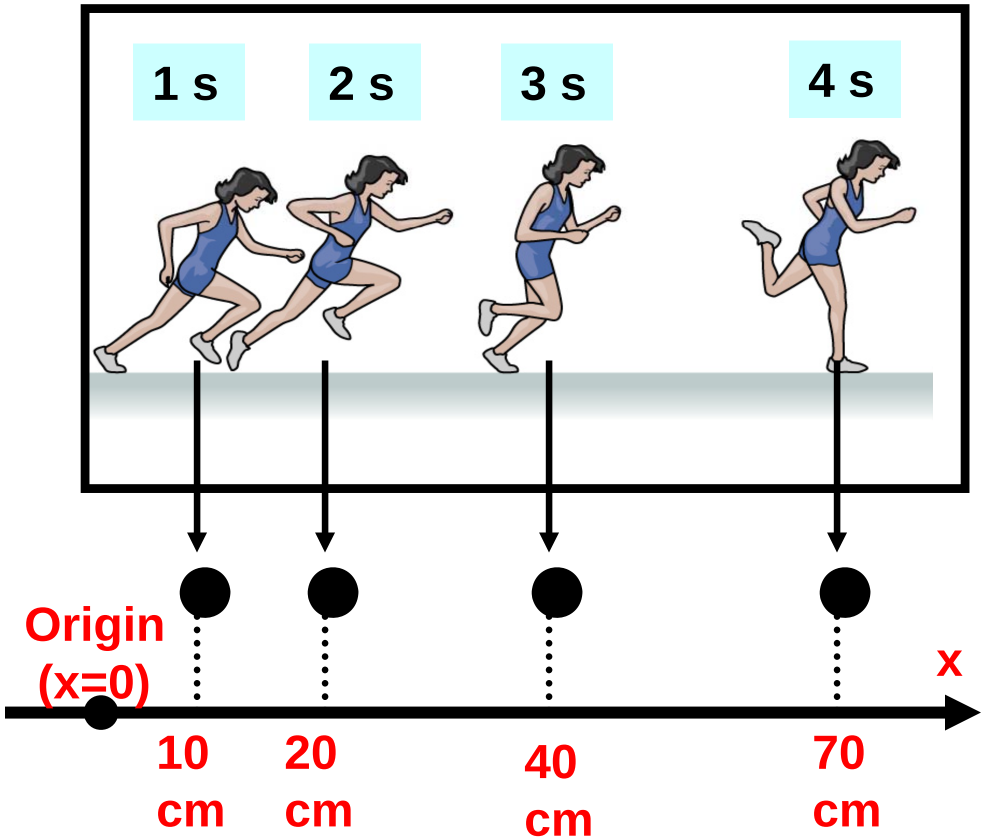
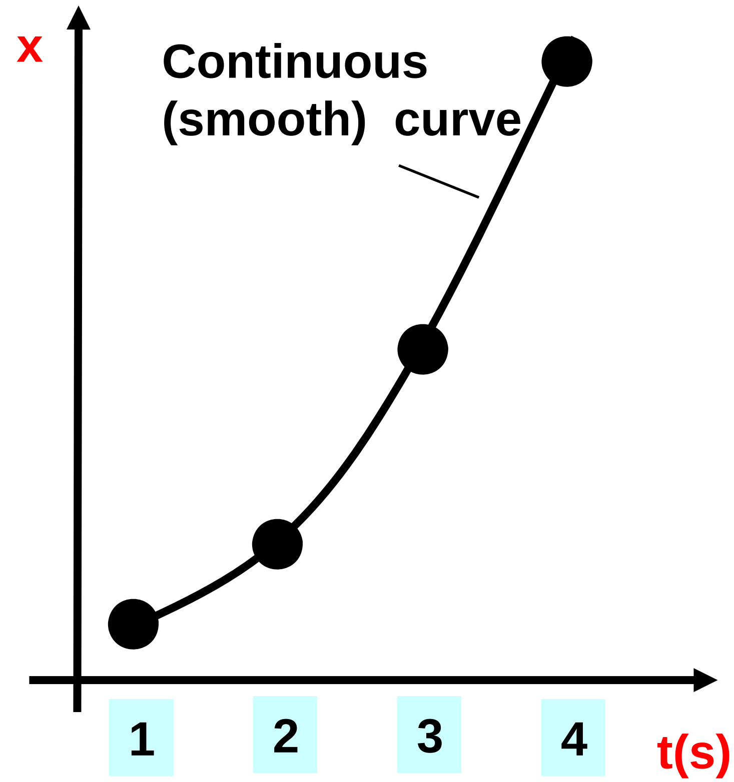
Kinematics: The Mathematics of Motion

Readings: Chapter 2

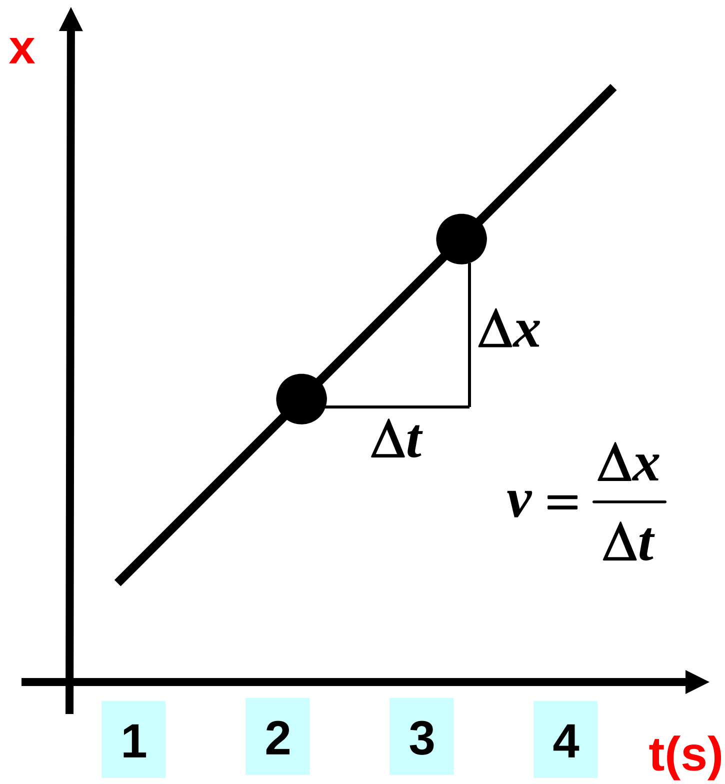
Motion along a straight line

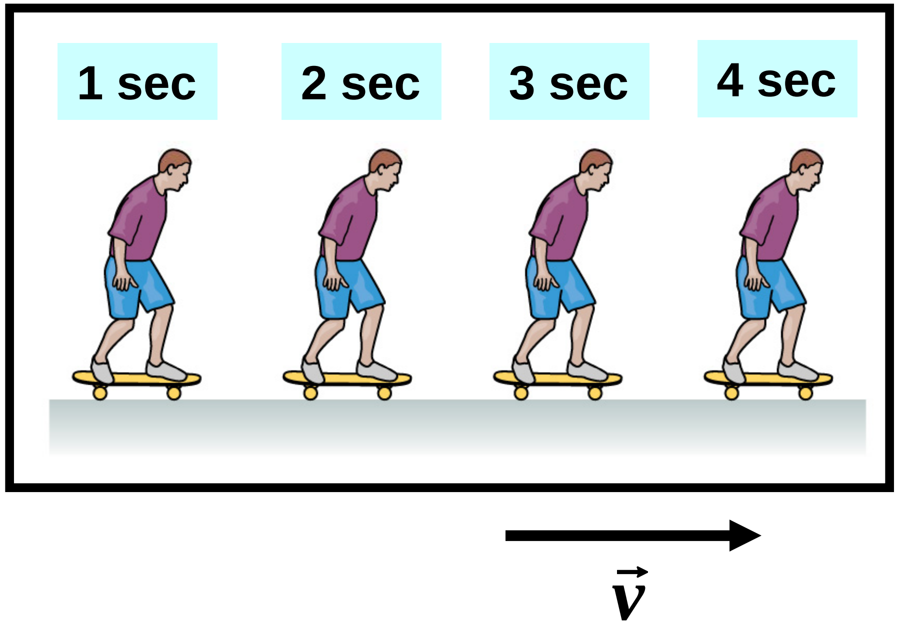


Can be illustrated by position-versus-time graph:



Position graph





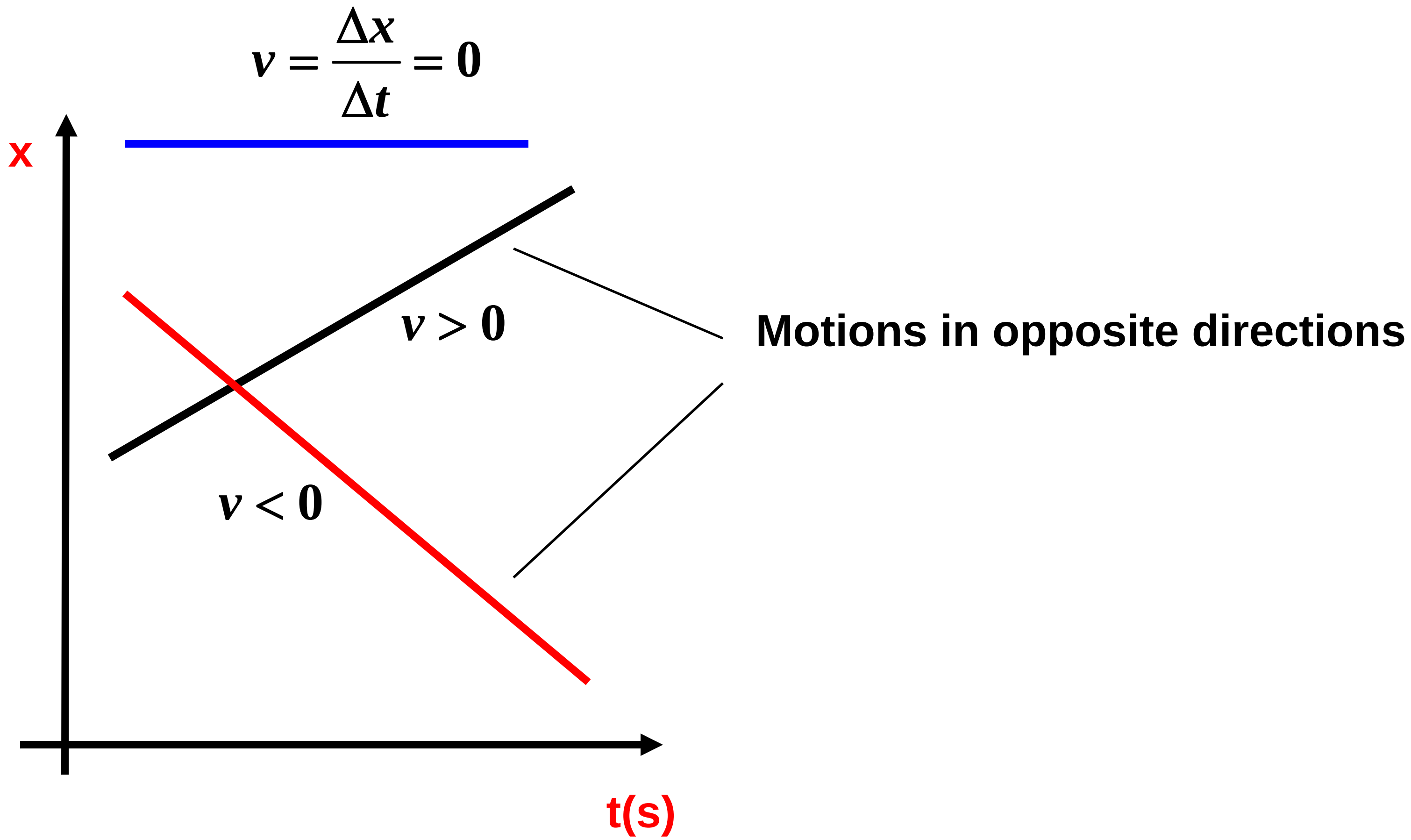
Velocity is the same – zero acceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = 0$$

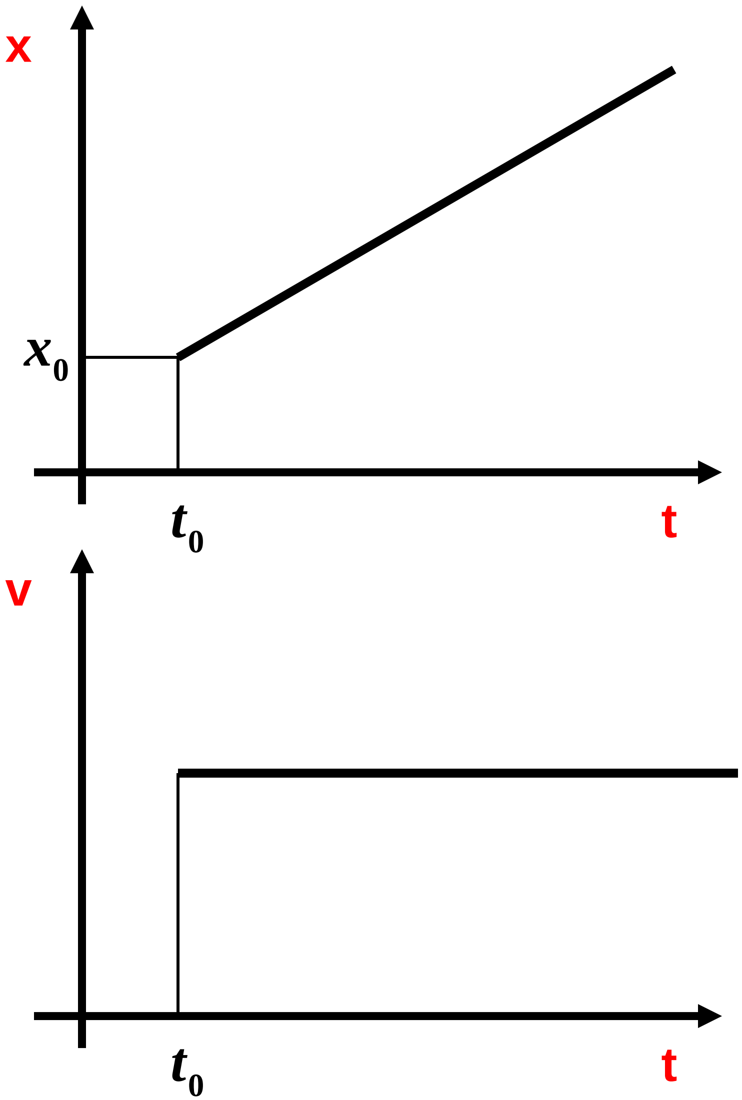
Straight line – uniform motion – the same velocity

Velocity is the slope

Position graph: uniform motion



Uniform motion: motion with constant velocity



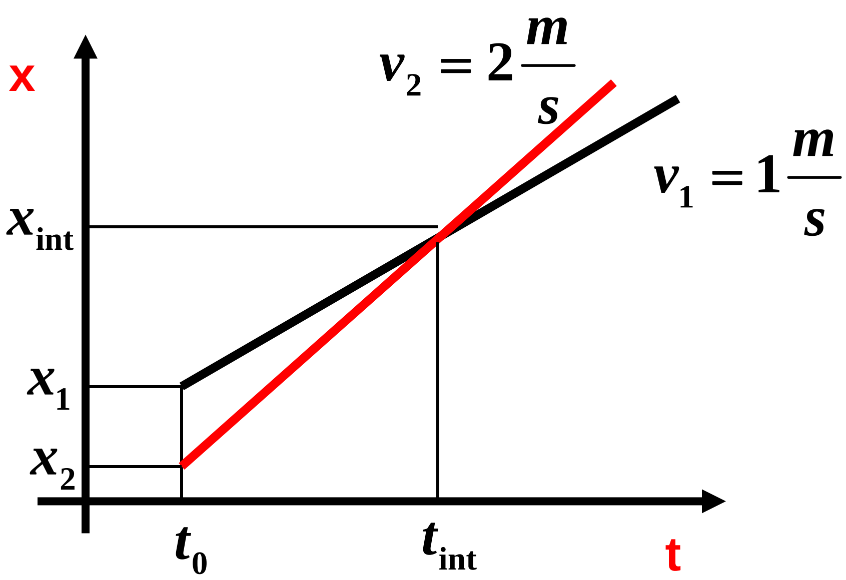
$$v = \frac{\Delta x}{\Delta t} = \frac{x_t - x_0}{t - t_0}$$

$$x_t = x_0 + v(t - t_0)$$

Uniform motion: application

$$x_t = x_0 + v(t - t_0)$$

Example:



$x_1 = 5m$ $\quad x_2 = 1m$ $\quad t_0 = 2s$

Find x_{int} and t_{int}

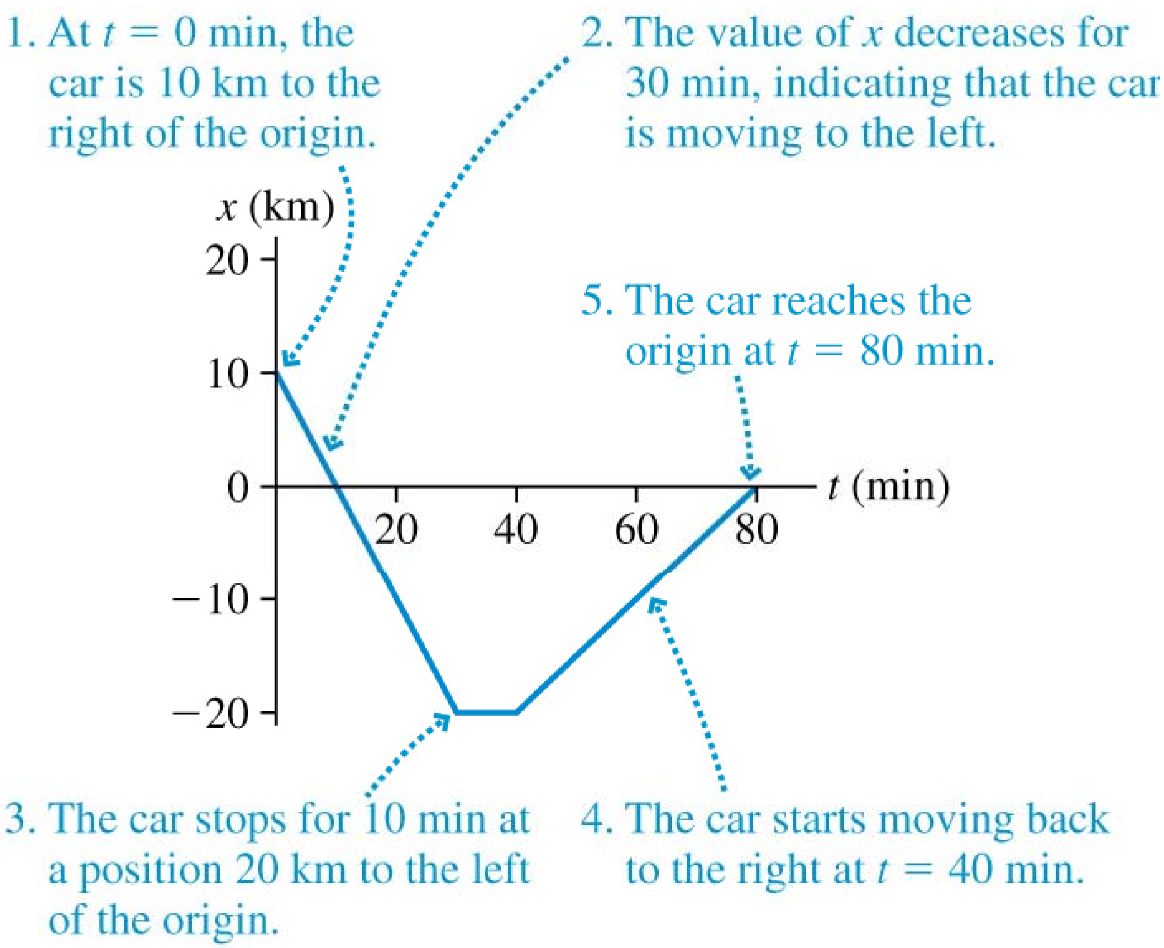
$$x_{int} = x_1 + v_1(t_{int} - t_0) = 5 + t_{int} - 2 = 3 + t_{int}$$

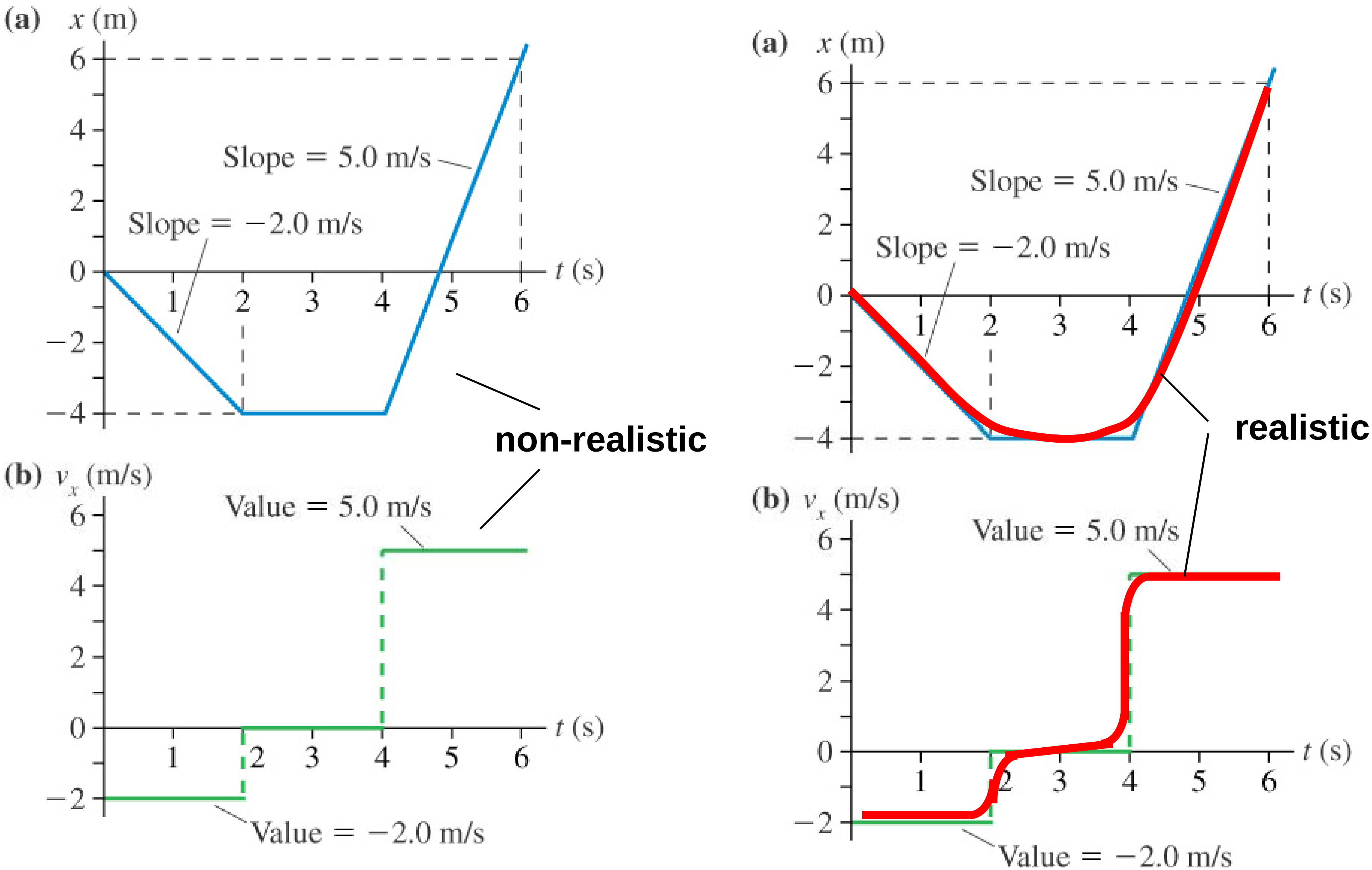
$$x_{int} = x_2 + v_2(t_{int} - t_0) = 1 + 2t_{int} - 4 = -3 + 2t_{int}$$

$$3 + t_{int} = -3 + 2t_{int} \qquad t_{int} = 6s$$

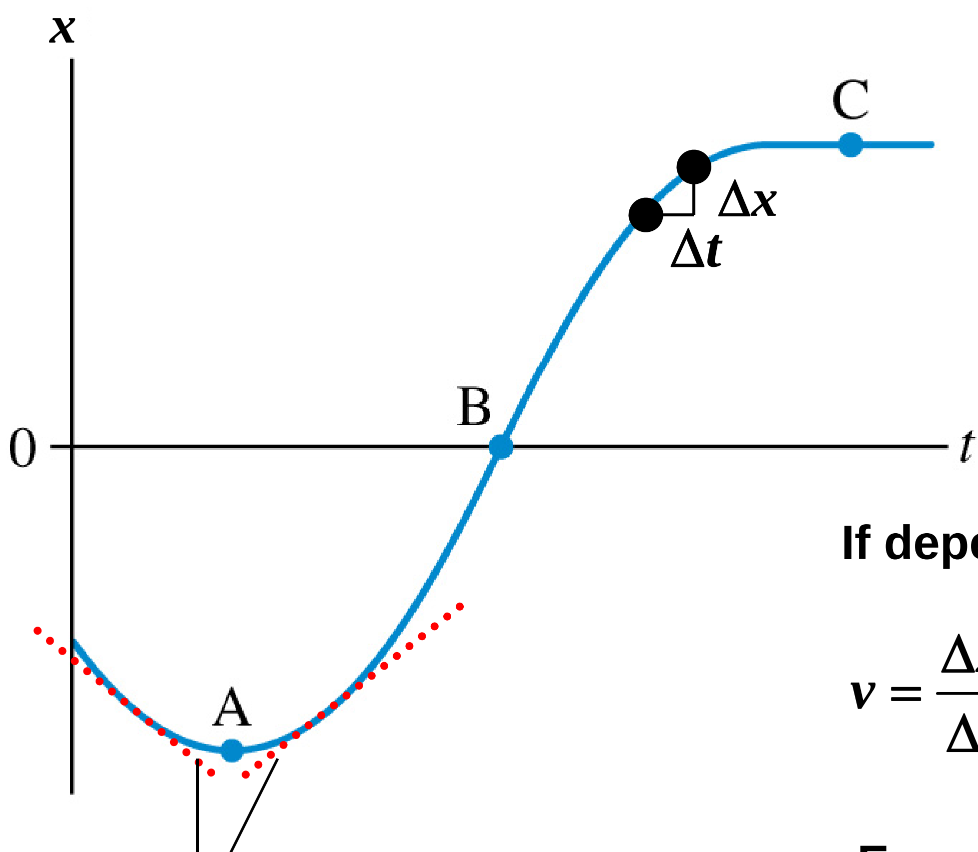
$$x_{int} = 3 + t_{int} = 3 + 6 = 9m$$

Position graph




(a) x (m)
Slope = 5.0 m/s
Slope = −2.0 m/s
t (s)
(b) v_x (m/s)
Value = 5.0 m/s
Value = −2.0 m/s
non-realistic
realistic

Instantaneous velocity



Instantaneous velocity - slope

Very small Δt

Instantaneous velocity:

$$v = \frac{\Delta x}{\Delta t}$$

If dependence $x(t)$ is given, then

$$v = \frac{\Delta x}{\Delta t} = \frac{dx(t)}{dt}$$ - derivative

Example:

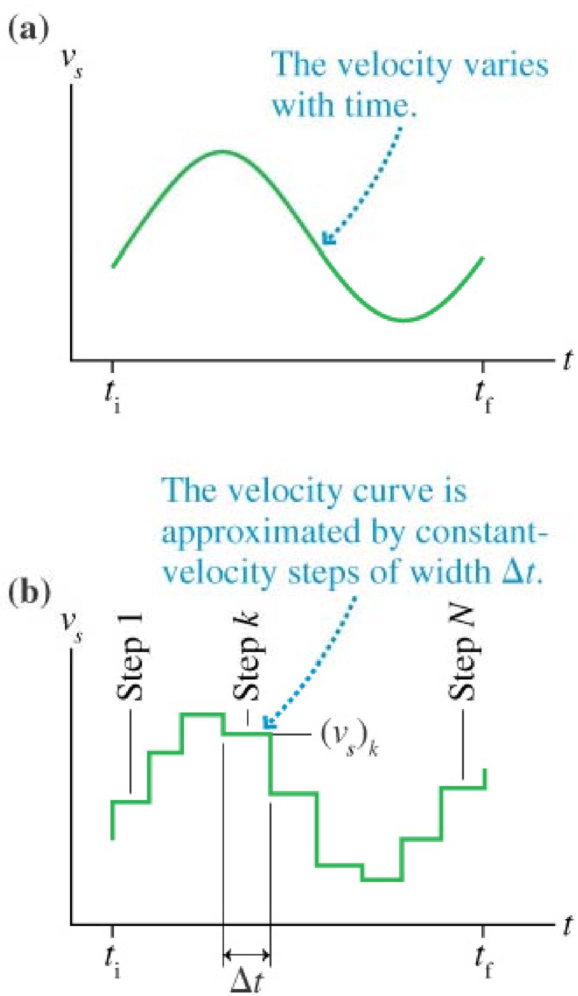
$$x(t) = 5t^4 \qquad v = \frac{dx(t)}{dt} = 20t^3$$

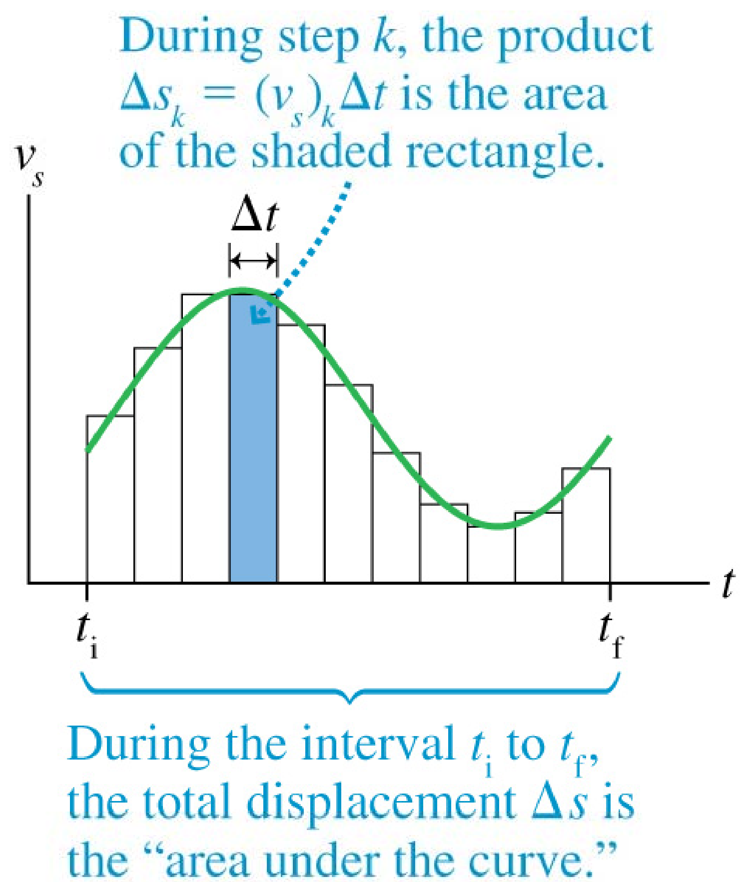
$$x(t) = 5\sin(t) \qquad v = \frac{dx(t)}{dt} = 5\cos(t)$$

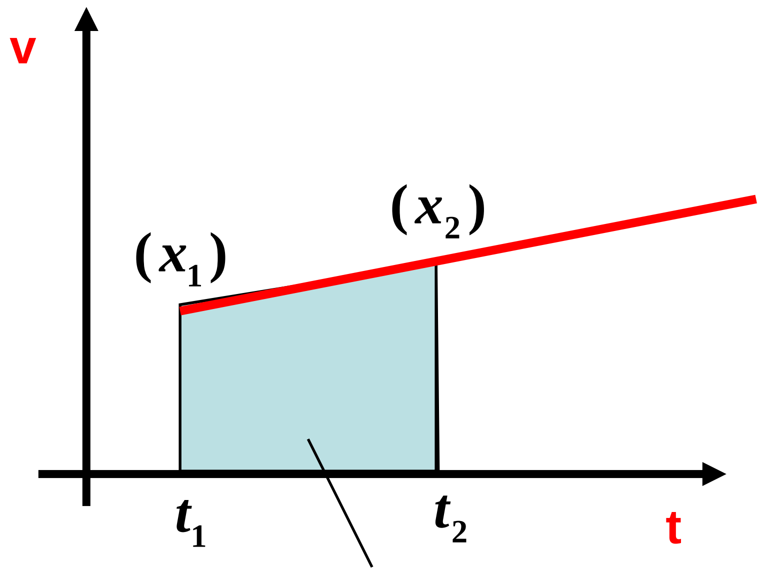
Finding position from velocity

$$v = \frac{dx(t)}{dt}$$

$$x(t) = \int v(t)dt$$





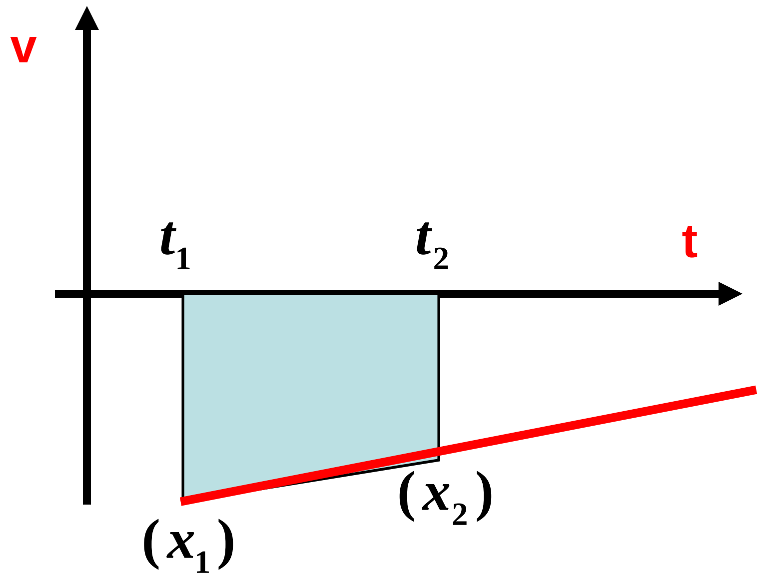


$$x_{2-1} = x_2 - x_1 = \int_{t_1}^{t_2} v(t)dt$$

The displacement is the area, displacement is positive

$$x_{2-1} = x_2 - x_1 = \int_{t_1}^{t_2} v(t)dt > 0$$

$$v(t) > 0$$



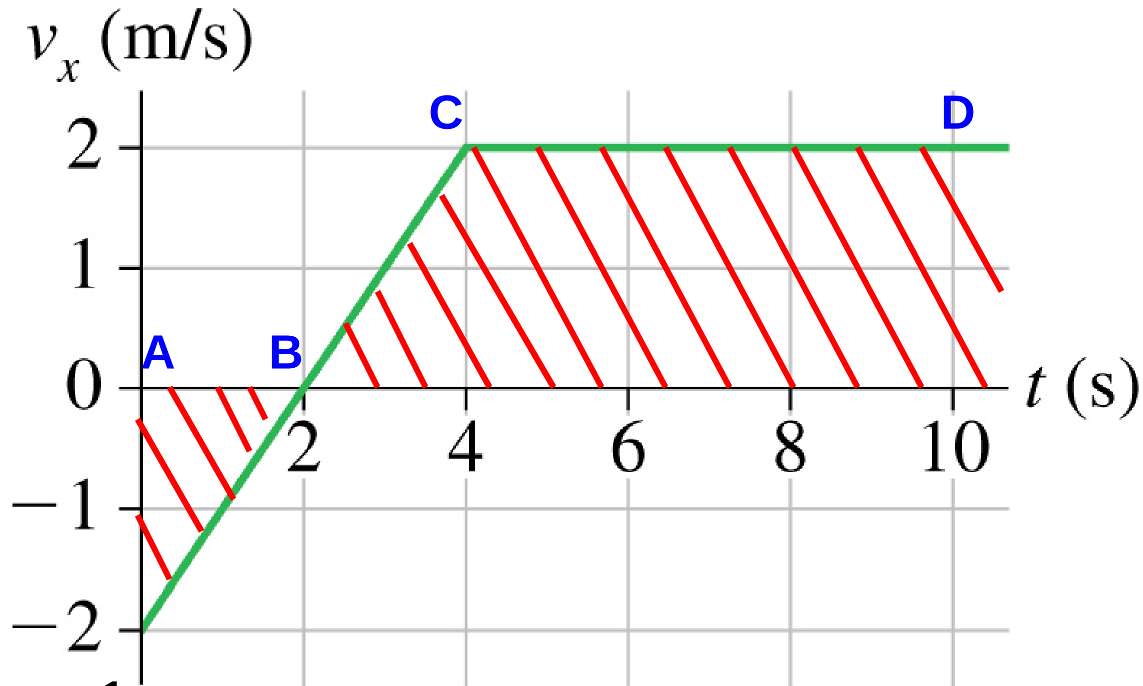
Displacement is negative

$$x_{2-1} = x_2 - x_1 = \int_{t_1}^{t_2} v(t)dt < 0$$

$$v(t) < 0$$

Example:

Find the net displacement



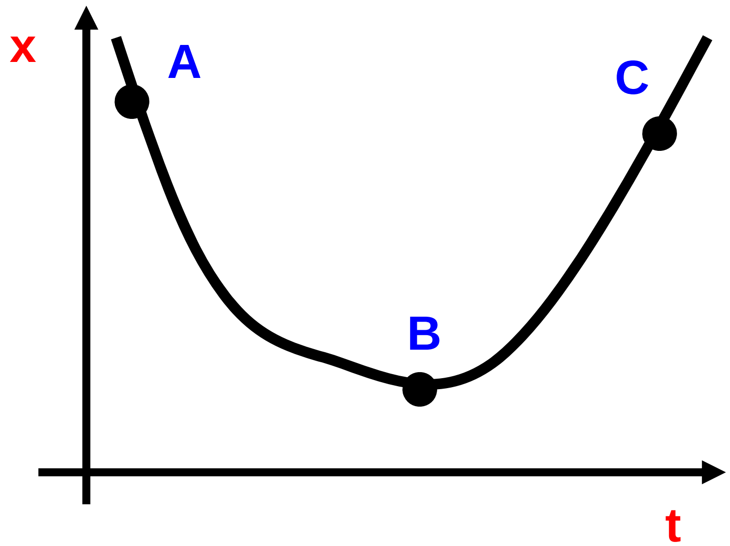
$$x_{A-B} = x_B - x_A = \int_{t_A}^{t_B} v(t)dt = -\frac{1}{2}2 \bullet 2 = -2m$$

area

$$x_{B-C} = x_C - x_B = \int_{t_B}^{t_C} v(t)dt = \frac{1}{2}2 \bullet 2 = 2m$$

$$x_{C-D} = x_D - x_C = \int_{t_C}^{t_D} v(t)dt = 2 \bullet (10-4) = 12m$$

$$x_{A-D} = x_D - x_A = x_D - x_C + x_C - x_B + x_B - x_A = x_{C-D} + x_{B-C} + x_{A-B}$$
$$= -2 + 2 + 12 = 12m$$

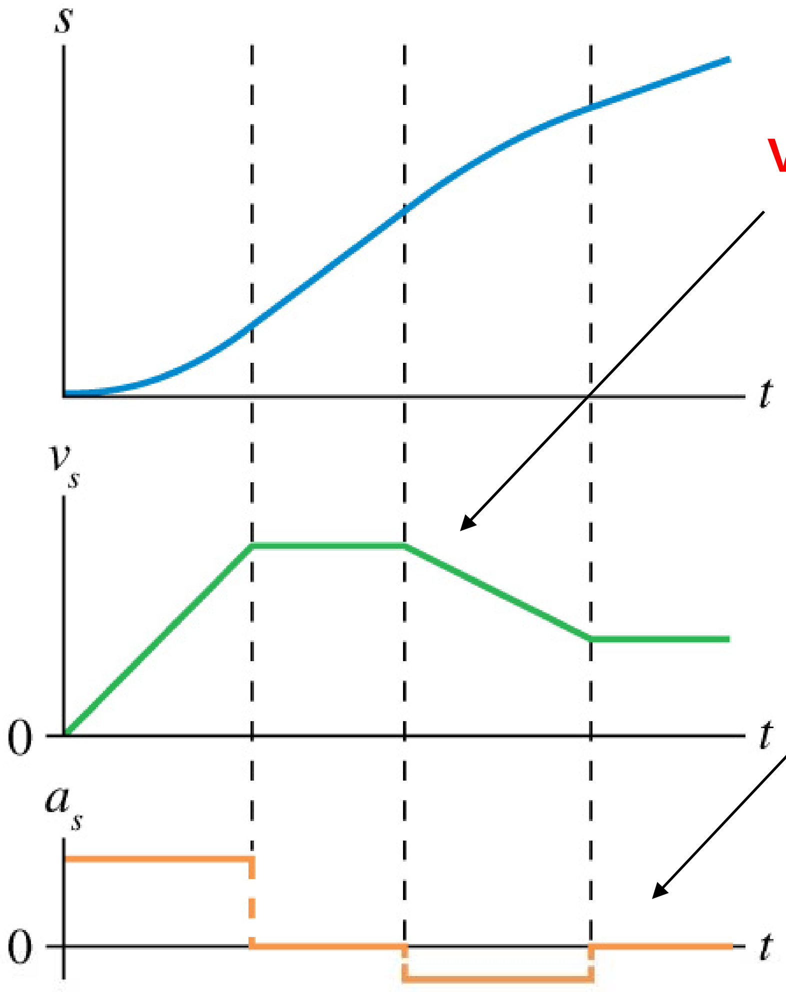


$$v_A < v_B < v_C$$

Velocity is the slope

$$v = \frac{ds}{dt}$$

$$v_A < 0 \qquad v_C > 0$$



Velocity is the slope

$$v = \frac{ds}{dt}$$

Acceleration is the slope

$$a = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Motion with constant acceleration

Motion with constant velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_t - x_0}{t - t_0}$$

$$x_t = x_0 + v(t - t_0)$$

Motion with constant acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_t - v_0}{t - t_0}$$

$$v_t = v_0 + a(t - t_0)$$

$$x_t = ?$$

$$v_t = \frac{dx_t}{dt}$$

$$x_t = x_0 + \int_{t_0}^{t} v_t dt = x_0 + \int_{t_0}^{t} (v_0 + a(t - t_0))dt = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

Motion with constant velocity or constant acceleration

Motion with constant velocity:

$$x_t = x_0 + v(t - t_0)$$

Motion with constant acceleration:

$$v_t = v_0 + a(t - t_0)$$

$$x_t = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

Useful relation:

$$(t - t_0) = \frac{v_t - v_0}{a}$$

$$x_t = x_0 + v_0\frac{v_t - v_0}{a} + \frac{1}{2}\frac{(v_t - v_0)^2}{a}$$

then

$$v_t^2 - v_0^2 = 2a(x_t - x_0)$$

Motion with constant velocity or constant acceleration

$$t_0 = 0$$

Motion with constant velocity:

$$x_t = x_0 + v\,t$$

Motion with constant acceleration:

$$v_t = v_0 + a\,t$$

$$x_t = x_0 + v_0\,t + \frac{1}{2}a\,t^2$$

$$v_t^2 - v_0^2 = 2a(x_t - x_0)$$

Free fall motion - motion with constant acceleration

y

$$a = -g = -9.8\frac{m}{s^2}$$

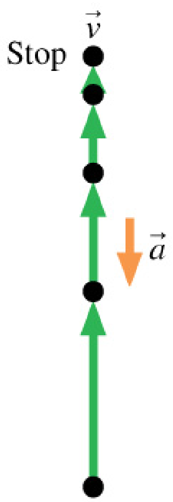
$\vec{g} = \text{constant}$

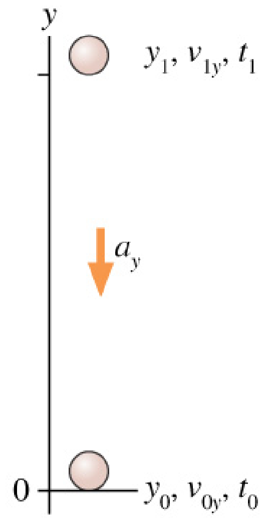
$$v_t = v_0 - g\,t$$

$$v_t^2 - v_0^2 = -2g(y_t - y_0)$$

$$y_t = y_0 + v_0\,t - \frac{1}{2}g\,t^2$$

Free fall motion - motion with constant acceleration





$$v_{final} = 0$$

$$v_0 = 2\,m/s$$
$$y_0 = 0$$

$$g = 9.8\frac{m}{s^2}$$

$$v_t = v_0 - g\,t$$
$$v_t^2 - v_0^2 = -2g(y_t - y_0)$$
$$y_t = y_0 + v_0\,t - \frac{1}{2}g\,t^2$$

$$v_t = 2 - 9.8\,t$$
$$v_t^2 - 2^2 = -2\times 9.8\times y_t$$
$$y_t = 2\,t - 4.9\,t^2$$

At final point:

$$v_{final} = 0$$

$$0 = 2 - 9.8\,t_f$$
$$-2^2 = -2\times 9.8\times y_f$$
$$y_f = 2\,t_f - 4.9\,t_f^2$$