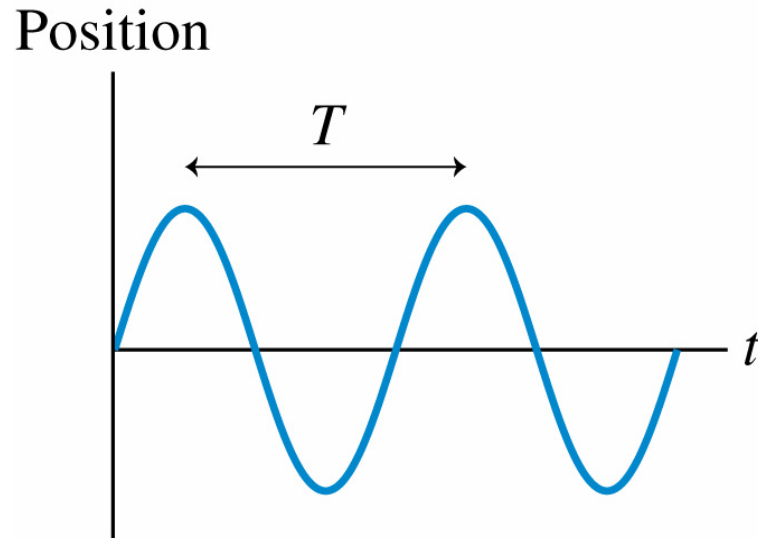
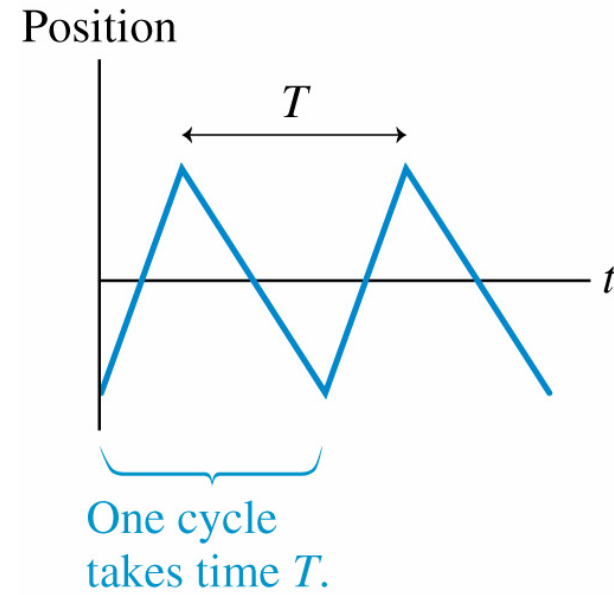
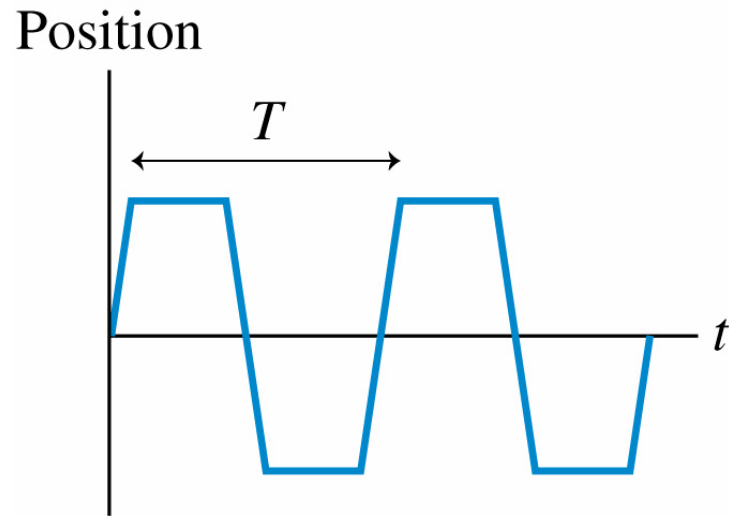


Oscillations

Readings: Chapter 14

Oscillation: Periodic Motion

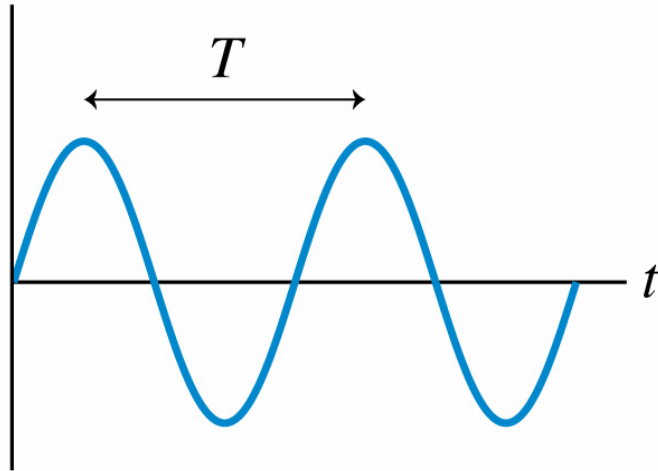


T – period of motion

$$f = \frac{1}{T} \text{ - frequency}$$

Oscillation: Periodic Motion: Simple Harmonic Motion

Position



Simple Harmonic Motion – sinusoidal oscillation

$$x(t) = A \cos\left(\frac{2\pi}{T}t\right) = A \cos(2\pi ft) \quad \text{or} \quad x(t) = A \sin\left(\frac{2\pi}{T}t\right) = A \sin(2\pi ft)$$

The most general expression for sinusoidal motion

$$x(t) = A \cos\left(\frac{2\pi}{T}t + \varphi_0\right) = A \cos(2\pi ft + \varphi_0)$$

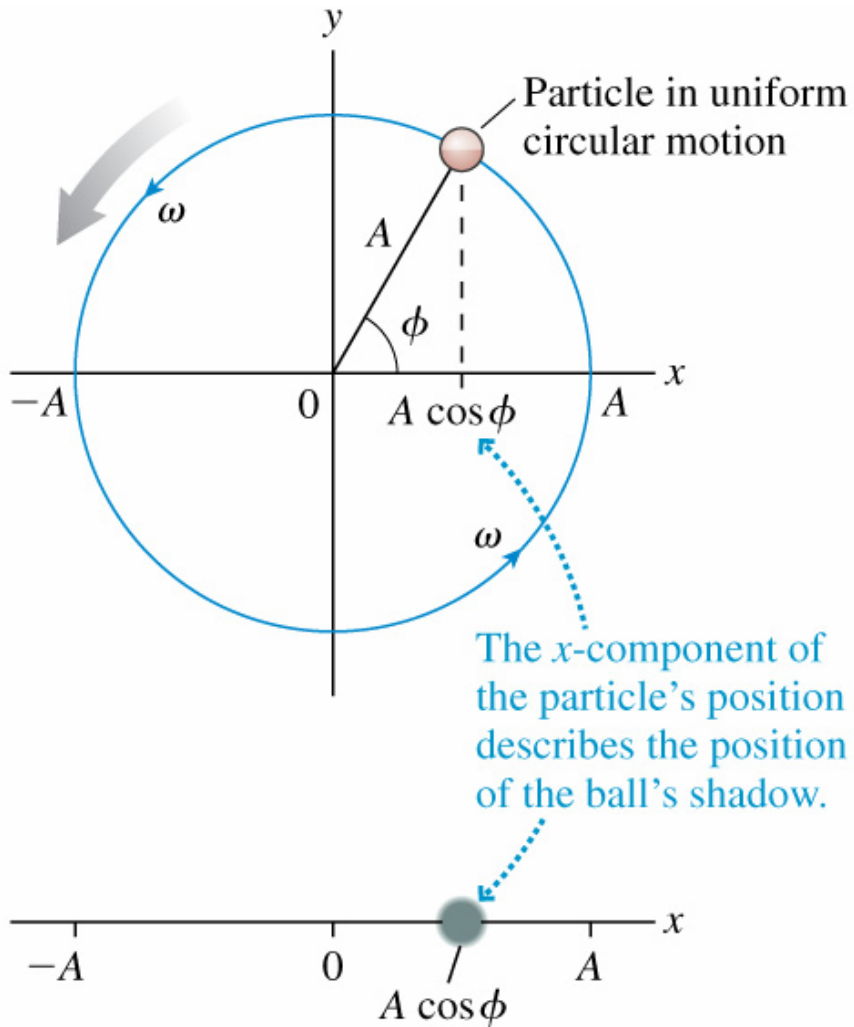
φ_0 - phase constant

A - amplitude

If $\varphi_0 = 90^\circ$ then $x(t) = A \cos(2\pi ft + 90^\circ) = A \sin(2\pi ft)$

Oscillation: Periodic Motion: Simple Harmonic Motion

Example: Uniform circular motion

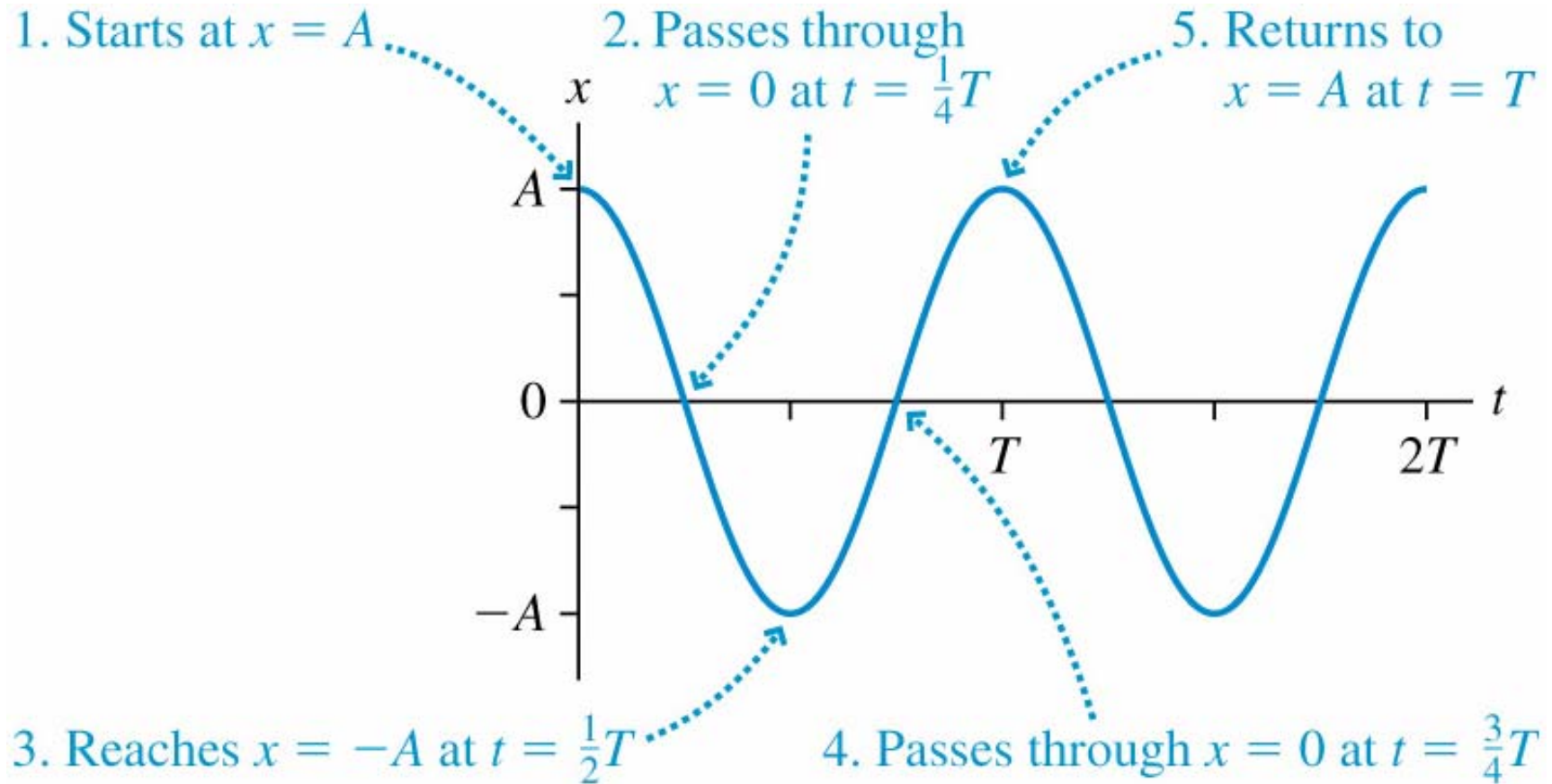


x- component:

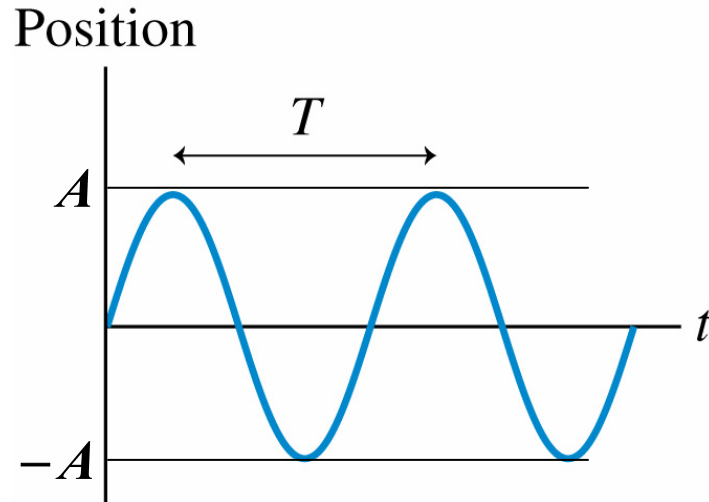
$$x(t) = A \cos(\phi) = A \cos(\omega t)$$

ω - angular frequency

$$x(t) = A \cos\left(\frac{2\pi}{T}t\right) = A \cos(2\pi ft)$$



Oscillation: Periodic Motion: Simple Harmonic Motion



$$x(t) = A \cos\left(\frac{2\pi}{T}t + \varphi_0\right) = A \cos(2\pi ft + \varphi_0) = A \cos(\omega t + \varphi_0)$$

A - amplitude

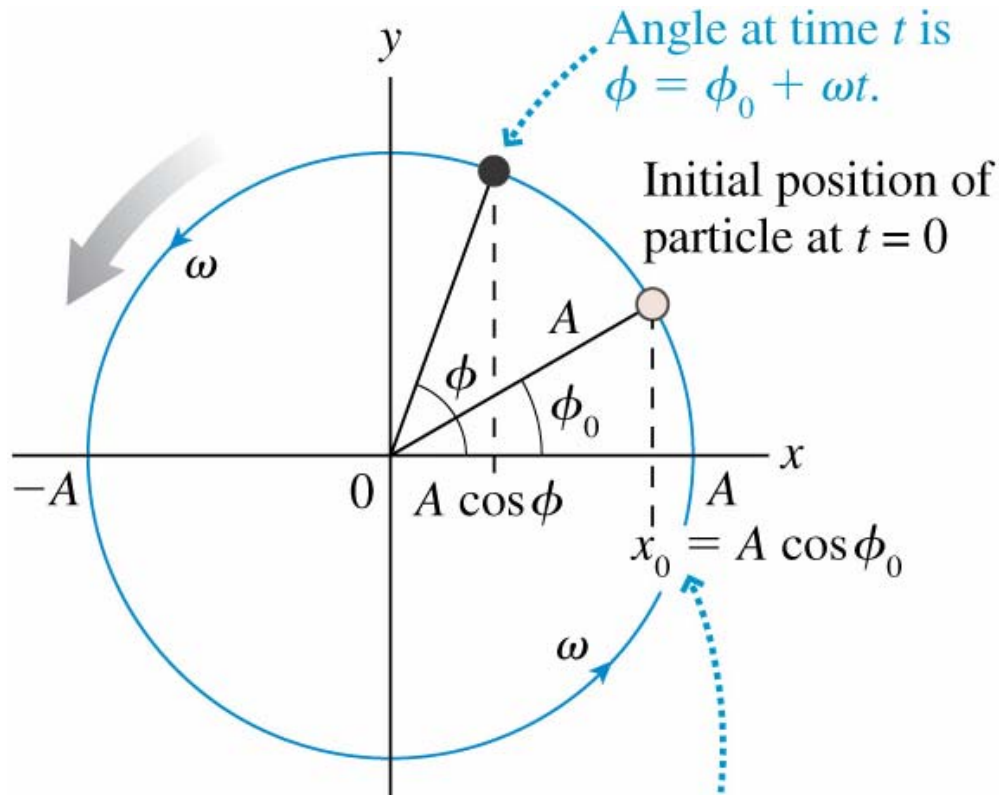
φ_0 - phase constant

T – period of motion (units – s)

$f = \frac{1}{T}$ - frequency (units – hertz – Hz = 1/s)

$\omega = 2\pi f = 2\pi \frac{1}{T}$ - angular frequency
(units – rad/s)

$$x(t) = A \cos\left(\frac{2\pi}{T}t + \varphi_0\right) = A \cos(2\pi ft + \varphi_0) = A \cos(\omega t + \varphi_0)$$



φ_0 - phase constant

Phase constant specifies the initial position of the oscillator

The initial x -component of the particle's position can be anywhere between $-A$ and A , depending on ϕ_0 .

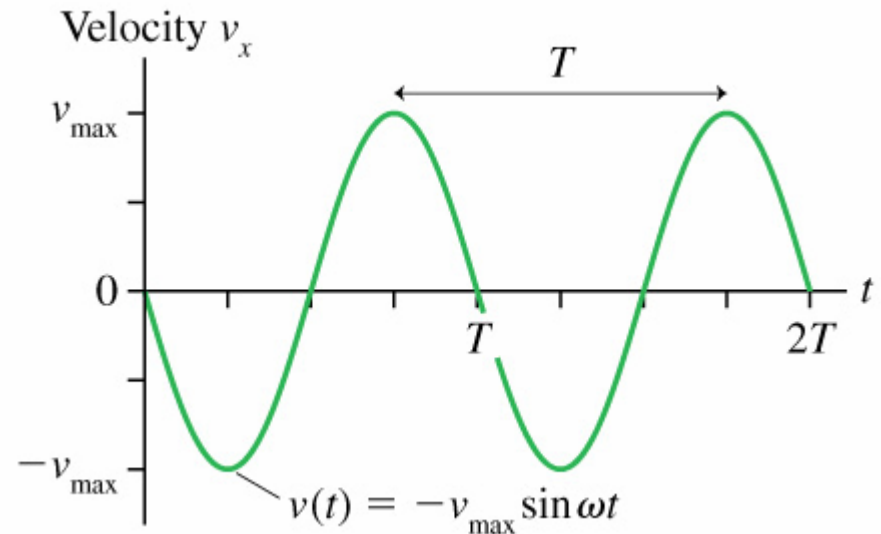
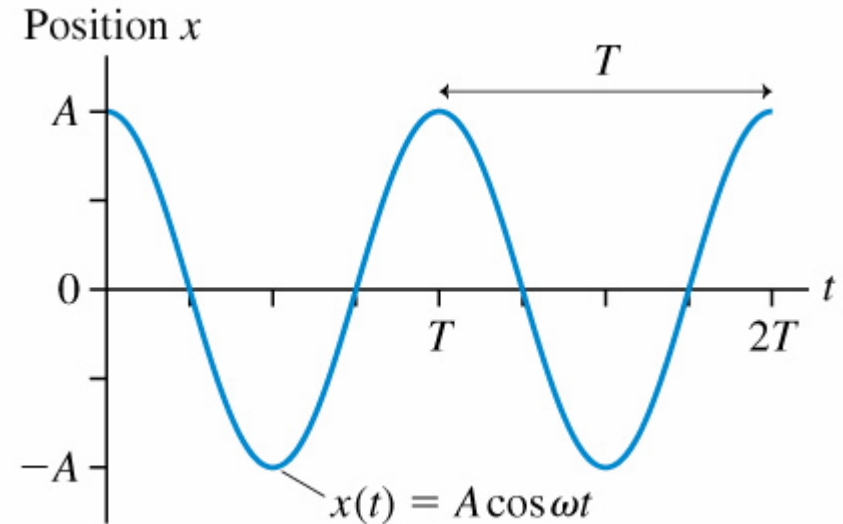
Simple Harmonic Motion: Position, Velocity, and Acceleration

$$x(t) = A \cos(\omega t + \varphi_0)$$

$$\begin{aligned} v(t) &= \frac{dx(t)}{dt} = A \frac{d \cos(\omega t + \varphi_0)}{dt} = \\ &= -\omega A \sin(\omega t + \varphi_0) = -v_{\max} \sin(\omega t + \varphi_0) \end{aligned}$$

$$v_{\max} = \omega A$$

$$\begin{aligned} a(t) &= \frac{dv(t)}{dt} = -\omega A \frac{d \sin(\omega t + \varphi_0)}{dt} = \\ &= -\omega^2 A \cos(\omega t + \varphi_0) = -\omega^2 x(t) \end{aligned}$$



Simple Harmonic Motion: object oscillating on a spring

$$x(t) = A \cos(\omega t + \varphi_0)$$

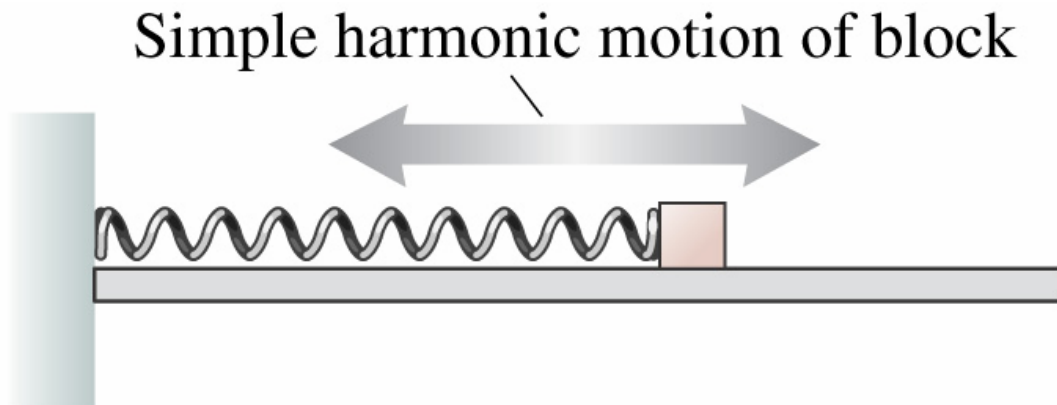
$$a(t) = -\omega^2 A \cos(\omega t + \varphi_0) = -\omega^2 x(t)$$

Newton's second law:

$$ma = F = -m\omega^2 x = -kx$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$F = -kx \quad \text{Hooke's law:}$$



Simple Harmonic Motion: Conservation of energy

$$x(t) = A \cos(\omega t + \varphi_0)$$

$$v(t) = -\omega A \sin(\omega t + \varphi_0)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Kinetic energy:

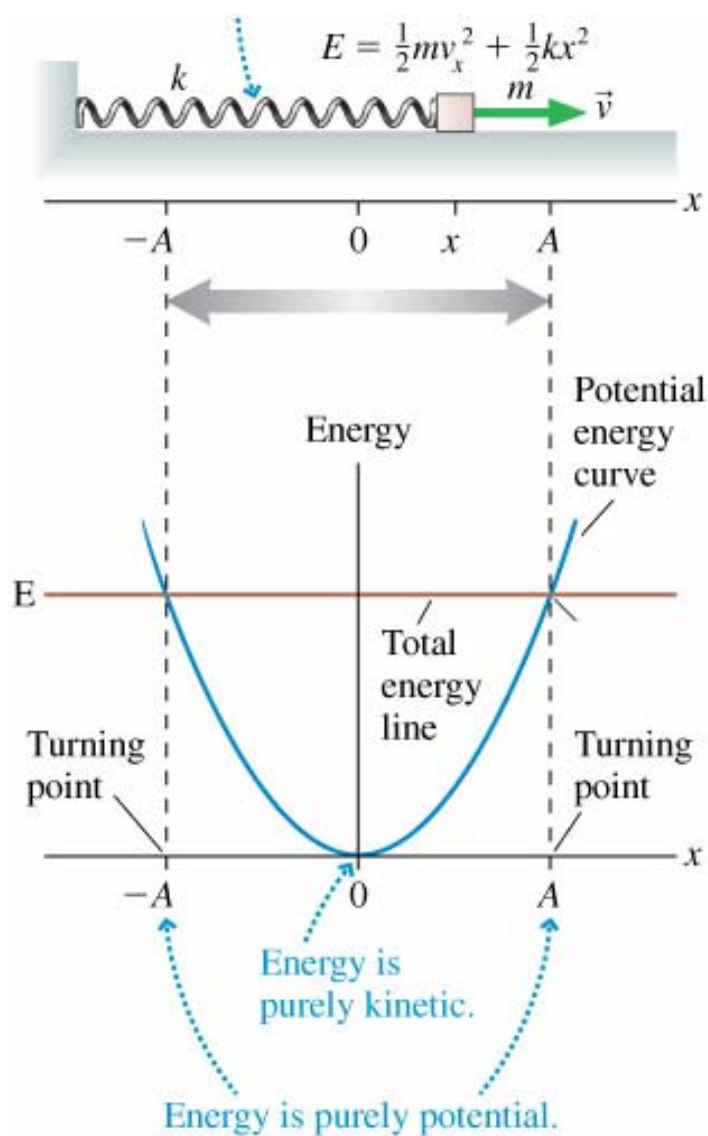
$$K = \frac{mv^2}{2} = \frac{m}{2} \omega^2 A^2 \sin^2(\omega t + \varphi_0)$$

Potential energy:

$$U = \frac{kx^2}{2} = \frac{k}{2} A^2 \cos^2(\omega t + \varphi_0)$$

Total energy:

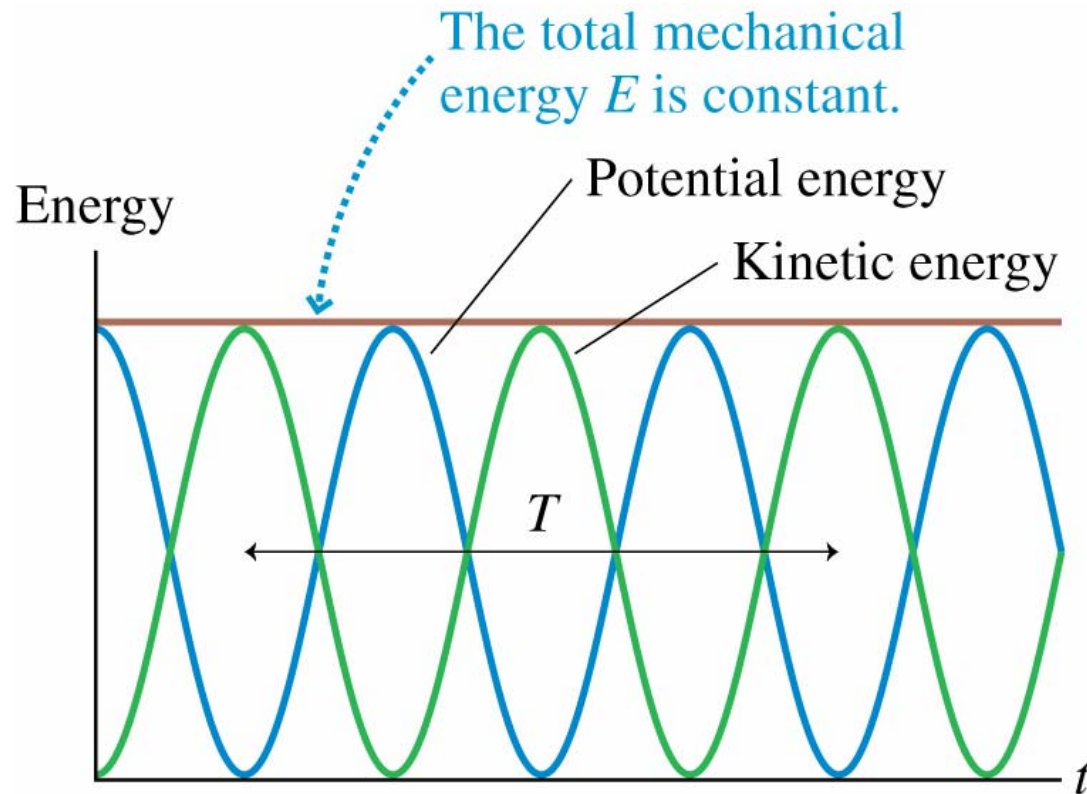
$$\begin{aligned} E = K + U &= \frac{m}{2} \omega^2 A^2 \sin^2(\omega t + \varphi_0) + \frac{k}{2} A^2 \cos^2(\omega t + \varphi_0) = \\ &= \frac{k}{2} A^2 (\sin^2(\omega t + \varphi_0) + \cos^2(\omega t + \varphi_0)) = \frac{k}{2} A^2 \end{aligned}$$



$$K = \frac{mv^2}{2} = \frac{m}{2} \omega^2 A^2 \sin^2(\omega t + \varphi_0) = \frac{k}{2} A^2 \sin^2(\omega t + \varphi_0)$$

$$U = \frac{kx^2}{2} = \frac{k}{2} A^2 \cos^2(\omega t + \varphi_0)$$

$$\omega = \sqrt{\frac{k}{m}}$$



Example: Find the relation between kinetic and potential energy at $x=A/3$.

$$x(t) = A \cos(\omega t + \varphi_0)$$

The total energy is the sum of kinetic and potential energy. At $x=A$ the kinetic energy is 0.

$$E = K + U = \frac{k}{2} A^2$$

At $x=A/3$ the potential energy is

$$U = \frac{kx^2}{2} = \frac{k}{2} \frac{A^2}{9} = \frac{kA^2}{18} = \frac{1}{9} E$$

Then the kinetic energy at this point is

$$K = E - U = \frac{k}{2} A^2 - \frac{k}{18} A^2 = \frac{4k}{9} A^2 = \frac{8}{9} E$$

Then

$$\frac{U}{K} = \frac{\frac{kx^2}{18}}{\frac{4kA^2}{9}} = \frac{1}{8}$$

Simple Harmonic Motion:

If force is proportional to displacement (it is not necessary the spring system)

$$F = -kx$$

then

$$ma = F = -kx$$

$$m \frac{d^2 x}{dt^2} = -kx$$

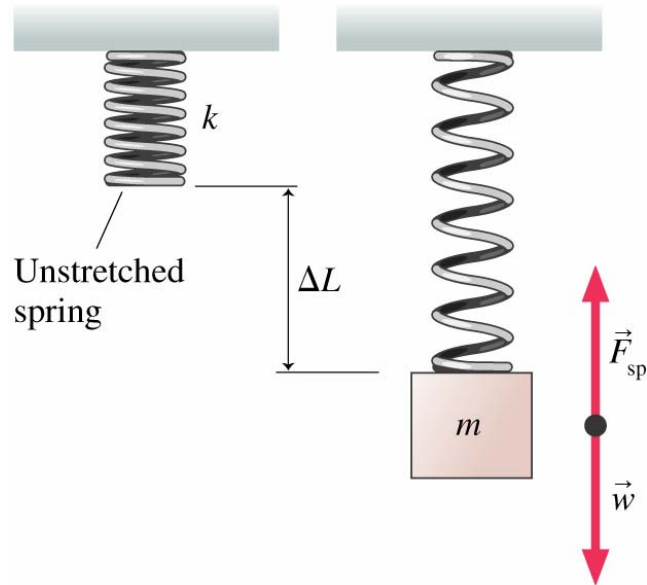
$$a = \frac{d^2 x}{dt^2}$$

Solution of this equation:

$$x(t) = A \cos(\omega t + \varphi_0)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Oscillations about equilibrium position



Equilibrium:

$$k\Delta L = w$$

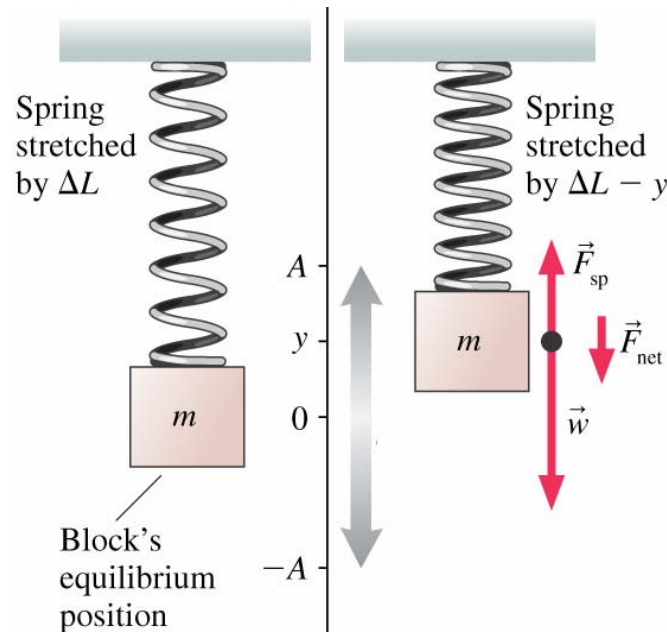
$$\Delta L = \frac{w}{k}$$

Net force:

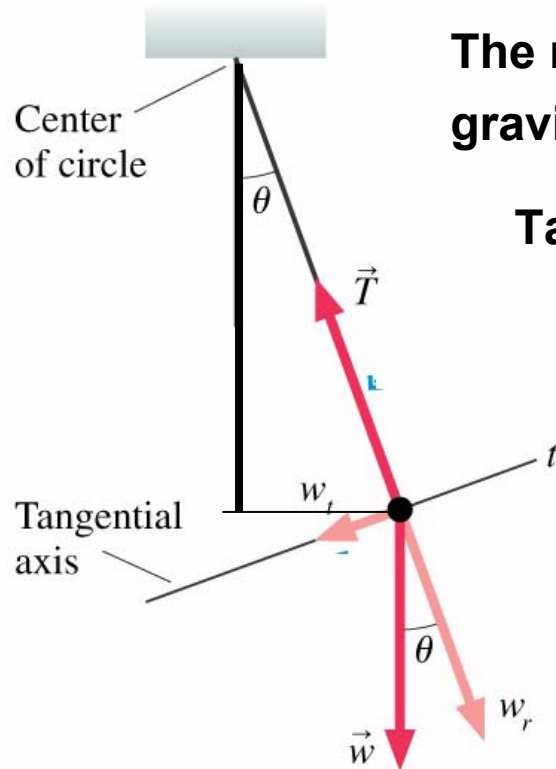
$$F_{net} = k(\Delta L - y) - w = -ky + (k\Delta L - w) = -ky$$

$$y(t) = A \cos(\omega t + \varphi_0)$$

Oscillations about equilibrium position



The pendulum: small-angle approximation



The net force is the sum of two forces: tension and gravitational force.

Tangential component of the net force is

$$F_{net,t} = w_t = -w \sin \theta$$

If y is the arc length then

$$\theta = \frac{y}{l}$$

$$F_{net,t} = w_t = -w \sin \frac{y}{l}$$

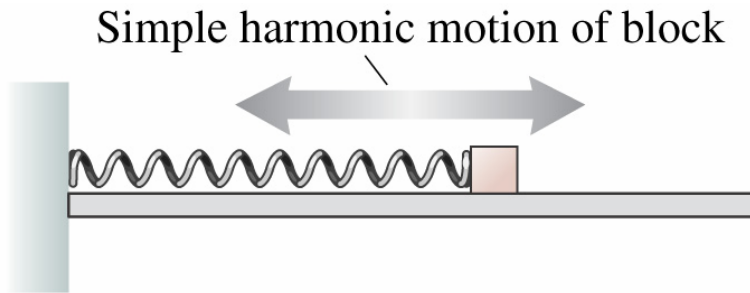
If $y \ll l$ then

$$F_{net,t} = w_t = -w \sin \frac{y}{l} \approx -w \frac{y}{l} = -k_0 y$$

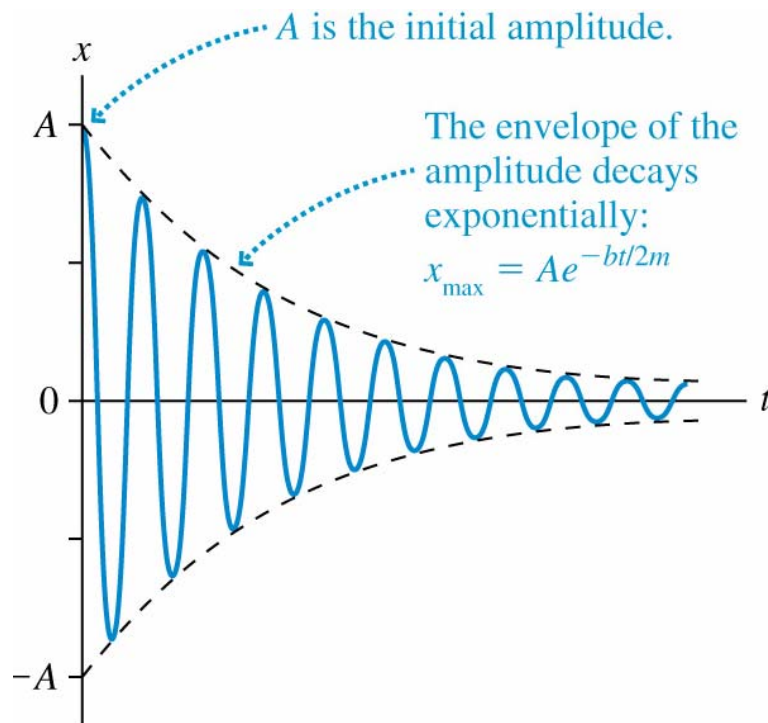
$$k_0 = \frac{w}{l} = m \frac{g}{l}$$

then $y(t) = A \cos(\omega t + \varphi_0)$

$$\omega = \sqrt{\frac{k_0}{m}} = \sqrt{\frac{mg}{ml}} = \sqrt{\frac{g}{l}}$$



If there is a friction then there will be energy loss



$$E = \frac{k}{2} A^2$$

The energy is determined by the amplitude of oscillations, so the energy loss means that the amplitude is decreasing