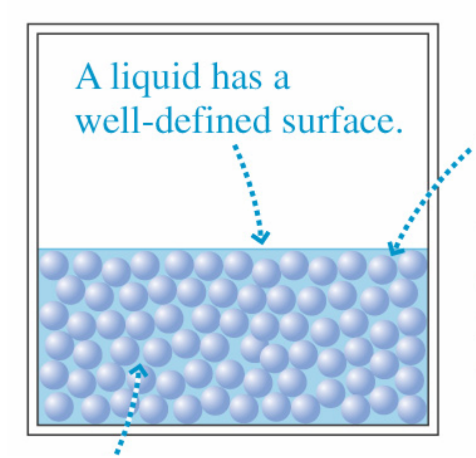
Fluids and Elasticity

Readings: Chapter 15

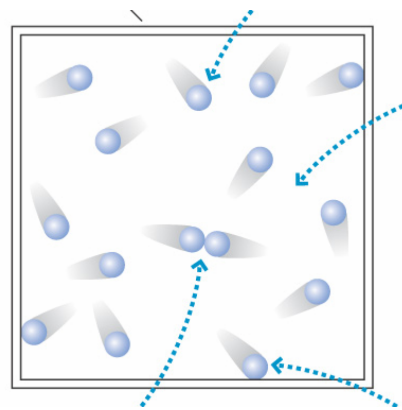
Solid, Liquid, Gas

-Solid has well-defined shape and well-defined surface. Solid is (nearly) incompressible. Specific positions of the molecules



-Liquid has well-defined surface (nor shape), It is (nearly) incompressible.

- Gases are compressible. They occupy all the volume.



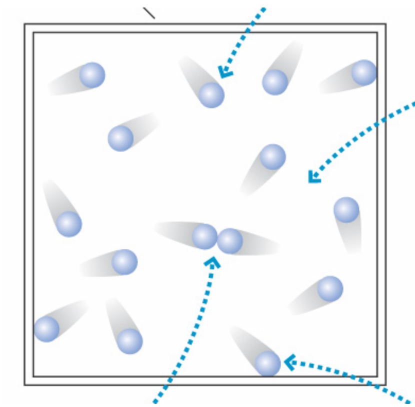
Liquid, Gas: Density

-Density is defined as a ratio of the mass of the object and occupied volume

$$\rho = \frac{m}{V}$$

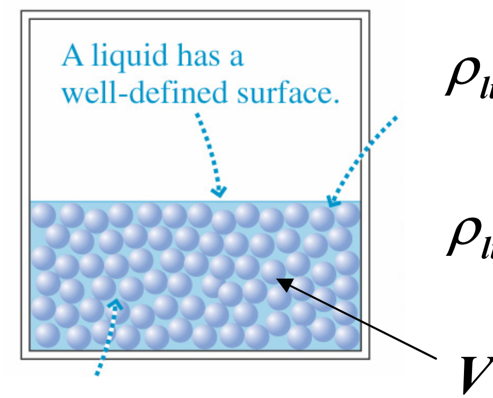
The mass of unit volume (1m x 1m x 1m)

Units: kg/m^3



$$\rho_{air} = \frac{m}{V}$$

$$\rho_{air} \sim 1\frac{kg}{m^3}$$

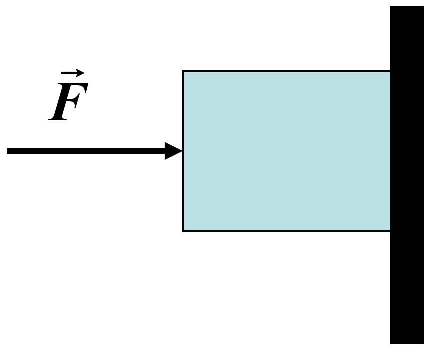


$$\rho_{liquid} = \frac{m}{V}$$

$$\rho_{liquid} \sim 1000\frac{kg}{m^3}$$

$$\rho_{air} << \rho_{liquid}$$

Liquid, Gas: Pressure

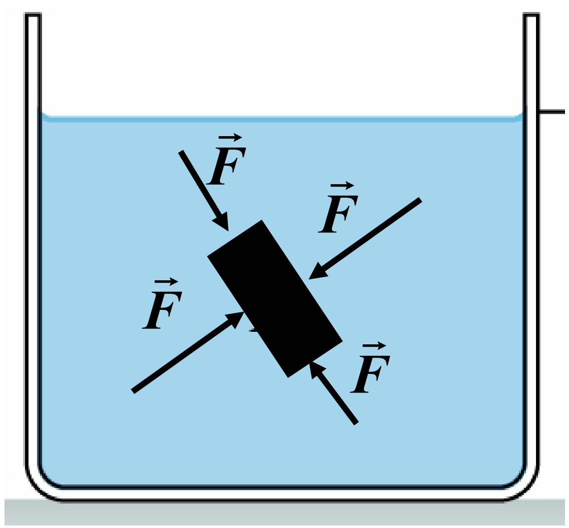


The force will be distributed over the area *A*,

We can define the force per unit area - pressure

$$p = \frac{F}{A}$$

If we place some object inside liquid (gas) then there will be normal force acting on the object

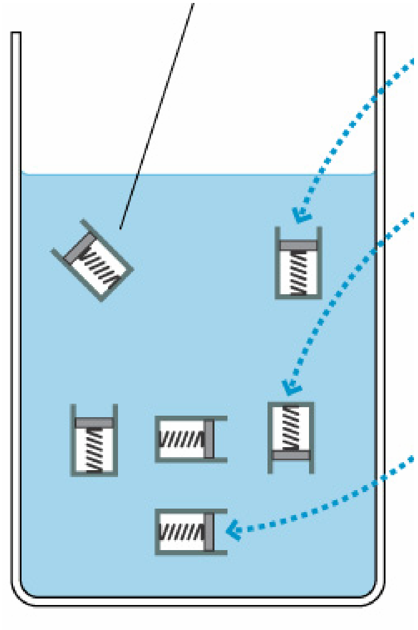


The force will depend on the area. We can define pressure. The pressure will be the same for all orientation of the object.

$$p = \frac{F}{A}$$

Liquid, Gas: Pressure

$$p = \frac{F}{A}$$

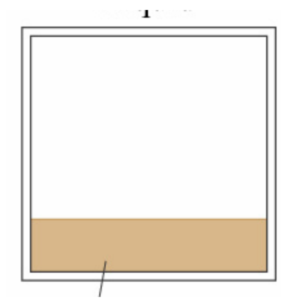


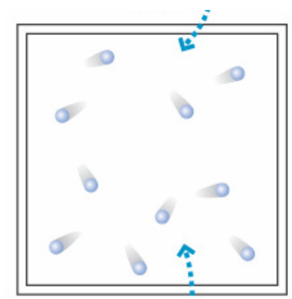
Pressure is a SCALAR. The force due to pressure will be perpendicular to the surface.

Units: Pascal (Pa)

$$Pa = \frac{N}{m^2}$$

Origin of pressure:





1. **Gravitational – in liquid - as gravity pull down the liquid exerts a force on the bottom and sides.**
2. **Thermal motion – in gasses – motion of atoms (molecules) results in collision with the walls.**

Atmospheric pressure – 1atm = 101300 Pa

Liquid: Pressure

$$F_{net} = 0$$

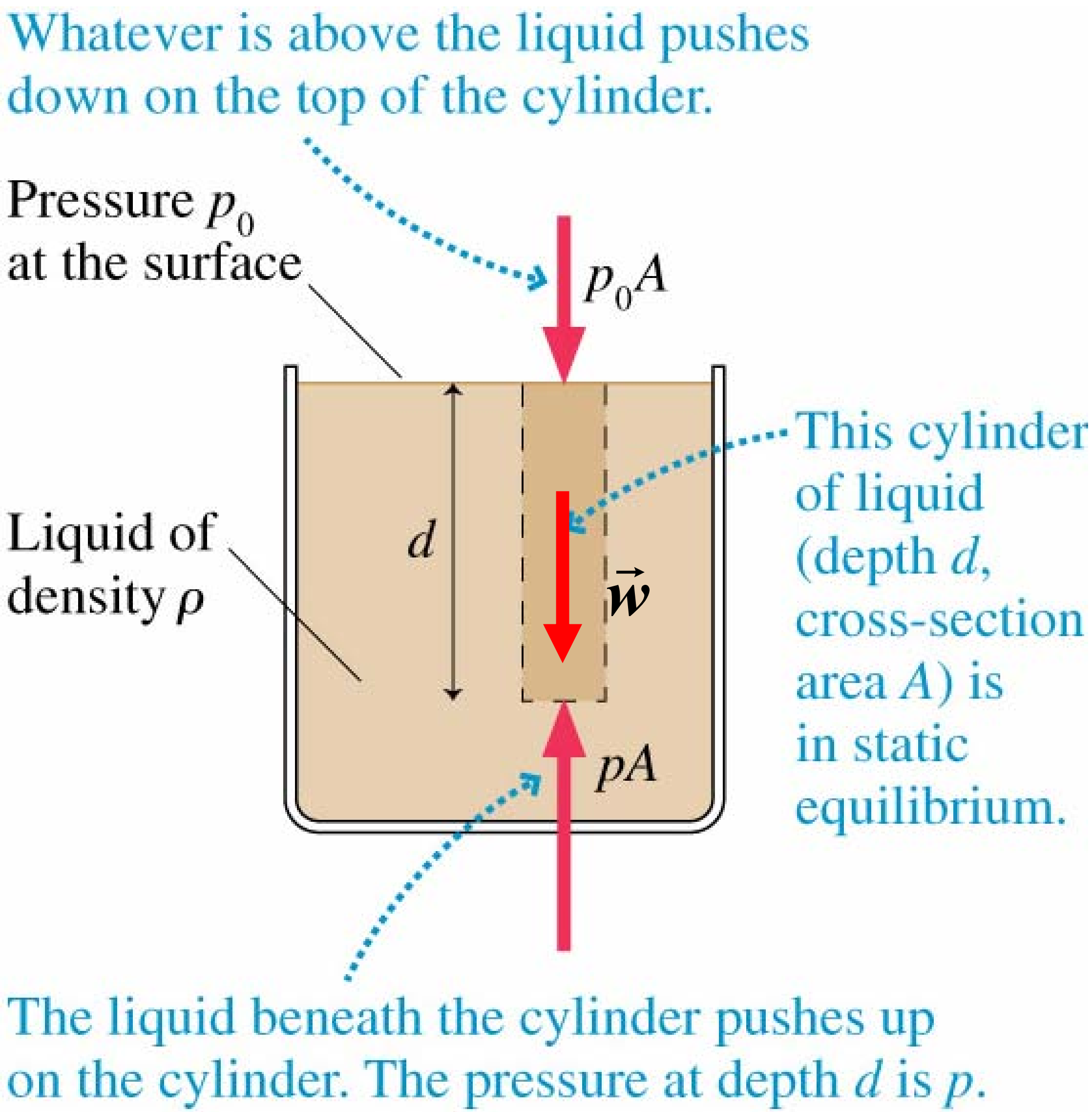
$$p_0 A + w = pA$$

$$w = mg = \rho A d g$$

Then

$$p_0 A + \rho A d g = pA$$

$$\boxed{p = p_0 + \rho d g}$$



Hydrostatic pressure at depth *d*

Liquid: Pressure

Example: What is the hydrostatic pressure of water at depth 10 m, 100 m, 1000 m

$$\boxed{p = p_0 + \rho d g}$$

$$\rho = 1000 kg / m^3$$

$$p_0 = p_{atm} = 10^5 \, Pa$$

$$p_{10} = p_0 + \rho d g = 10^5 + 1000 \cdot 10 \cdot 10 = 2 \cdot 10^5 = 2 p_{atm}$$

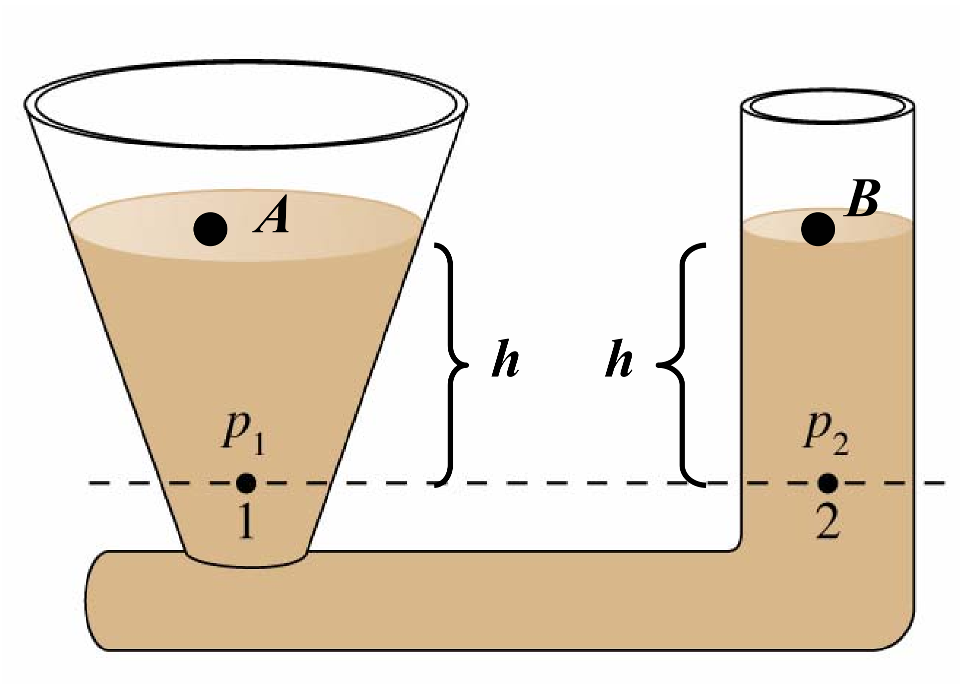
$$p_{100} = p_0 + \rho d g = 10^5 + 1000 \cdot 100 \cdot 10 = 11 \cdot 10^5 = 11 p_{atm}$$

$$p_{1000} = p_0 + \rho d g = 10^5 + 1000 \cdot 1000 \cdot 10 = 101 \cdot 10^5 = 101 p_{atm}$$

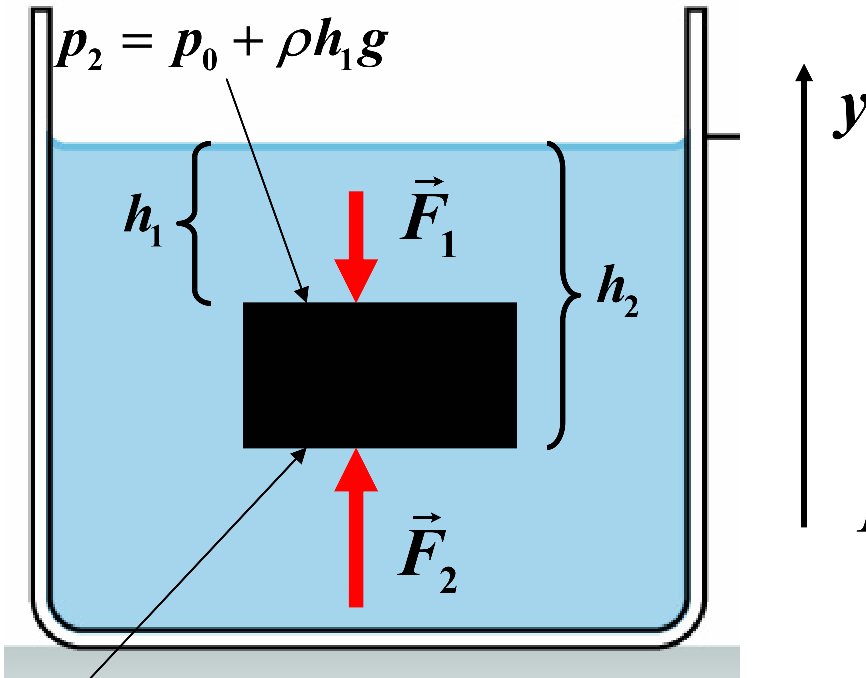
Pressure at point A is the same as the pressure at point B – atmospheric pressure, p_0

Pressure at point 1 is $p_1 = p_0 + \rho h g$

Pressure at point 2 is $p_2 = p_0 + \rho h g$



Liquid: Buoyancy force: Archimede's principle



$p_2 = p_0 + \rho h_2 g$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$F_{net,y} = F_2 - F_1$$

$$F_1 = p_1 A = (p_0 + \rho_{liquid} g h_1) A$$

$$F_2 = p_2 A = (p_0 + \rho_{liquid} g h_2) A$$

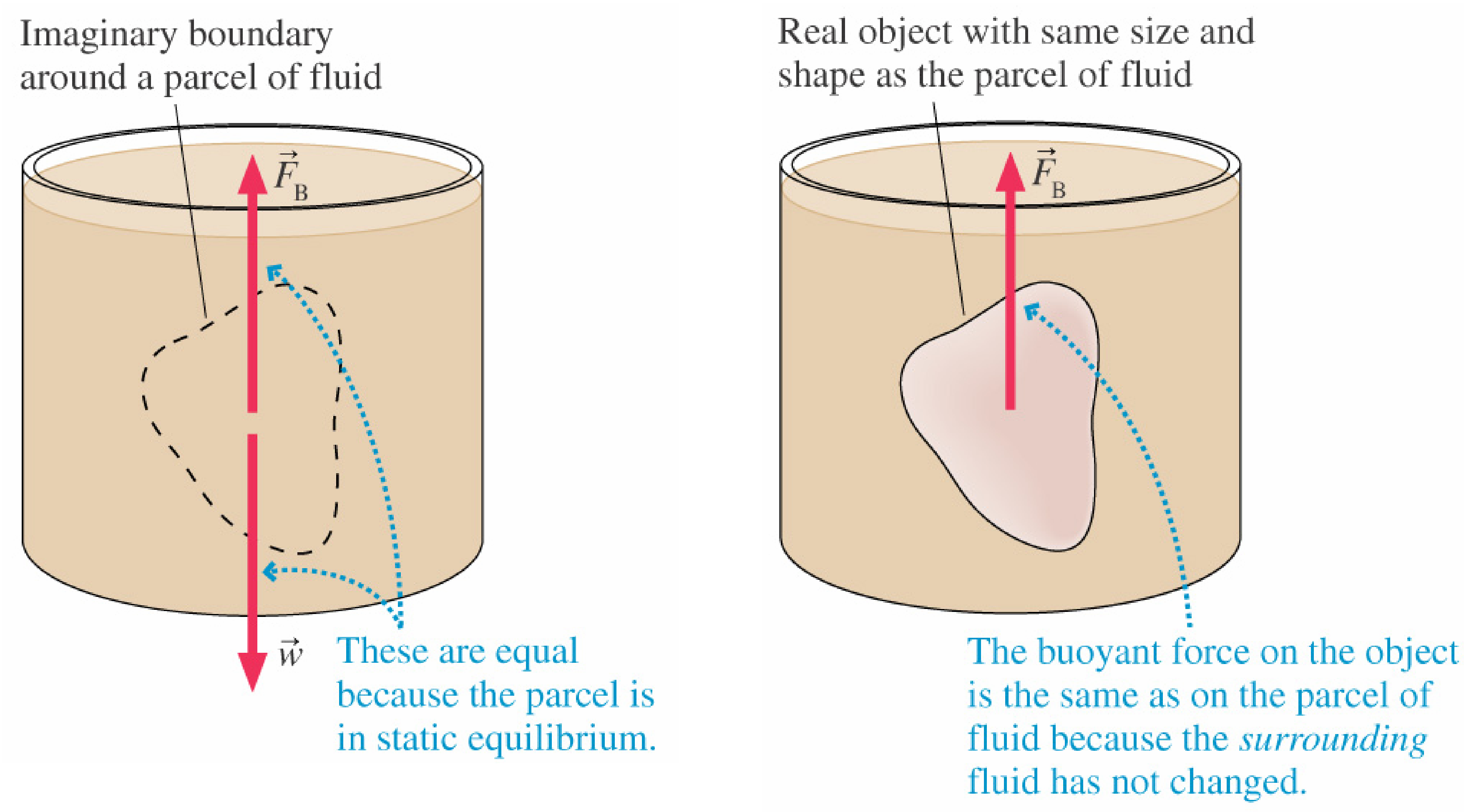
$$F_{net,y} = (p_0 + \rho_{liquid} g h_2) A - (p_0 + \rho_{liquid} g h_1) A =$$

$$= \rho_{liquid} g (h_2 - h_1) A = \rho_{liquid} V g$$

$$\boxed{F_B = \rho_{liquid} V g}$$

Archimede's principle

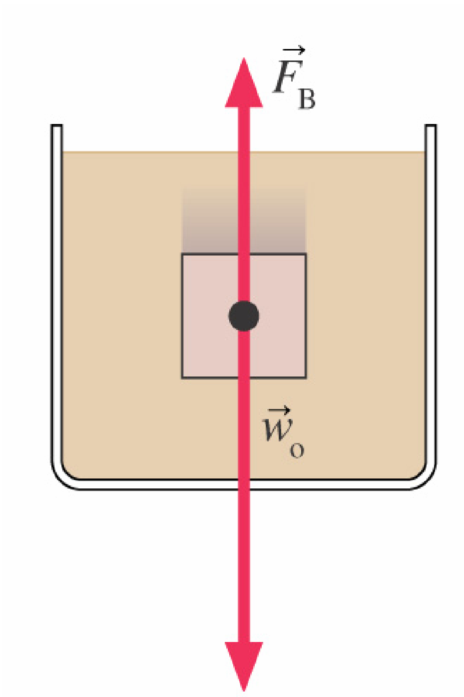
Liquid: Buoyancy force: Archimede's principle



$$F_B = \rho_{liquid} V g$$

Liquid: Buoyancy force: Archimede's principle

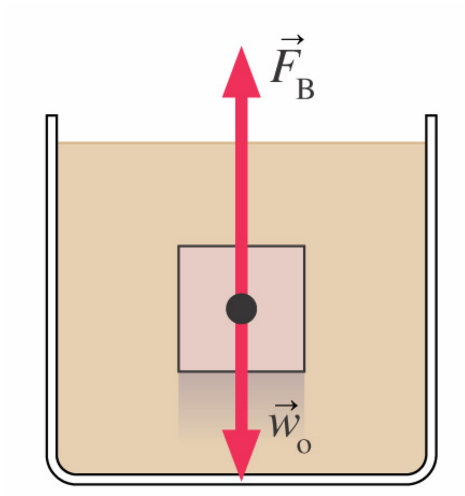
$$F_B = \rho_{liquid} V g \qquad w = m_{object} g = \rho_{object} V g$$



$$w > F_B$$

$$\rho_{object} > \rho_{liquid}$$

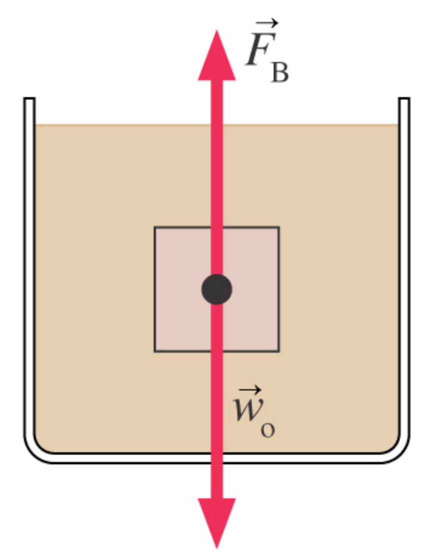
Object sinks



$$w < F_B$$

$$\rho_{object} < \rho_{liquid}$$

Object floats



$$w = F_B$$

$$\rho_{object} = \rho_{liquid}$$

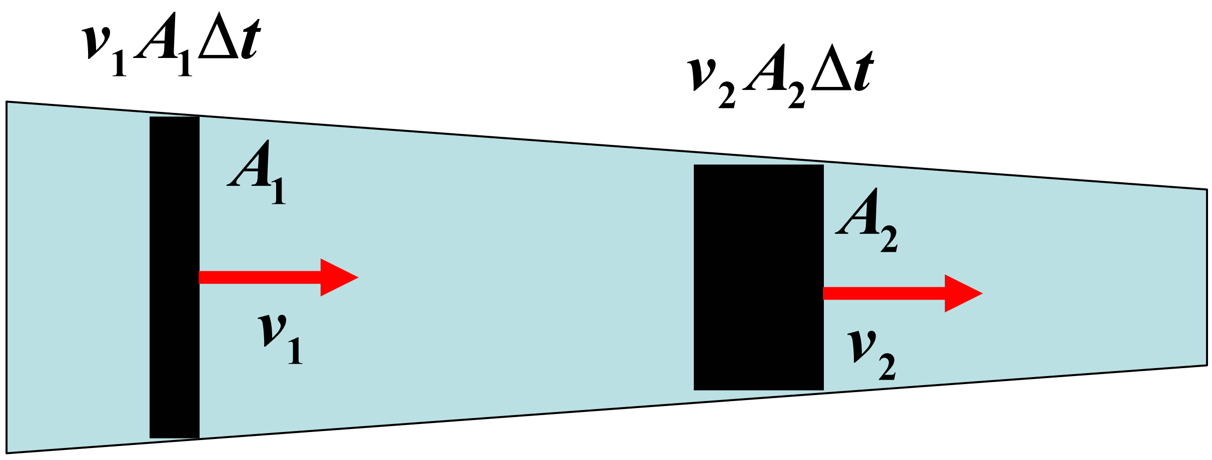
Equilibrium

Liquid: Fluid Dynamics: Equation of Continuity

Ideal-fluid model:

1. **Liquid is incompressible**
2. **Liquid is nonviscous (no friction)**
3. **Flow is steady**

Liquid is incompressible: Equation of Continuity $v_1 A_1 = v_2 A_2$

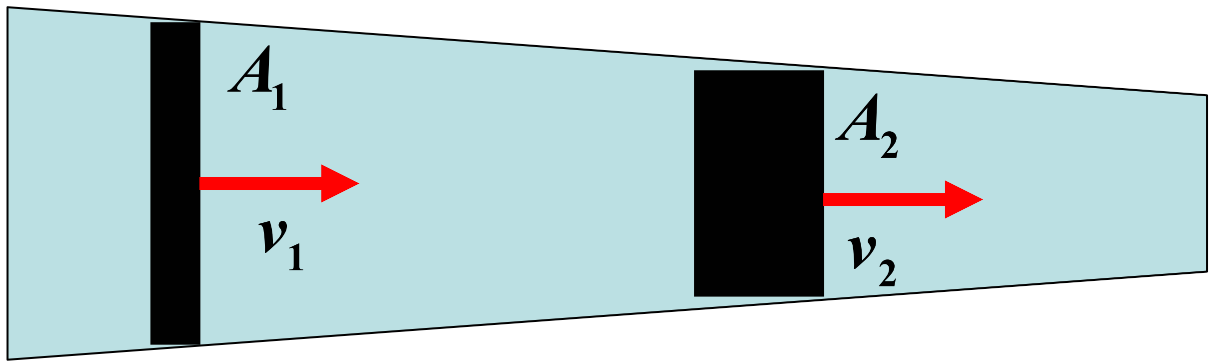


The same volume of fluid crosses both planes

Liquid: Fluid Dynamics: Bernoulli's Equation

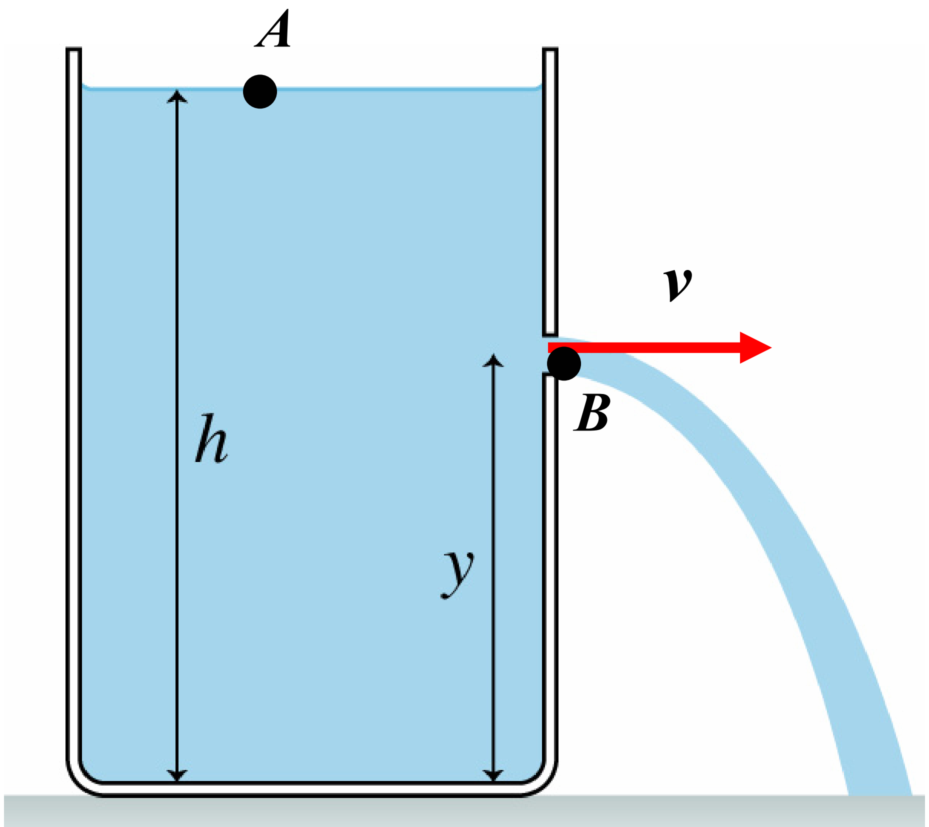
No friction: conservation of energy

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$



$$v_1 A_1 = v_2 A_2$$

Example: find v-?



At point A: $p_A = p_0$

$v_A = 0$

$h_A = h$

At point B: $p_B = p_0$

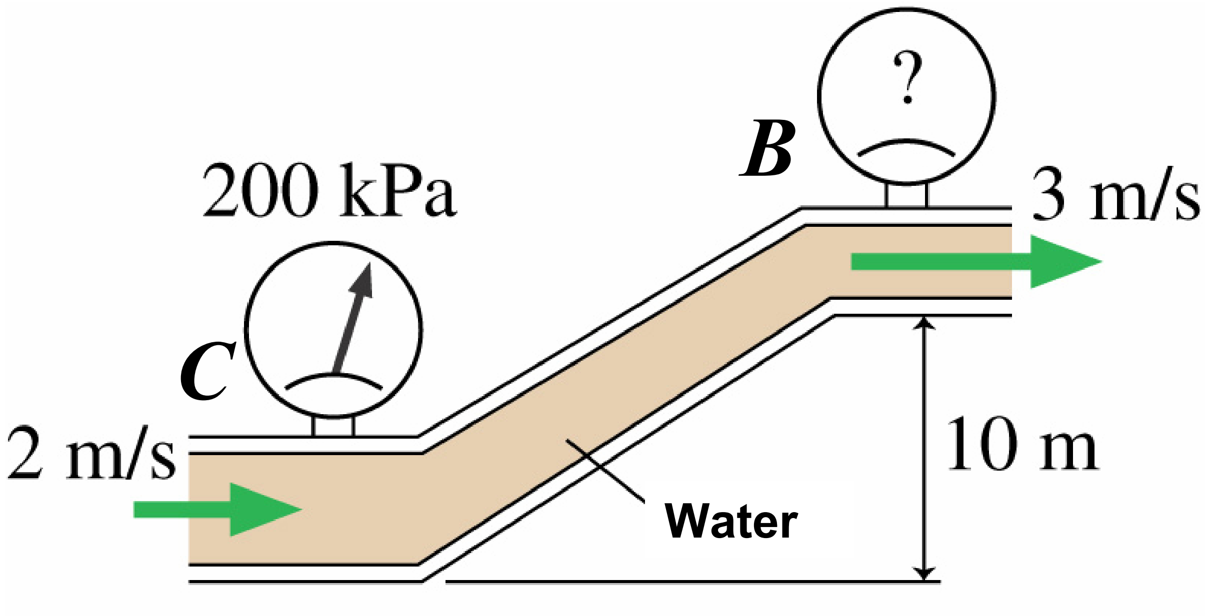
$v_B = v$

$h_B = y$

$$p_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

$$\rho g h = \frac{1}{2}\rho v^2 + \rho g y \qquad v = \sqrt{2g(h-y)}$$

Example: find *p* at point B



$$v_1 A_1 = v_2 A_2$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 =$$

$$= p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Continuity equation: $v_C A_C = v_B A_B$

Bernoulli's equation: $p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C = p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$

$$h_B = 10m$$

$$h_c = 0$$

$$\rho = 1000 kg / m^3$$

$$p_B = p_C + \frac{1}{2}\rho(v_C^2 - v_B^2) - \rho g h_B = 102500 Pa$$