Newton's Second Law:

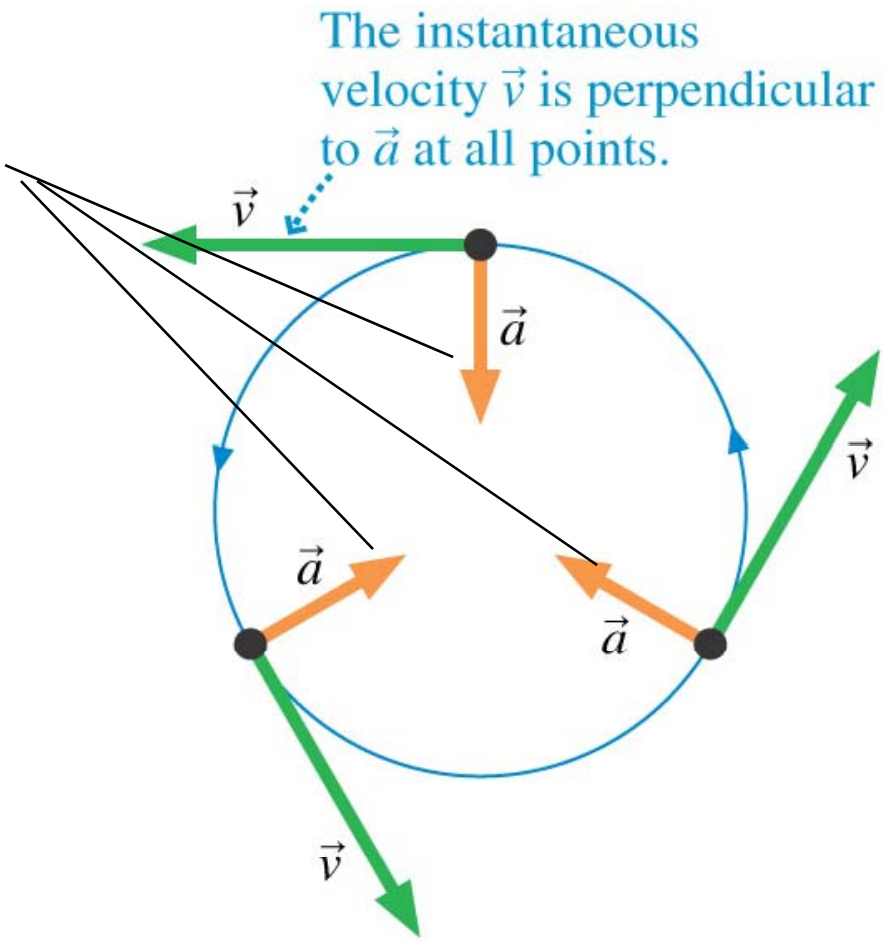
Motion in a Circle

Readings: Chapter 8

Uniform Circular Motion: Acceleration

$$\omega = \frac{v}{r}$$

centripetal acceleration

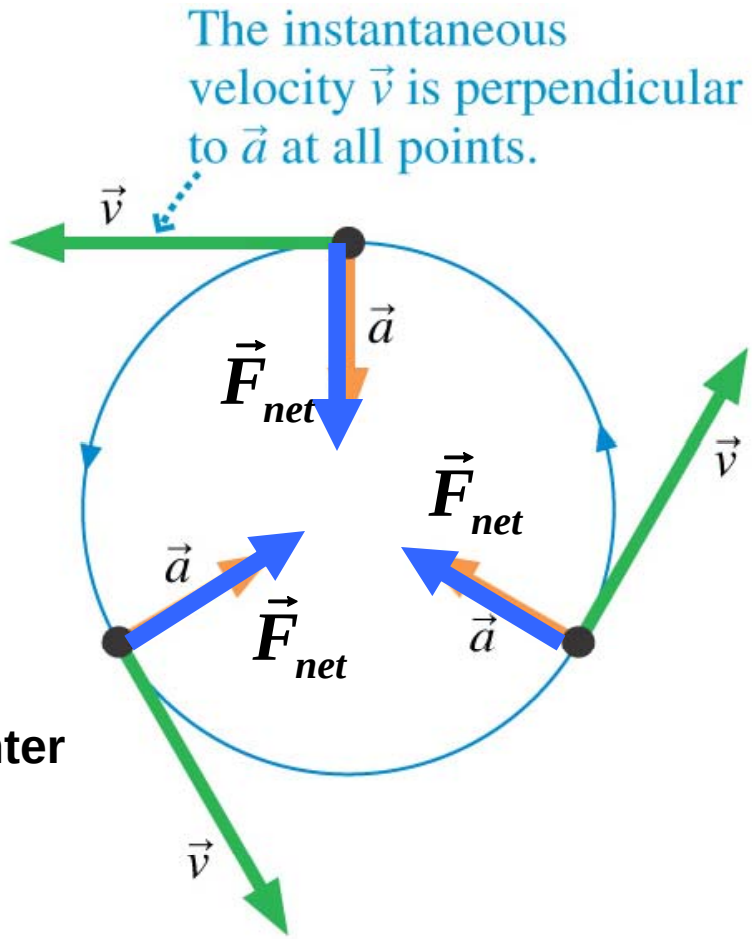


The magnitude of acceleration:

$$a = \frac{v^2}{r} = \omega^2 r = v\omega$$

Uniform Circular Motion: Dynamics

$$\omega = \frac{v}{r}$$



The magnitude of acceleration:

$$a = \frac{v^2}{r} = \omega^2 r = v\omega$$

Direction of acceleration: toward the center

The second Newton's law:

$$\vec{F}_{net} = m\vec{a}$$

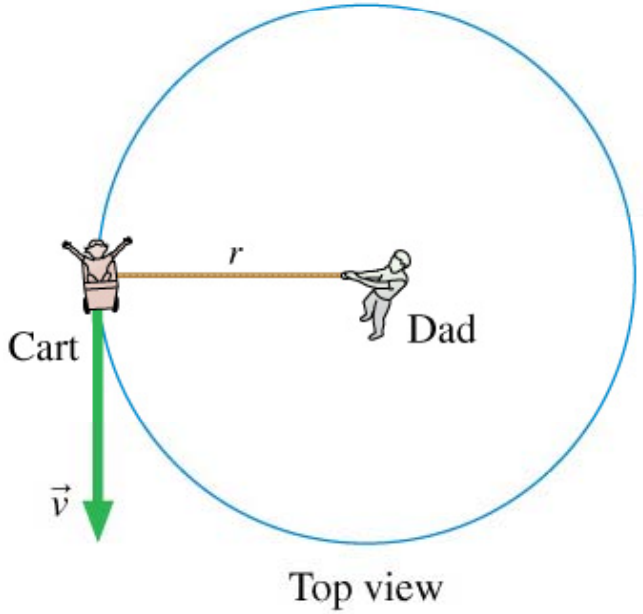
Uniform Circular Motion: Dynamics

$$a = \frac{v^2}{r} = \omega^2 r = v\omega$$

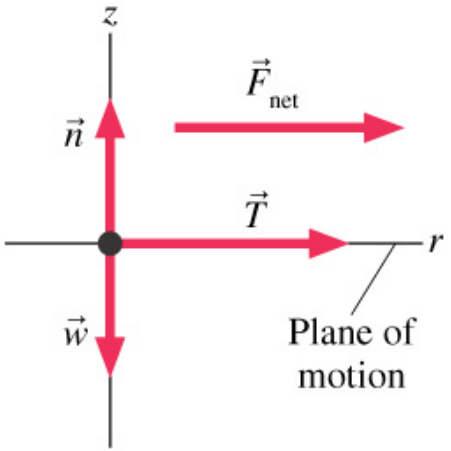
The magnitude of the net force

$$F_{net} = m\frac{v^2}{r}$$

Direction of the net force: toward the center



Top view



Edge view

$$\vec{F}_{net} = \vec{n} + \vec{T} + \vec{w} = m\vec{a}$$

$$n - w = 0$$

$$T = ma = m\frac{v^2}{r}$$

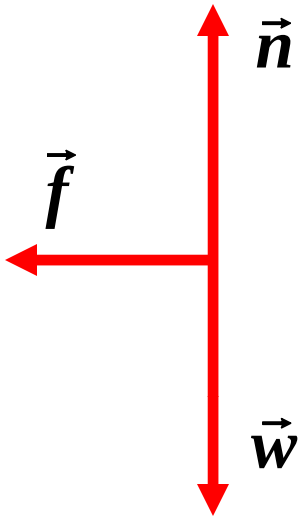
Uniform Circular Motion: Dynamics

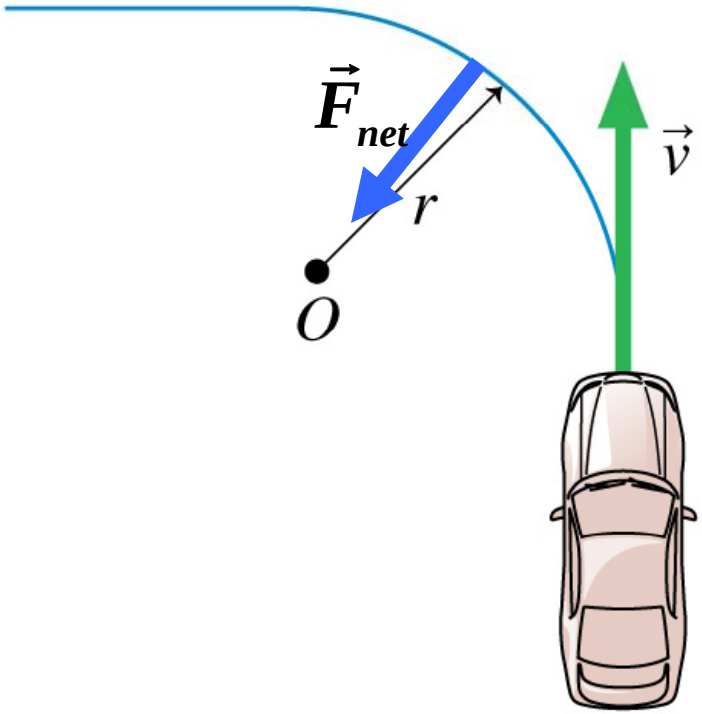
$$\vec{F}_{net} = \vec{n} + \vec{f} + \vec{w} = m\vec{a}$$

$$n - w = 0$$

$$f = ma = m\frac{v^2}{r}$$

$$f_{max} = \mu n = \mu w = \mu mg$$

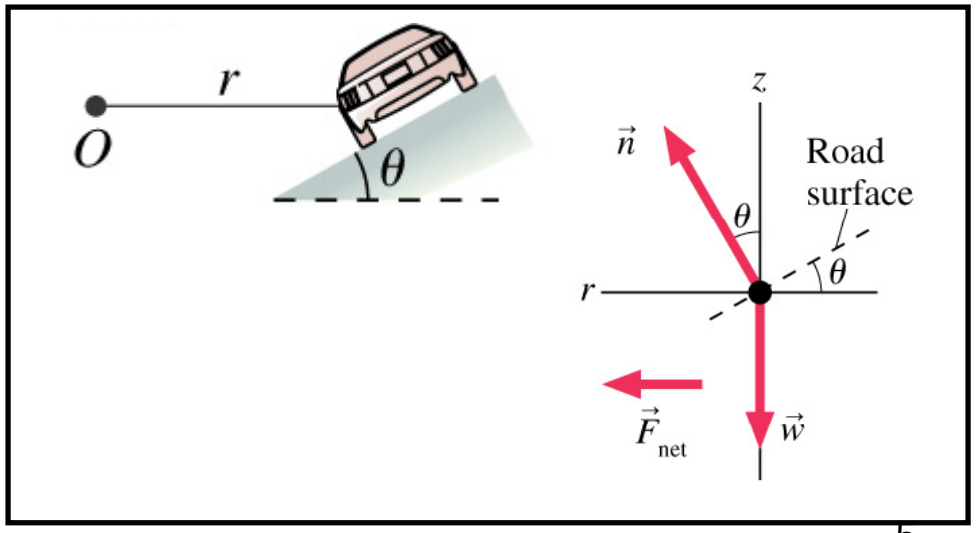




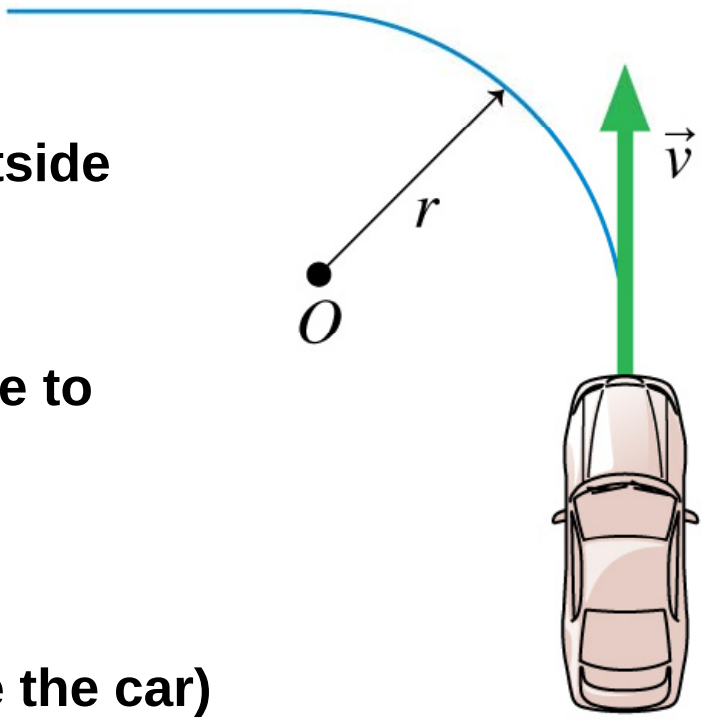
then

$$m\frac{v^2}{r} \le \mu mg$$

$$v \le \sqrt{\mu rg}$$



Fictitious Forces



The force that seems to push the object outside of the circle is called centrifugal force

The direction of centrifugal force is opposite to the direction of acceleration

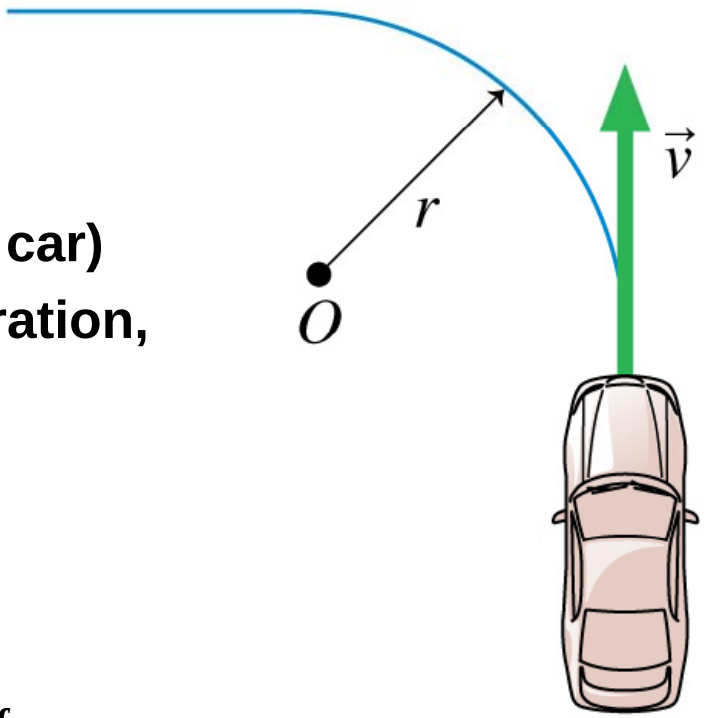
The centrifugal force exists only inside the systems which is moving in a circle (inside the car)

The centrifugal force is not a REAL FORCE

The magnitude of centrifugal force is

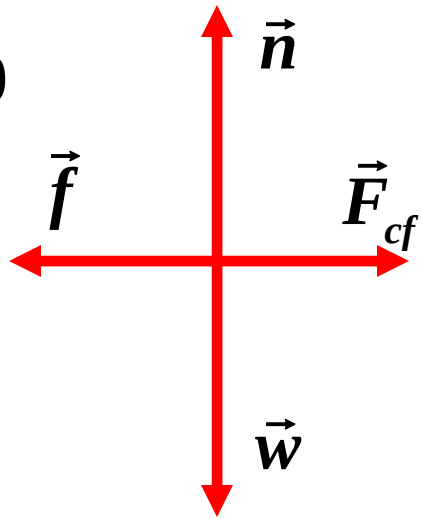
$$F_{cf} = m\frac{v^2}{r}$$

Fictitious Forces: generalized second Newton's law



In the reference frame of the car (inside the car) there are no motion, so there are no acceleration, but THERE IS CENTRIFUGAL FORCE

$$\vec{F}_{net} = \vec{n} + \vec{f} + \vec{w} + \vec{F}_{cf} = 0$$

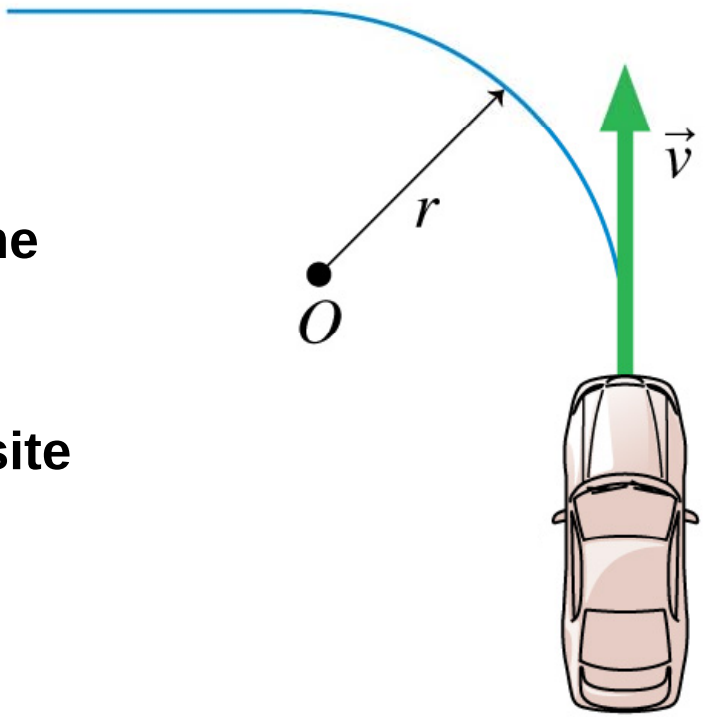


$$n - w = 0$$

$$f - F_{cf} = 0$$

$$F_{cf} = m\frac{v^2}{r}$$

Fictitious Forces



The fictitious force exists only inside the reference frame which are moving with some acceleration (non-inertial reference frame)

The direction of the fictitious force is opposite to the direction of acceleration

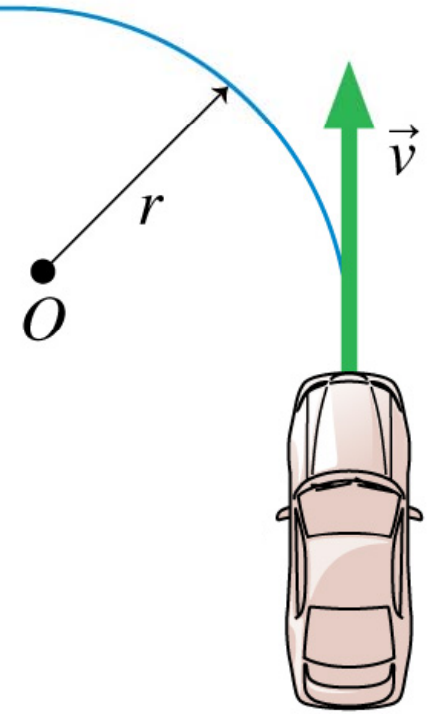
The magnitude of centrifugal force is

$$F_f = ma$$

Inside of the moving frame (inside of the car) there are no motion, so that acceleration is zero

Then the condition of equilibrium: the net force (which include the fictitious force is 0)

In the non-inertial reference frame (inside the car) there are no motion, so there are no acceleration, but THERE IS CENTRIFUGAL FORCE

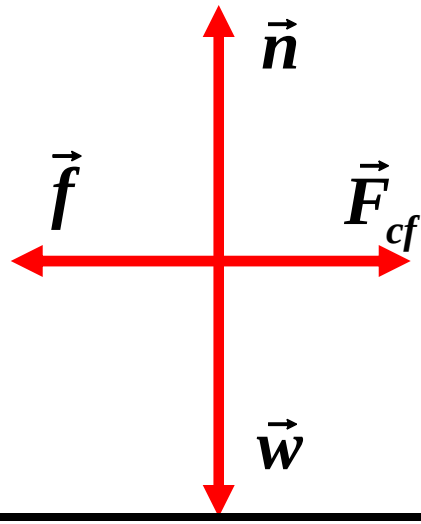


$$\vec{F}_{net} = \vec{n} + \vec{f} + \vec{w} + \vec{F}_{cf} = 0$$

$$n - w = 0$$

$$f - F_{cf} = 0 \qquad F_{cf} = m\frac{v^2}{r}$$

$$f = F_{cf} = m\frac{v^2}{r}$$



In the inertial reference frame (outside the car) this is the motion with acceleration are no motion, and there are fictitious forces

$$\vec{F}_{net} = \vec{n} + \vec{f} + \vec{w} = m\vec{a}$$

$$n - w = 0$$

$$f = ma = m\frac{v^2}{r}$$