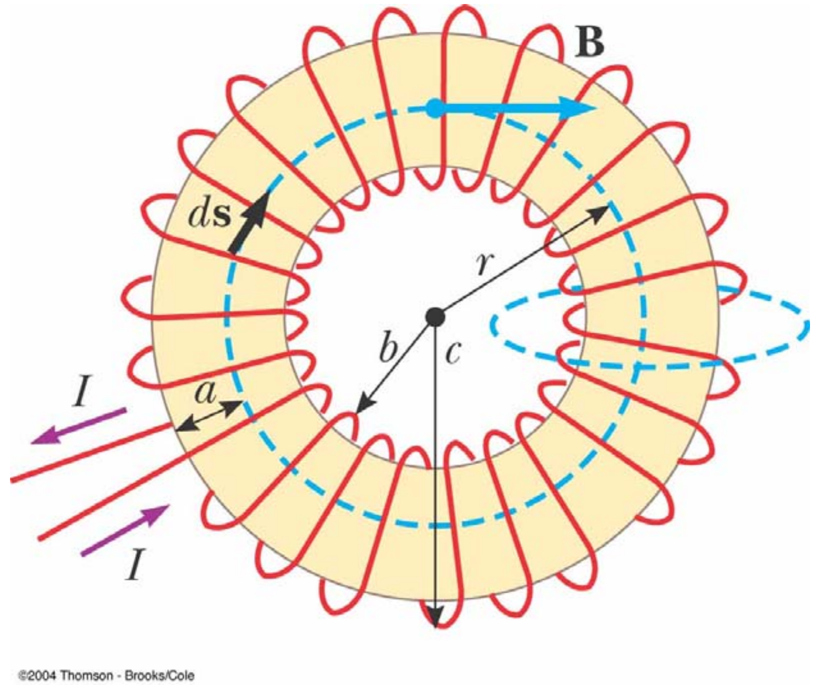




$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

Magnetic Field of a Toroid

- Find the field at a point at distance ***r*** from the center of the toroid
- The toroid has ***N*** turns of wire

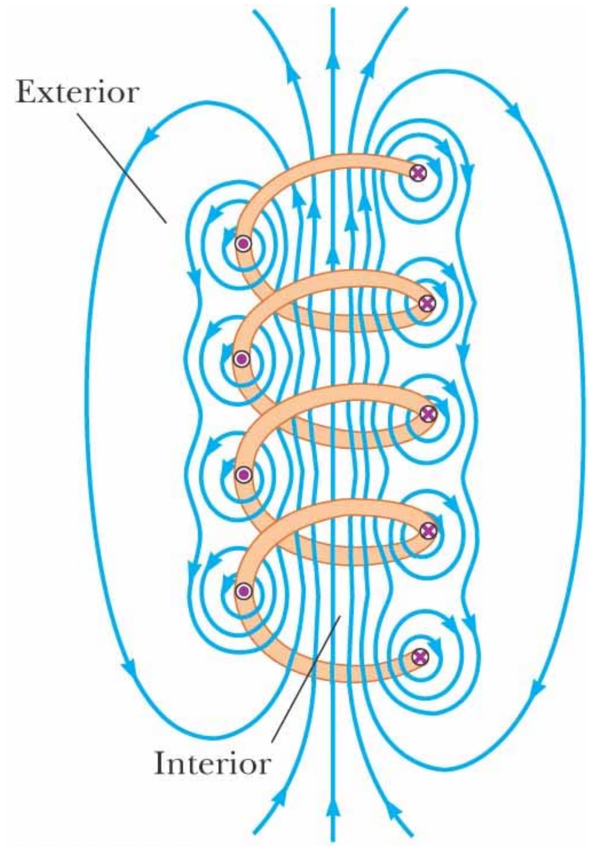


$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

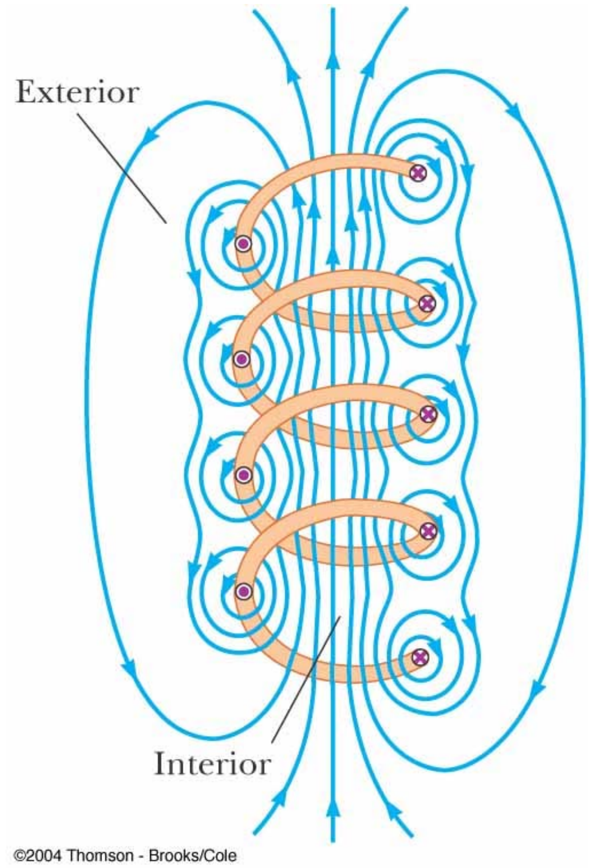
Magnetic Field of a Solenoid

- A **solenoid** is a long wire wound in the form of a helix
- A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire



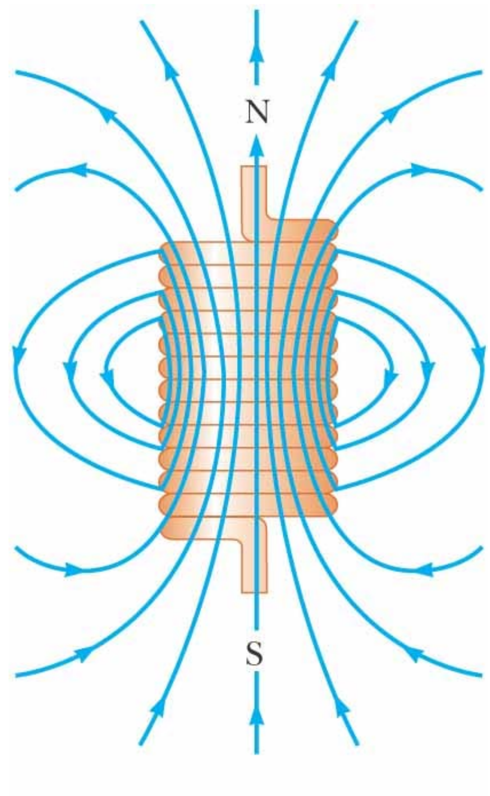
Magnetic Field of a Solenoid

- The field lines in the interior are
  - approximately parallel to each other
  - uniformly distributed
  - close together
- This indicates the field is strong and almost uniform



Magnetic Field of a Solenoid

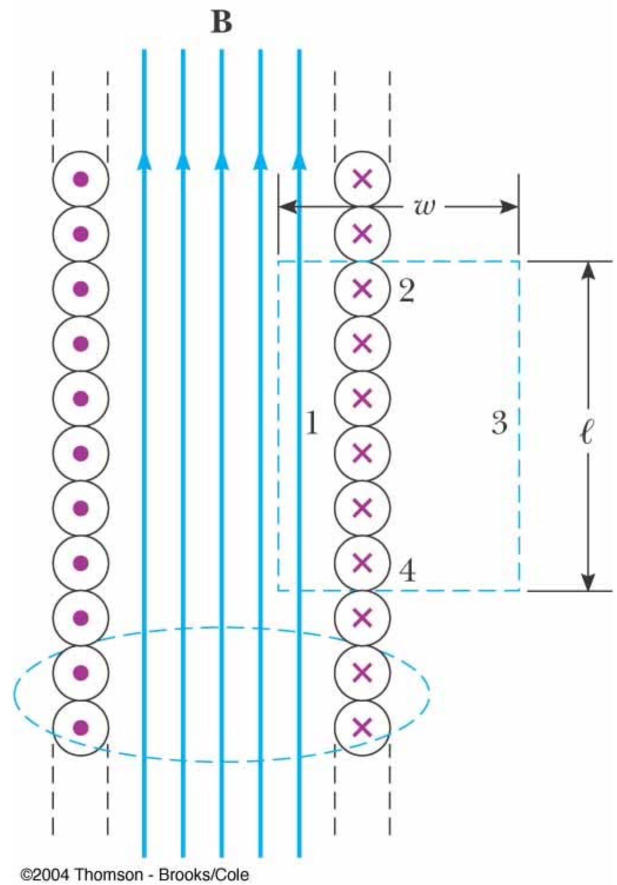
- The field distribution is similar to that of a bar magnet
- As the length of the solenoid increases
  – the interior field becomes more uniform
  – the exterior field becomes weaker



(a)

Magnetic Field of a Solenoid

- An *ideal solenoid* is approached when:
  - the turns are closely spaced
  - the length is much greater than the radius of the turns
- Consider a rectangle with side ***ℓ*** parallel to the interior field and side ***w*** perpendicular to the field
- The side of length ℓ inside the solenoid contributes to the field
  - This is path 1 in the diagram



Magnetic Field of a Solenoid

- Applying Ampere's Law gives

$$\oint \mathbf{B}\cdot d\mathbf{s} = \int_{\text{path1}} \mathbf{B}\cdot d\mathbf{s} = B\int_{\text{path1}} ds = B\ell$$

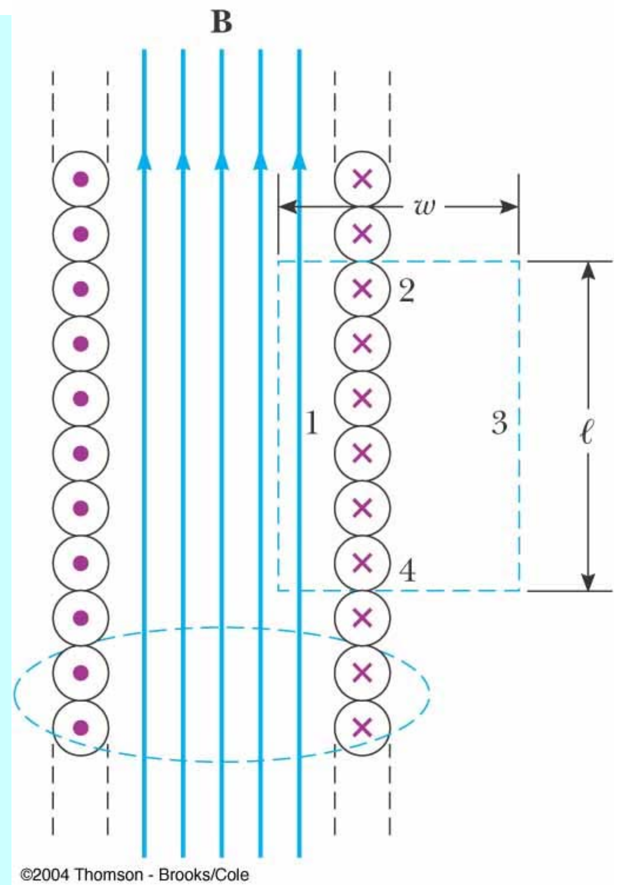
- The total current through the rectangular path equals the current through each turn multiplied by the number of turns

$$\oint \mathbf{B}\cdot d\mathbf{s} = B\ell = \mu_o NI$$

- Solving Ampere's law for the magnetic field is

$$B = \mu_o \frac{N}{\ell} I = \mu_o n I$$

  - $n = N / \ell$ is the number of turns per unit length
- This is valid only at points near the center of a very long solenoid



Chapter 32

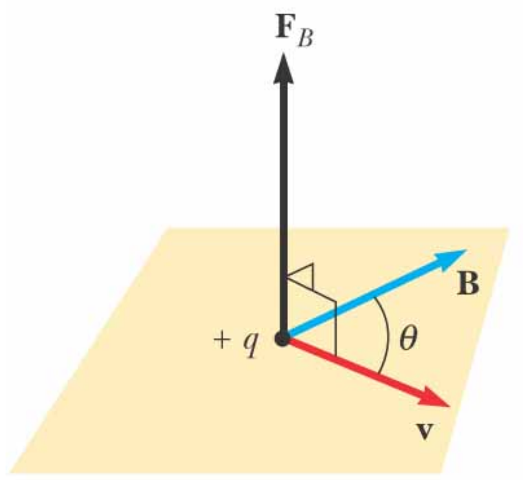
Interaction of Charged Particles with Magnetic Field

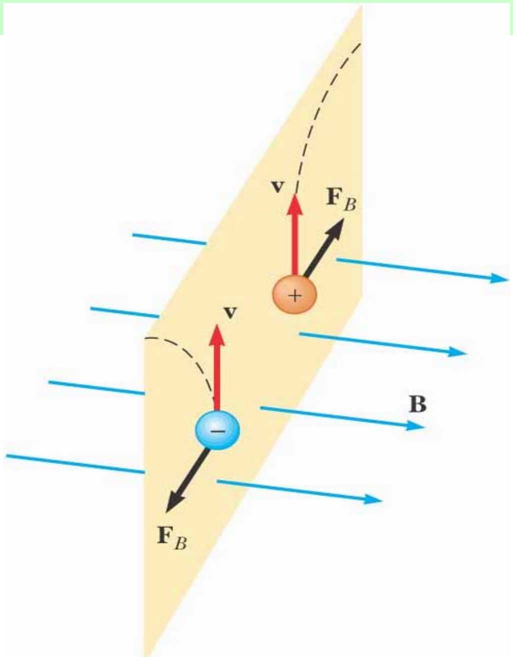
Interaction of Charged Particles with Magnetic Field

- The properties can be summarized in a vector equation:

$$\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$$

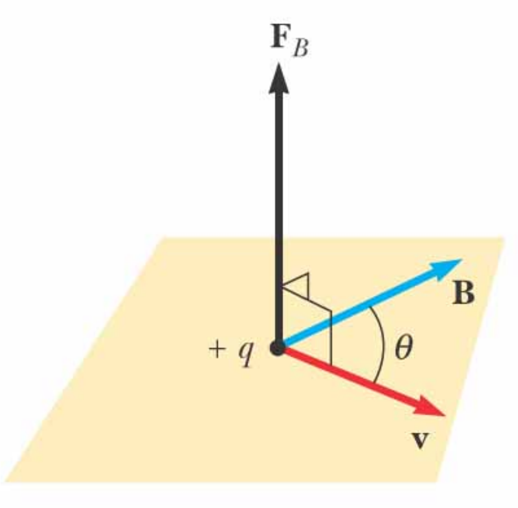
  - $\mathbf{F}_B$ is the magnetic force
  - q is the charge
  - $\mathbf{v}$ is the velocity of the moving charge
  - $\mathbf{B}$ is the magnetic field

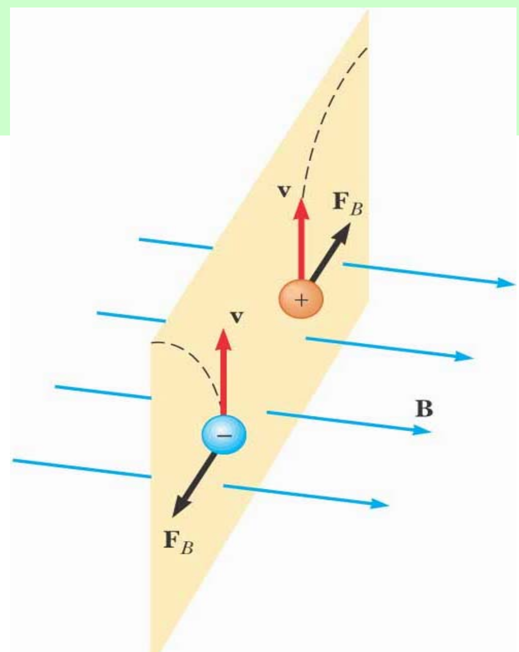




Interaction of Charged Particle with Magnetic Field

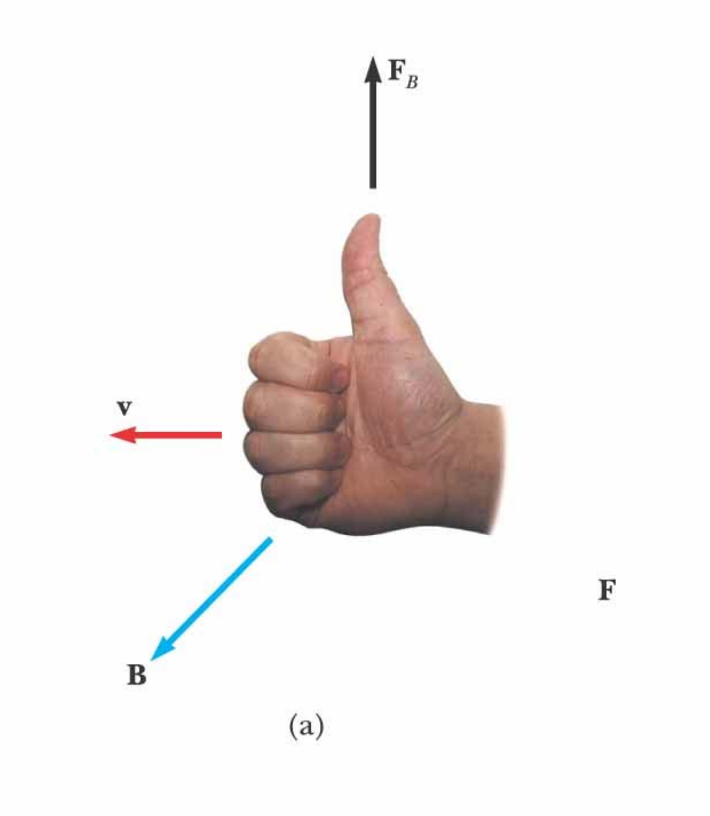
- The magnitude of the magnetic force on a charged particle is $F_B = |q|\, vB \sin\theta$
  - ➢ θ is the smallest angle between **v** and **B**
  - – F_B is zero when **v** and **B** are parallel or antiparallel
    - ➢ $\theta = 0$ or 180^o
  - – F_B is a maximum when **v** and **B** are perpendicular
    - ➢ $\theta = 90^o$





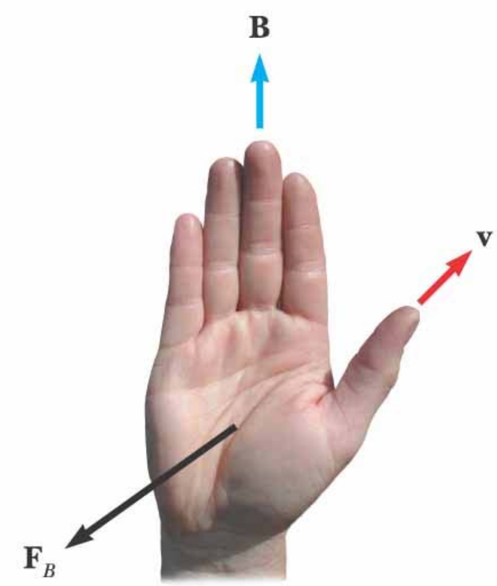
Direction of Magnetic Force

- The fingers point in the direction of **v**
- **B** comes out of your palm
  - Curl your fingers in the direction of **B**
- The thumb points in the direction of **v** x **B** which is the direction of $\mathbf{F}_B$



Direction of Magnetic Force

- Thumb is in the direction of **v**
- Fingers are in the direction of **B**
- Palm is in the direction of $\mathbf{F}_B$
  - On a positive particle
  - You can think of this as your hand pushing the particle



(b)

Differences Between Electric and Magnetic Fields

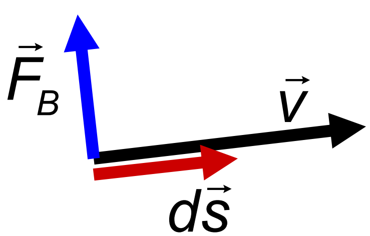
$$\vec{F}_E = q\vec{E} \qquad\qquad \vec{F}_B = q\vec{v} \times \vec{B}$$

- Direction of the force
  - The electric force acts along the direction of the electric field
  - The magnetic force acts perpendicular to the magnetic field
- Motion
  - The electric force acts on a charged particle regardless of whether the particle is moving
  - The magnetic force acts on a charged particle only when the particle is in motion

Differences Between Electric and Magnetic Fields

$$\vec{F}_E = q\vec{E} \qquad \vec{F}_B = q\vec{v} \times \vec{B}$$

- Work
  - The electric force does work in displacing a charged particle
  - The magnetic force associated with a steady magnetic field does no work when a particle is displaced
    - This is because the force is perpendicular to the displacement



$$W = \vec{F}_B \cdot d\vec{s} = 0$$

Work in Magnetic Field

- The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone
- When a charged particle moves with a velocity **v** through a magnetic field, the field can alter the ***direction*** of the velocity, but **not the speed** or the kinetic energy

Magnetic Field

- The SI unit of magnetic field is **tesla** (T)

$$T = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

Table 29.1

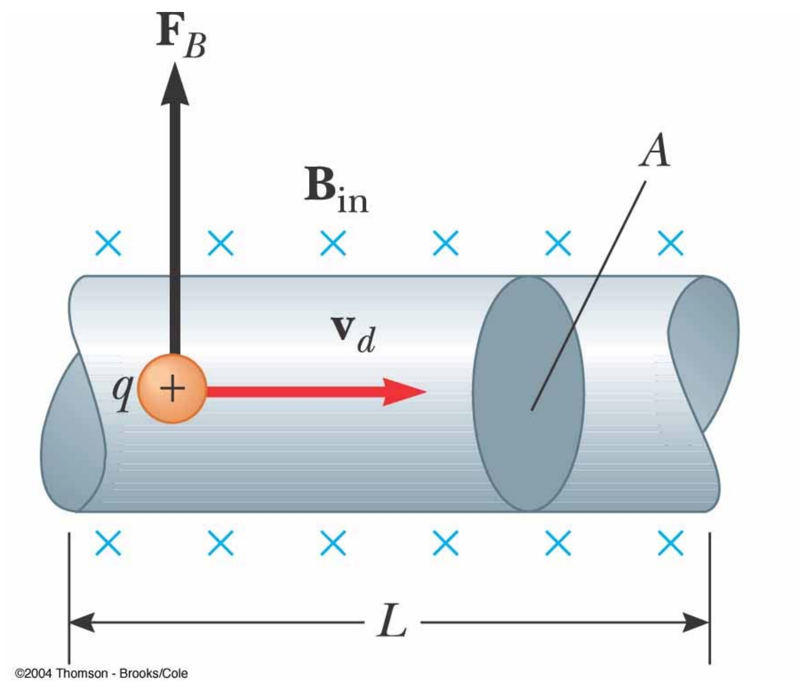
Some Approximate Magnetic Field Magnitudes	
Source of Field	**Field Magnitude (T)**
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain (due to nerve impulses)	10^{-13}

Force on a Wire

- The magnetic force is exerted on each moving charge in the wire
  - $\mathbf{F} = q\,\mathbf{v}_d \times \mathbf{B}$
- The total force is the product of the force on one charge and the number of charges
  - $\mathbf{F} = (q\,\mathbf{v}_d \times \mathbf{B})nAL$
- In terms of current:

$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B}$$

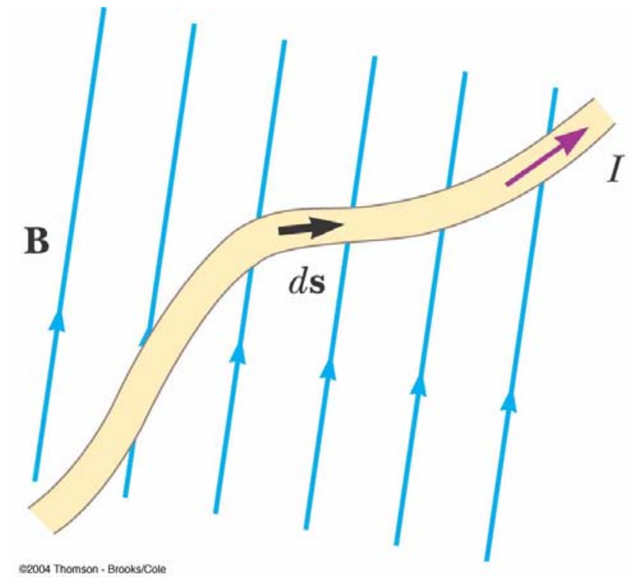
  - **L** is a vector that points in the direction of the current
    - Its magnitude is the length *L* of the segment



Force on a Wire

- Consider a small segment of the wire, $d\mathbf{s}$
- The force exerted on this segment is $\mathbf{F} = I\, d\mathbf{s} \times \mathbf{B}$
- The total force is

$$\mathbf{F} = I \int_{\mathbf{a}}^{b} d\mathbf{s} \times \mathbf{B}$$



Force on a Wire: Uniform Magnetic Field

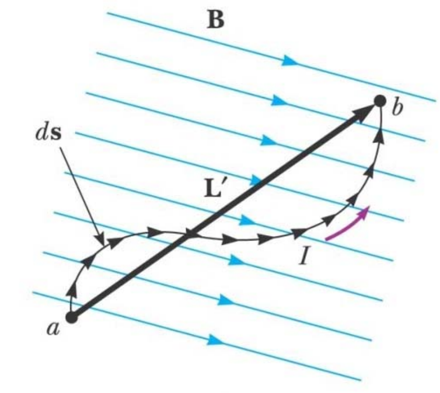
- **B** is a constant
- Then the total force is

$$\mathbf{F} = I\int_{\mathbf{a}}^{b} d\mathbf{s}\times\mathbf{B} = -I\,\mathbf{B}\times\int_{\mathbf{a}}^{b} d\mathbf{s}$$

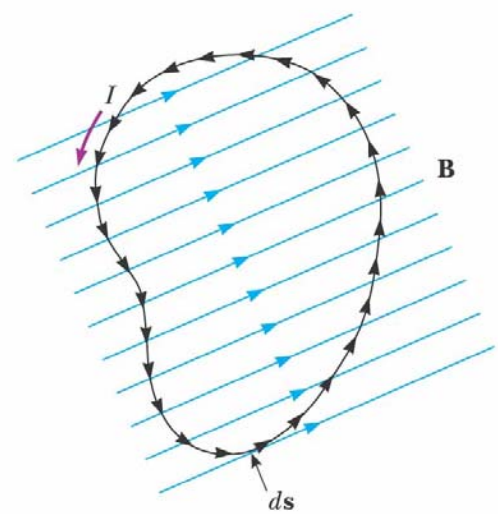
- For closed loop:

$$\int_{\mathbf{a}}^{b} d\mathbf{s} = 0$$

- The net magnetic force acting on any closed current loop in a uniform magnetic field **is zero**



(a)

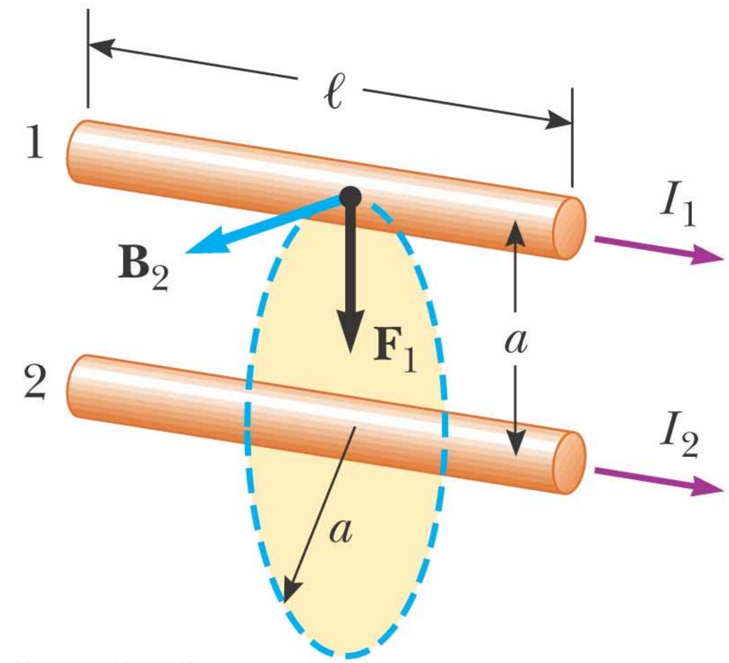


(b)

Magnetic Force between two parallel conductors

- Two parallel wires each carry a steady current
- The field $\mathbf{B}_2$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$
- Substituting the equation for B_2 gives

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a} \ell$$



Magnetic Force between two parallel conductors

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a}\ell$$

- Parallel conductors carrying currents in the same direction attract each other
- Parallel conductors carrying current in opposite directions repel each other
- The result is often expressed as the magnetic force *between* the two wires, F_B
- This can also be given as the *force per unit length*:

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$