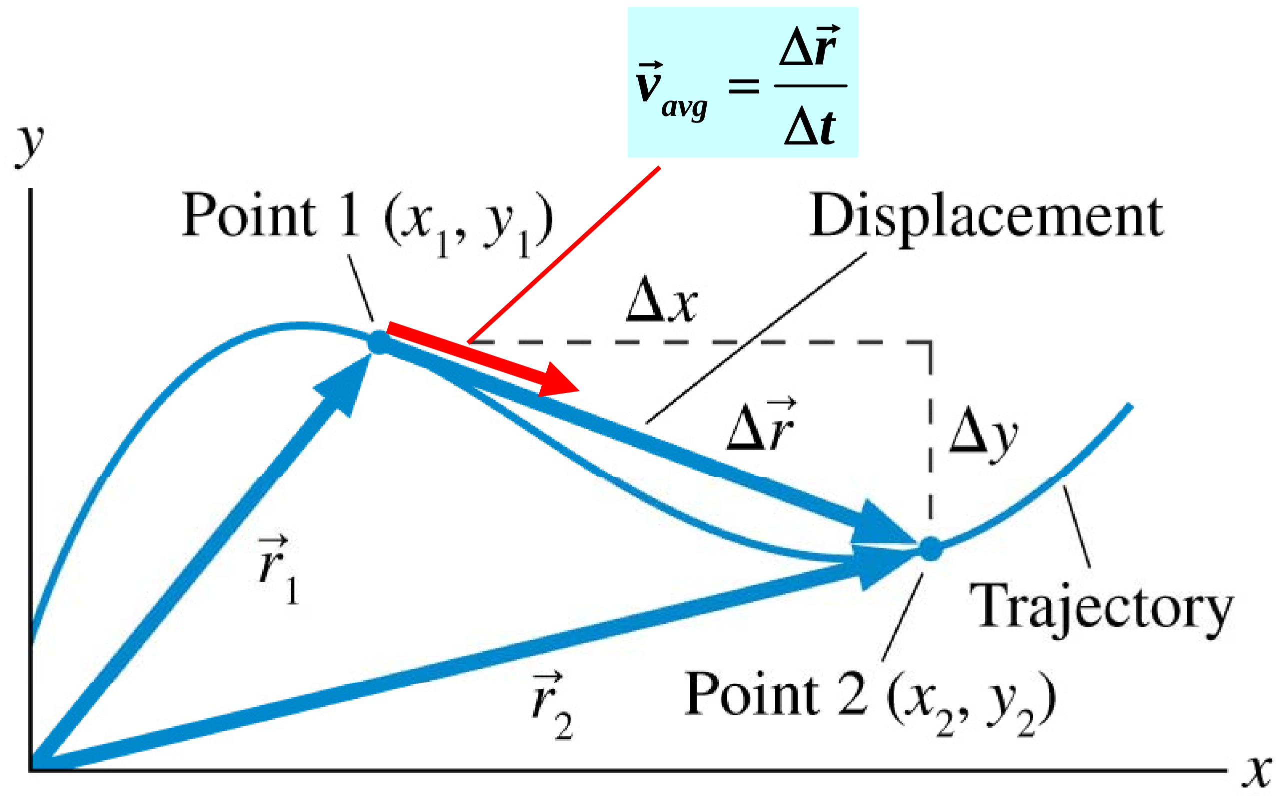
Motion in a Plane

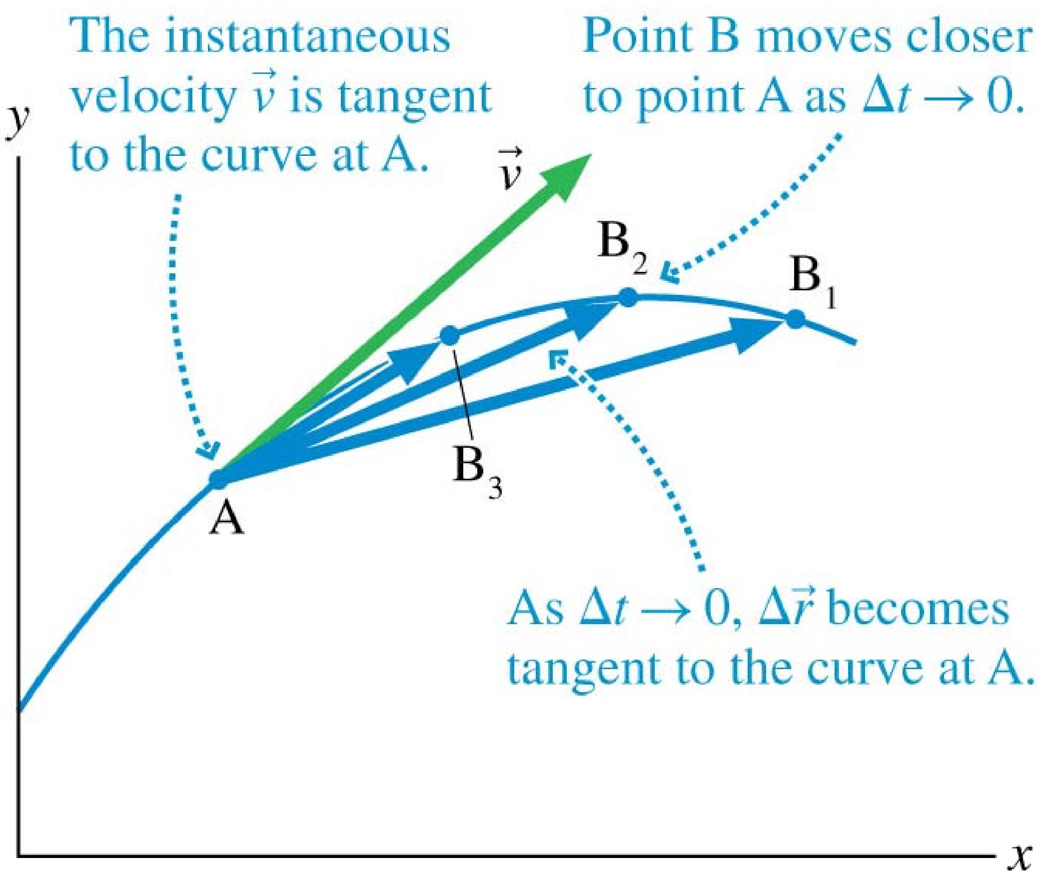
Readings: Chapter 4

Kinematics in Two Dimensions

Average velocity: $$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

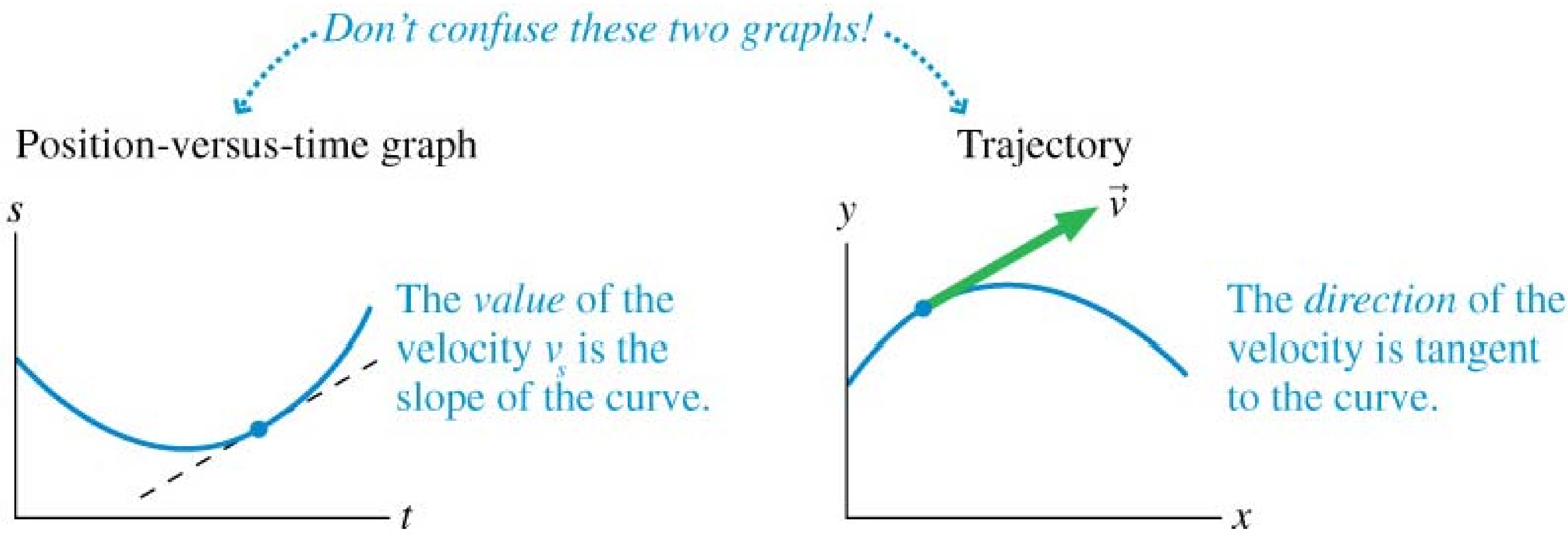


Kinematics in Two Dimensions: Instantaneous Velocity



$$\vec{v} = \left.\frac{\Delta\vec{r}}{\Delta t}\right|_{\Delta t \to 0}$$

Kinematics in Two Dimensions

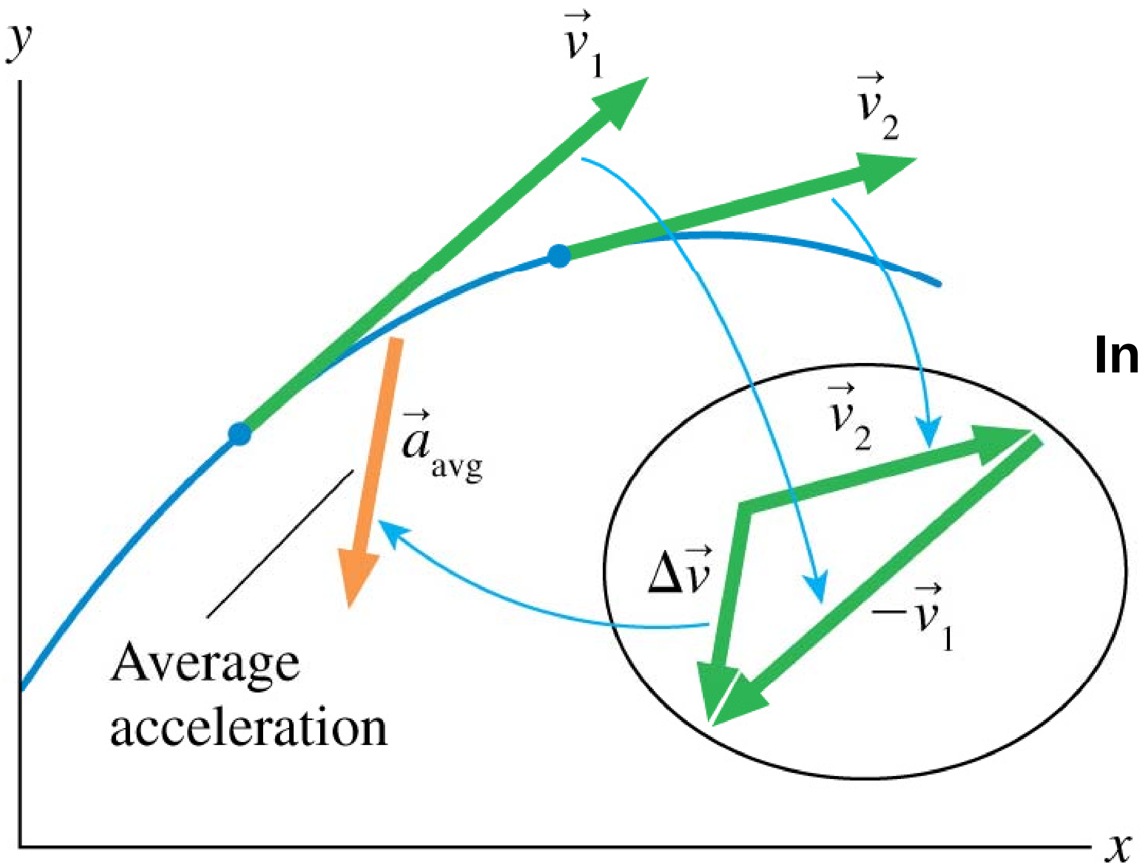


$$v = \frac{ds}{dt}$$

$$\vec{v} = \left.\frac{\Delta\vec{r}}{\Delta t}\right|_{\Delta t \to 0}$$

(motion along a line)

Kinematics in Two Dimensions: Instantaneous Acceleration



Average acceleration:

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

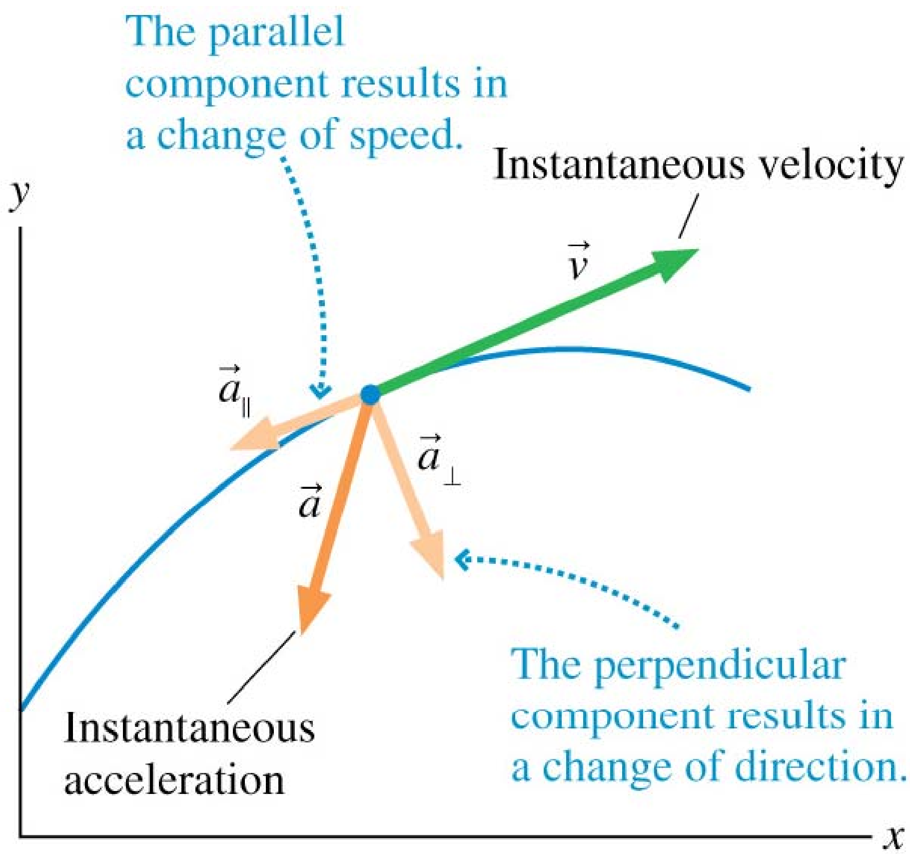
Instantaneous acceleration:

$$\vec{a} = \left.\frac{\Delta\vec{v}}{\Delta t}\right|_{\Delta t\to 0}$$

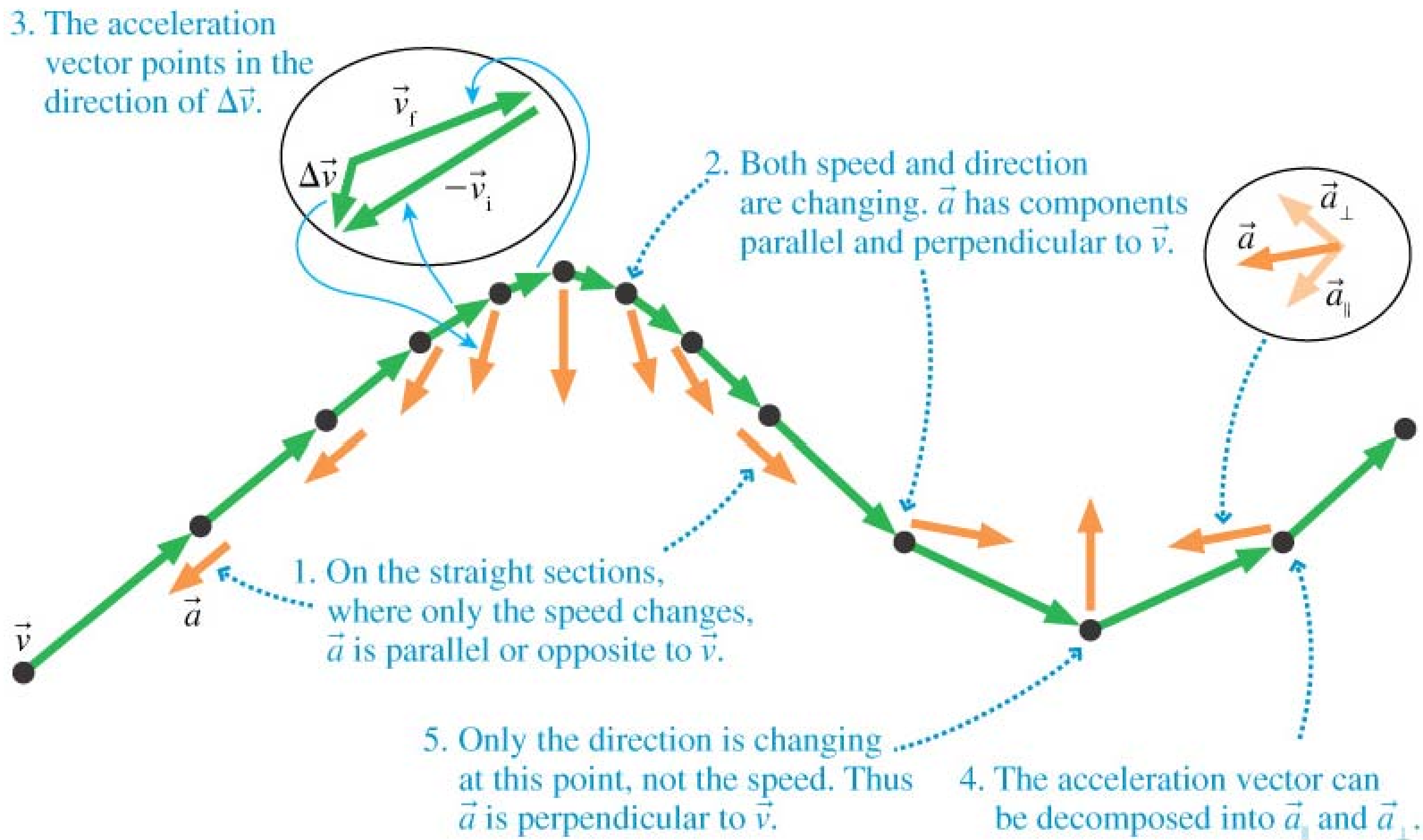
Kinematics in Two Dimensions: Instantaneous Acceleration

<u>Motion along a line</u>: acceleration results in change of speed (the magnitude of velocity)

<u>Motion in a plane</u>: acceleration can change the speed (the magnitude of velocity) and the direction of velocity.

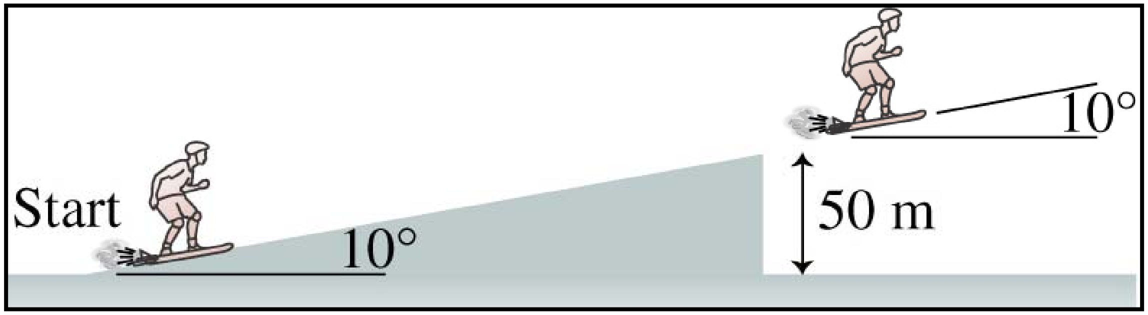


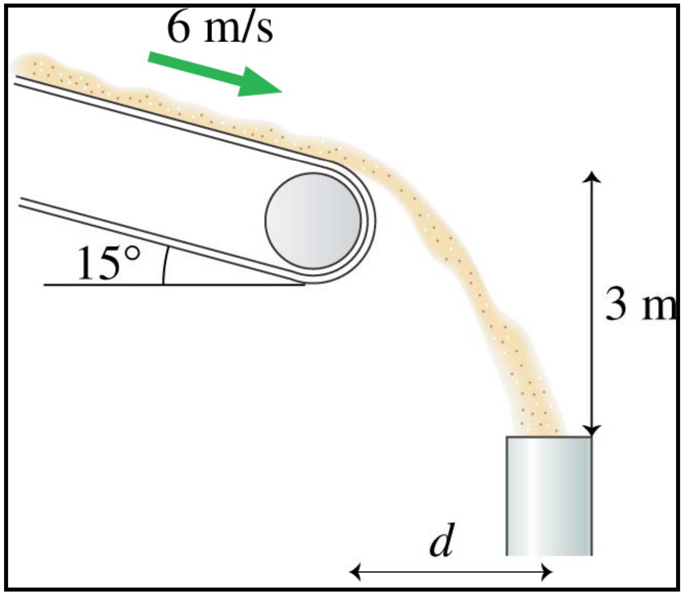
$$\vec{a} = \left.\frac{\Delta \vec{v}}{\Delta t}\right|_{\Delta t \to 0}$$

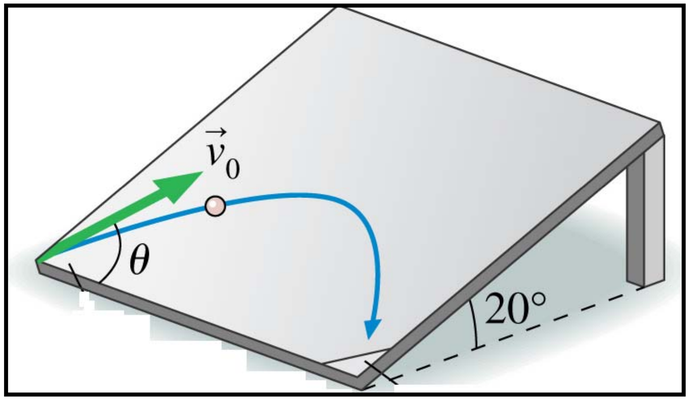


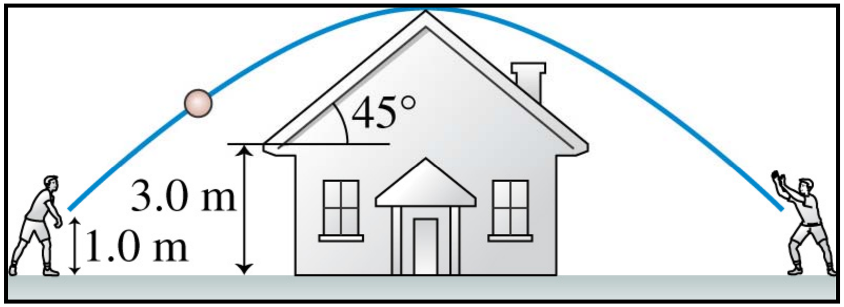
3. The acceleration vector points in the direction of $\Delta\vec{v}$.
$\vec{v}_f$
$\Delta\vec{v}$
$-\vec{v}_i$
2. Both speed and direction are changing. $\vec{a}$ has components parallel and perpendicular to $\vec{v}$.
$\vec{a}_\perp$
$\vec{a}$
$\vec{a}_\parallel$
$\vec{v}$
$\vec{a}$
1. On the straight sections, where only the speed changes, $\vec{a}$ is parallel or opposite to $\vec{v}$.
5. Only the direction is changing at this point, not the speed. Thus $\vec{a}$ is perpendicular to $\vec{v}$.
4. The acceleration vector can be decomposed into $\vec{a}_\parallel$ and $\vec{a}_\perp$.

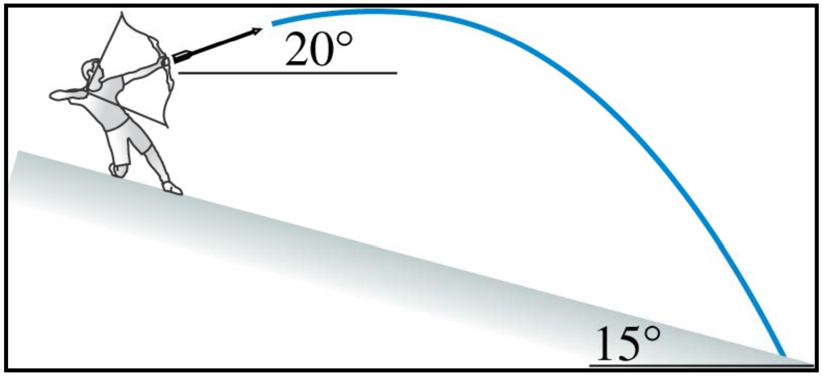
A Projectile is an object that moves in two dimensions (in a plane) under the influence of only the gravitational force.











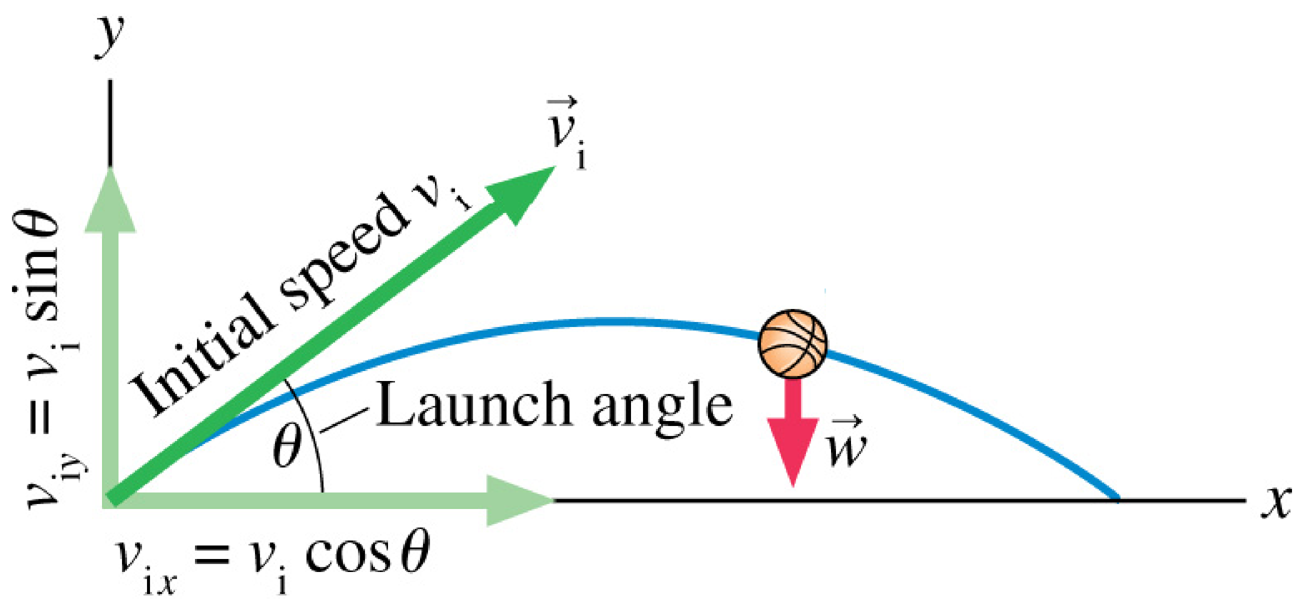
A **Projectile** is an object that moves in two dimensions with free fall acceleration

$$\vec{a} = \vec{g}$$

$a_x = 0$ - motion along x-axis – with zero acceleration – constant velocity

$v_x = \text{constant}$

$a_y = -g = -9.8 \frac{m}{s^2}$ - motion along y-axis – free fall motion with constant acceleration



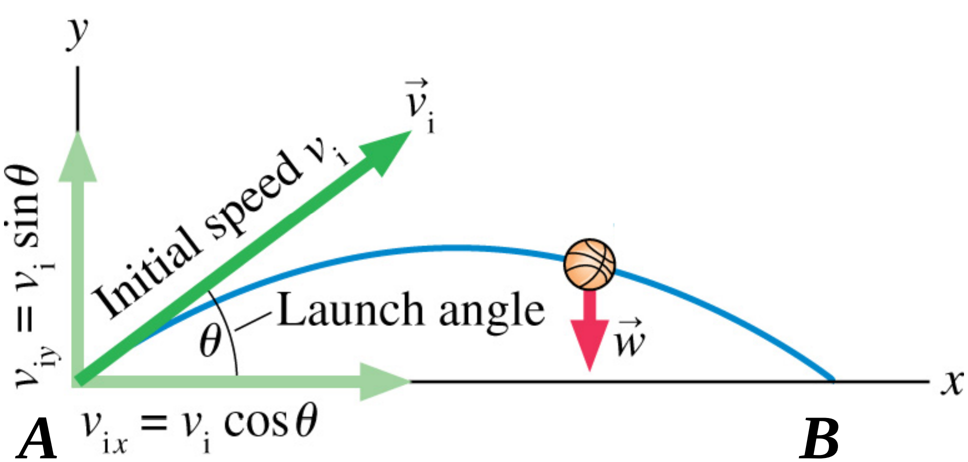
Example: Find the distance AB

$$v_x = \text{constant} = v_{i,x} = v_i \cos\theta \qquad x = v_{i,x} t = v_i t \cos\theta$$

$$a_y = -g = -9.8\frac{m}{s^2} \qquad v_y = v_{i,y} - gt \qquad v_{i,y} = v_i \sin\theta$$

Point A - $y_i = y_A = 0$

Point B - $y_{final} = y_B = 0$ **then** $y_{final} = v_{i,y} t_{AB} - g\frac{t_{AB}^2}{2}$ $\quad v_{i,y} t_{AB} = g\frac{t_{AB}^2}{2}$



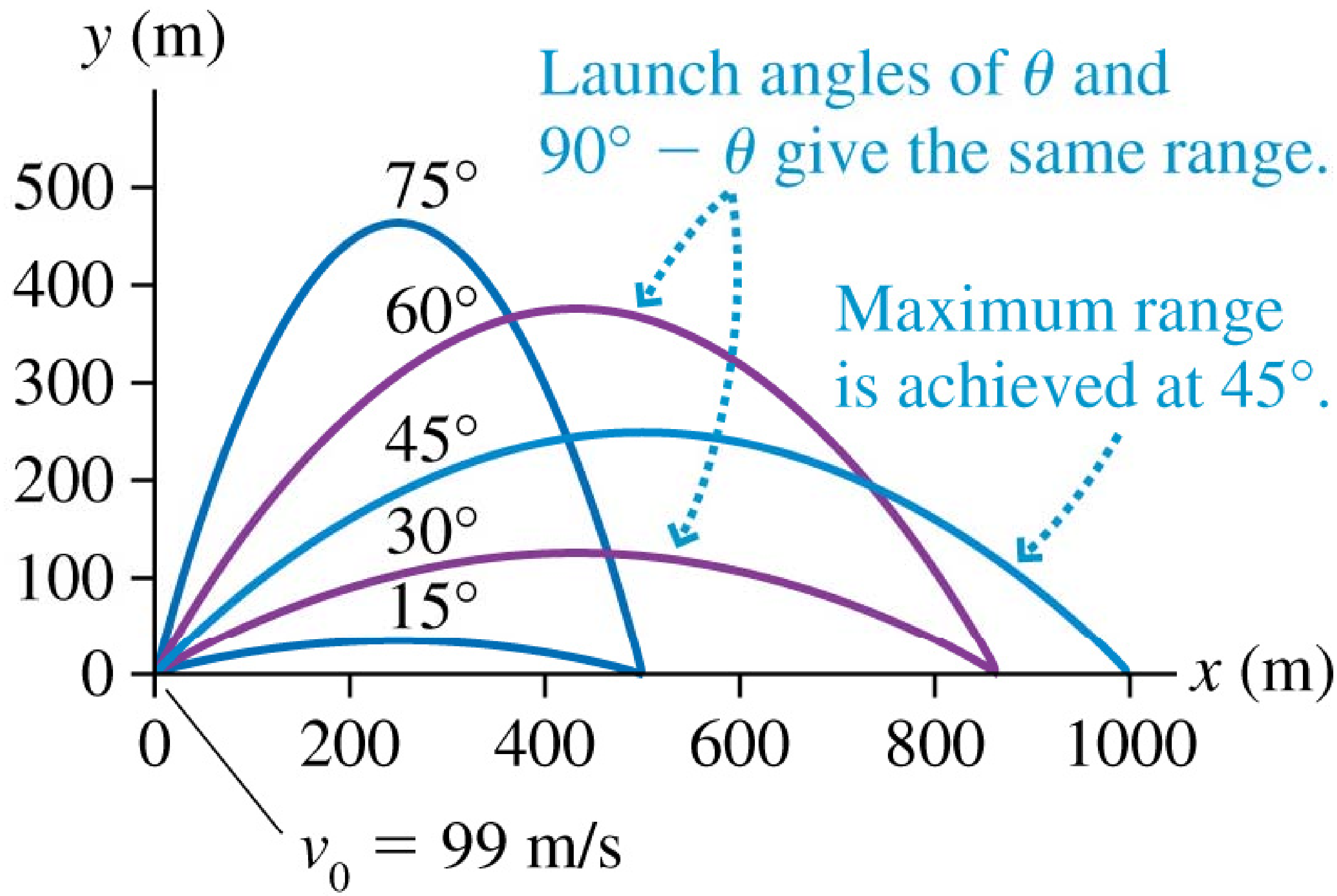
$$t_{AB} = \frac{2v_{i,y}}{g} = \frac{2v_i \sin\theta}{g}$$

then

$$x_{AB} = v_i t_{AB} \cos\theta =$$

$$= \frac{2v_i^2}{g}\sin\theta\cos\theta = \frac{v_i^2}{g}\sin 2\theta$$

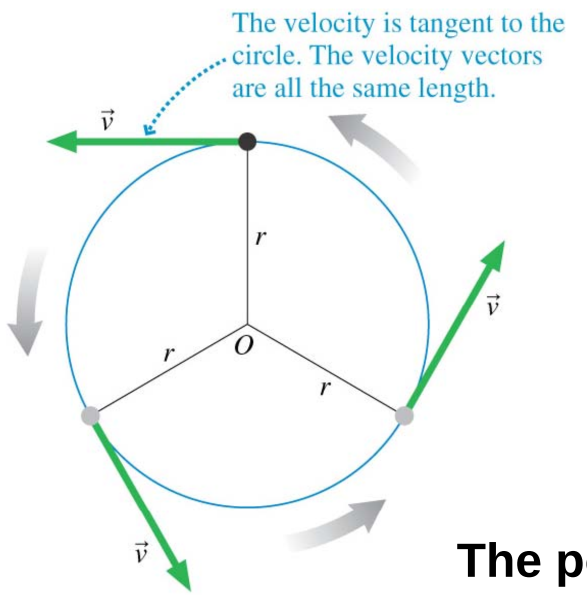
$$x_{AB} = \frac{v_i^2}{g}\sin 2\theta$$



Motion in a Circle

Kinematics in Two Dimensions: Uniform Circular Motion: Motion with Constant Speed

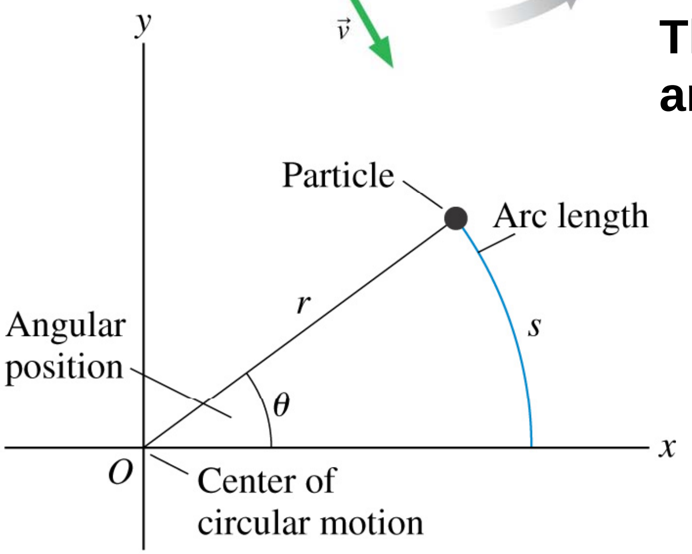
The speed (magnitude of velocity) is the same



Period:

$$T = \frac{2\pi r}{v}$$

The position of the particle is characterized by angle (or by arc length)



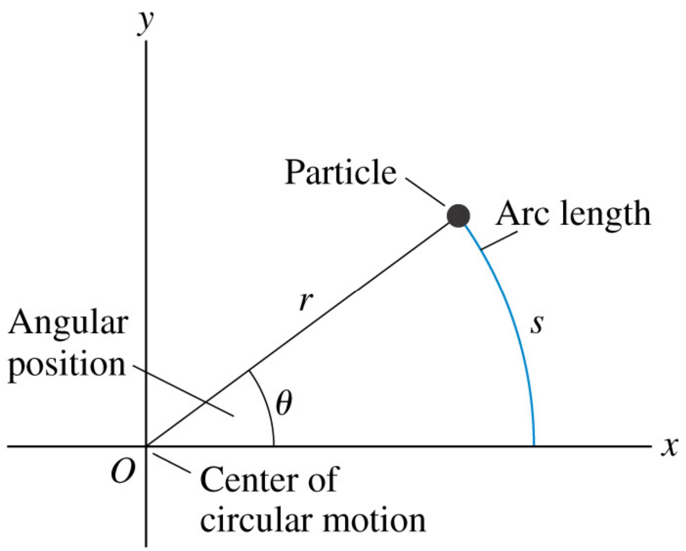
Arc length: $s = r\theta$

where angle θ **is in RADIANS**

$$1\ rad = \frac{360^0}{2\pi} = 57.3^0$$

Example:

What is the arc length for $\theta = 45^0, 90^0, 10^0$ **if** $r = 1m$



$$45^0 = 45^0 \frac{2\pi}{360^0} rad = \frac{\pi}{4} rad = 0.78\, rad$$

$$90^0 = 90^0 \frac{2\pi}{360^0} rad = \frac{\pi}{2} rad = 1.57\, rad$$

$$10^0 = 10^0 \frac{2\pi}{360^0} rad = \frac{\pi}{18} rad = 0.174\, rad$$

Arc length: $s = r\theta$

$$s_{45} = \frac{\pi r}{4} = 0.78m \qquad s_{10} = \frac{\pi r}{18} = 0.174m$$

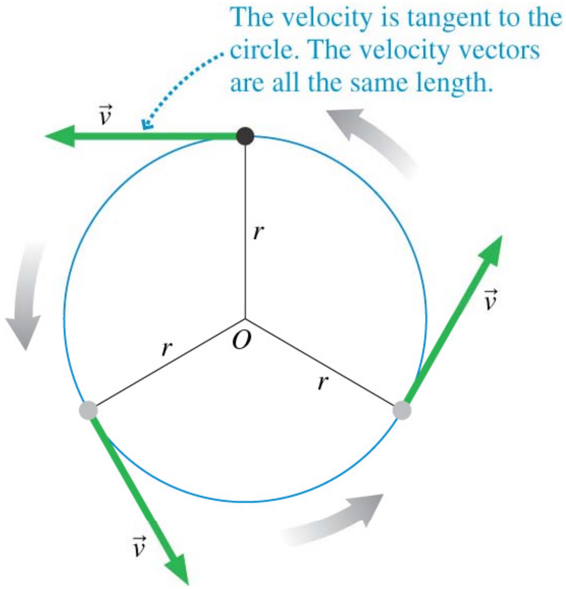
$$s_{90} = \frac{\pi r}{2} = 1.57m$$

Kinematics in Two Dimensions: Uniform Circular Motion: Motion with Constant Speed

The speed (magnitude of velocity) is the same

$$s = vt$$

$$\theta = \omega t$$

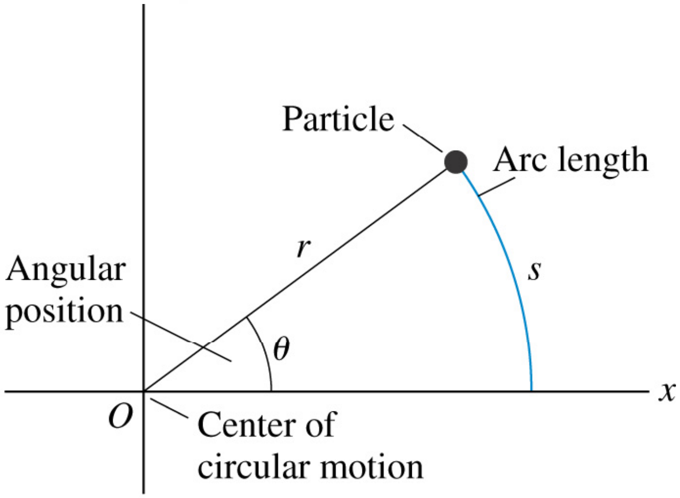


$$\omega = \frac{v}{r}$$

$$T = \frac{2\pi r}{v}$$

Then

$$\omega = \frac{2\pi}{T}$$

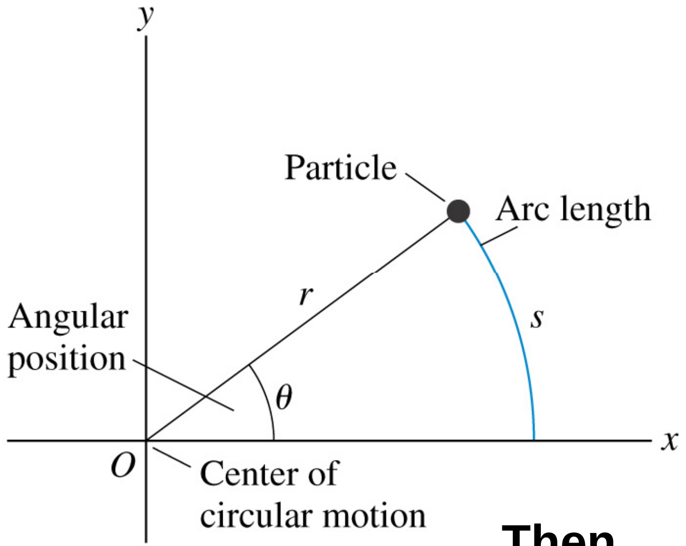


Uniform Circular Motion

$$s = vt$$

$$\omega = \frac{2\pi}{T}$$

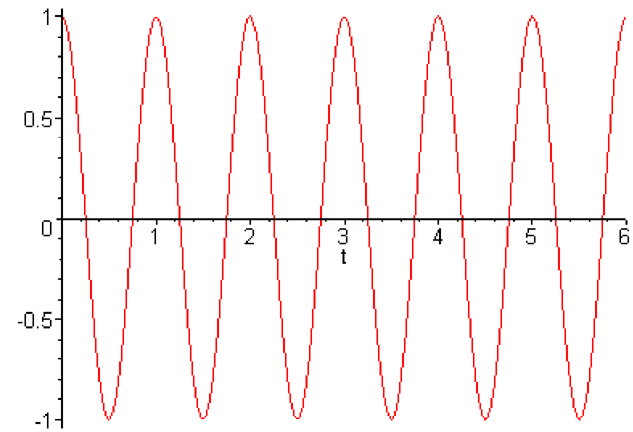
$$\omega = \frac{v}{r}$$



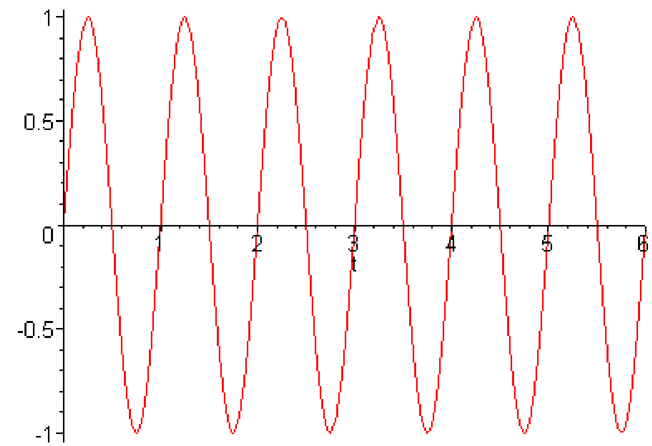
$$\theta = \omega t$$

Then

$$x = r\cos\theta = r\cos(\omega t)$$

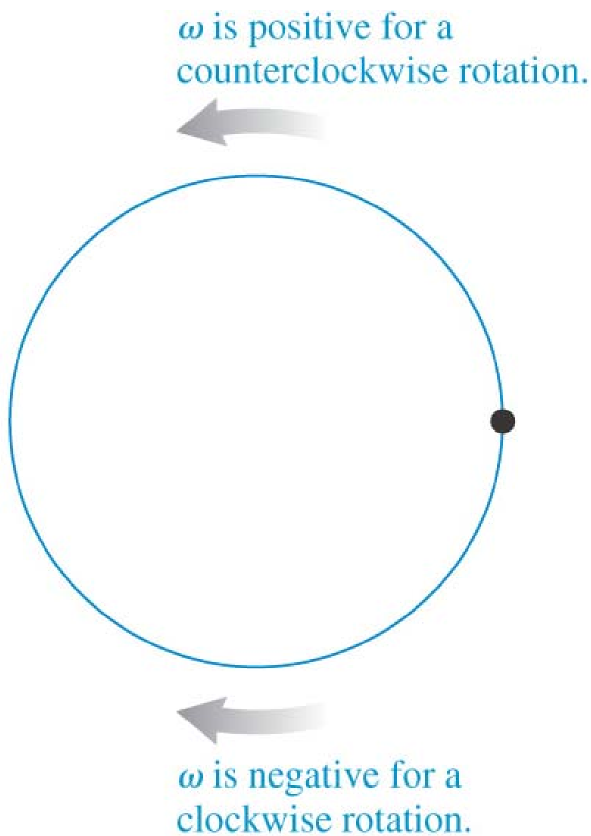


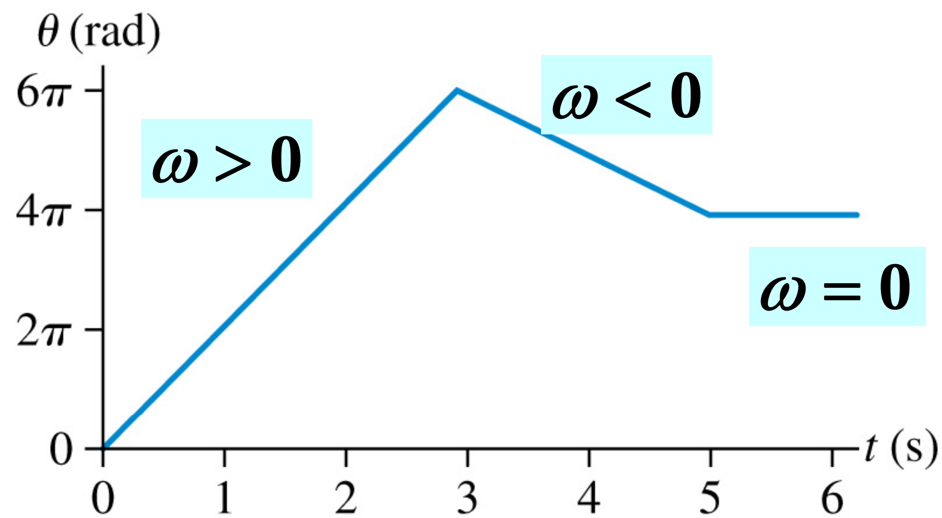
$$y = r\sin\theta = r\sin(\omega t)$$

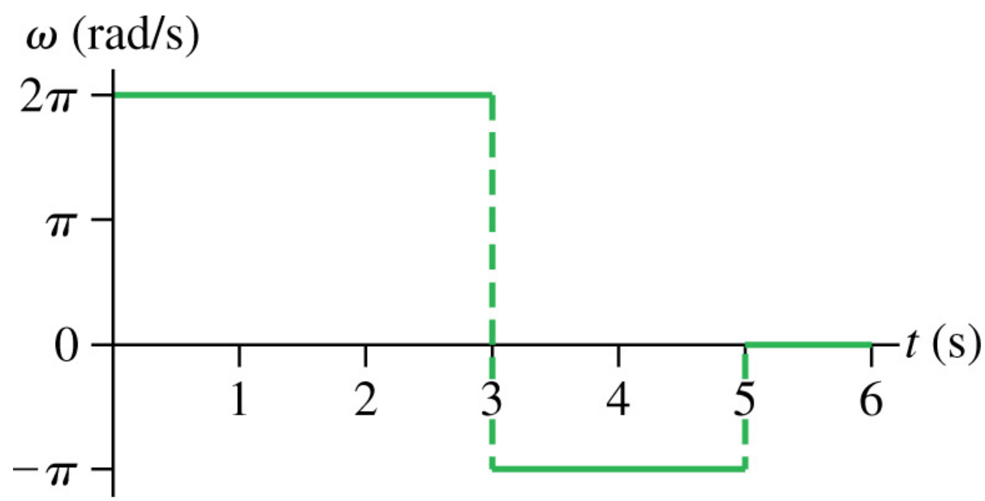


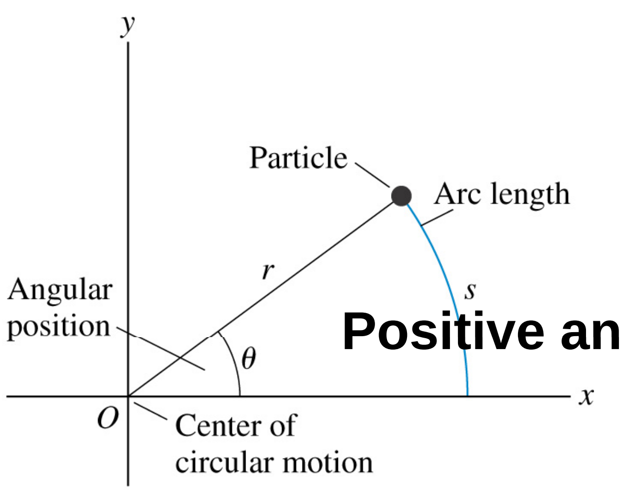
$$x^2 + y^2 = r^2\sin^2(\omega t) + r^2\cos^2(\omega t) = r^2$$

$\theta = \omega t$





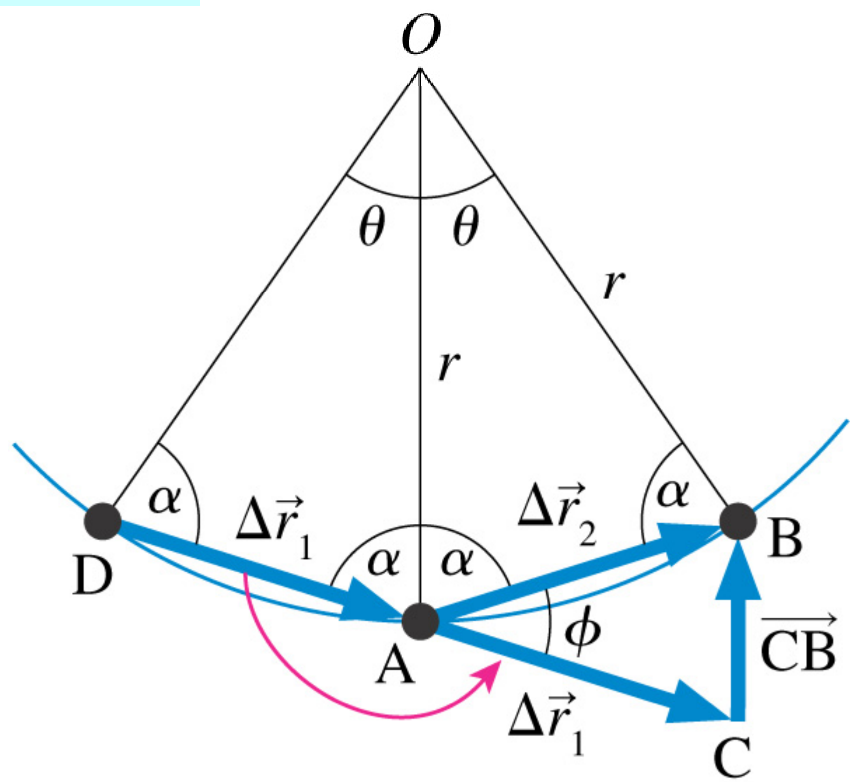




Positive angle

Uniform Circular Motion: Acceleration

$$\vec{a} = \left.\frac{\Delta \vec{v}}{\Delta t}\right|_{\Delta t \to 0} \qquad \vec{v} = \left.\frac{\Delta \vec{r}}{\Delta t}\right|_{\Delta t \to 0}$$



$$\Delta \vec{r}_1 = \vec{v}_1 \Delta t$$

$$\Delta \vec{r}_2 = \vec{v}_2 \Delta t$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{r}_2 - \Delta \vec{r}_1}{(\Delta t)^2}$$

Direction of acceleration is the same as direction of vector (toward the center)

$$\overrightarrow{CB} = \Delta \vec{r}_2 - \Delta \vec{r}_1$$

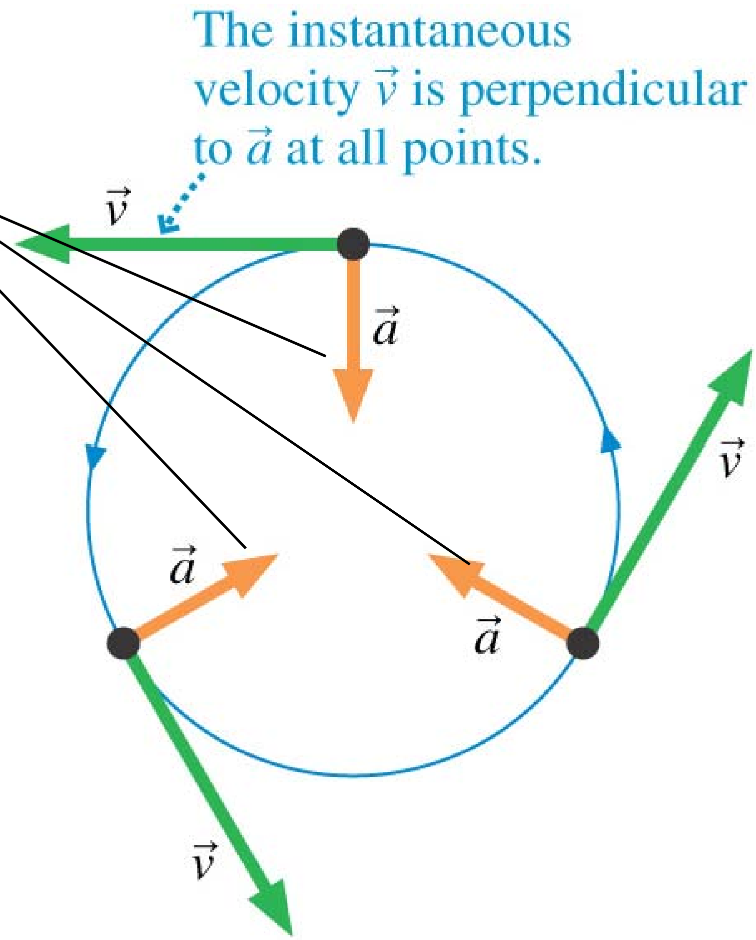
The magnitude of acceleration:

$$a = \frac{CB}{(\Delta t)^2} = \frac{\Delta r \phi}{(\Delta t)^2} = \frac{\Delta r(\pi - 2\alpha)}{(\Delta t)^2} = \frac{\Delta r \theta}{(\Delta t)^2} = \frac{v \Delta t \omega \Delta t}{(\Delta t)^2} = v\omega = \frac{v^2}{r}$$

Uniform Circular Motion: Acceleration

$$\omega = \frac{v}{r}$$

centripetal acceleration



The magnitude of acceleration:

$$a = \frac{v^2}{r} = \omega^2 r = v\omega$$