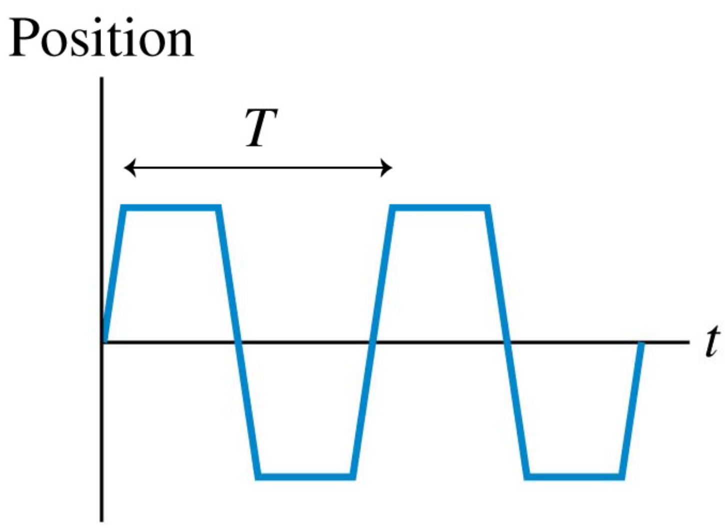
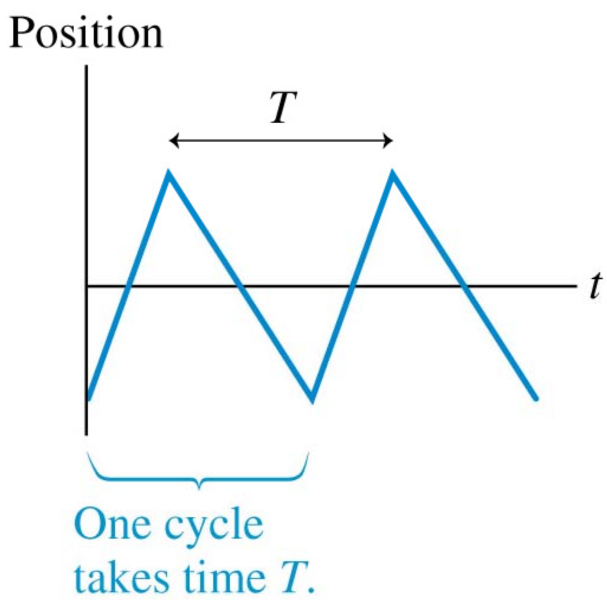
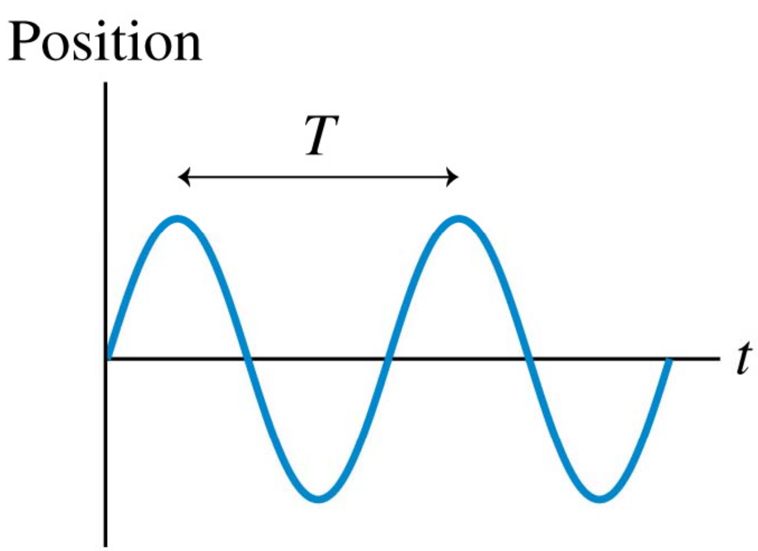
Oscillations

Readings: Chapter 14

Oscillation: Periodic Motion



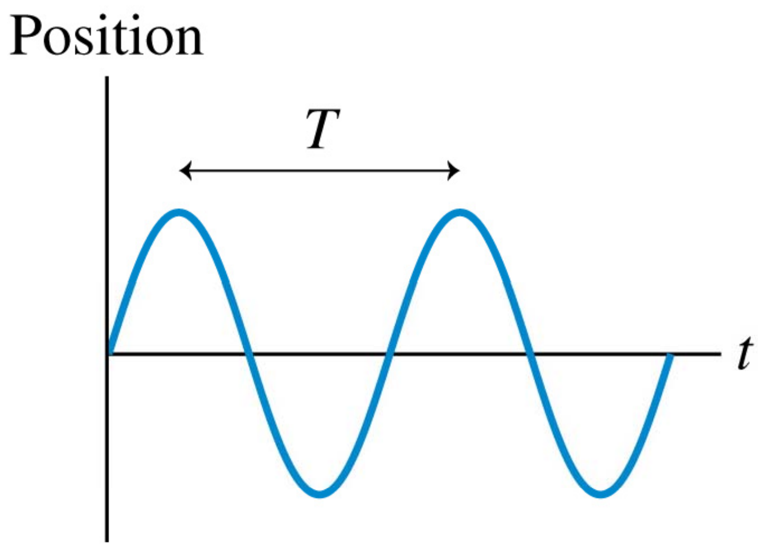




T **– period of motion**

$$f = \frac{1}{T}$$ **- frequency**

Oscillation: Periodic Motion: Simple Harmonic Motion



Simple Harmonic Motion **– sinusoidal oscillation**

$$x(t) = A\cos\left(\frac{2\pi}{T}t\right) = A\cos(2\pi ft)$$ **or** $$x(t) = A\sin\left(\frac{2\pi}{T}t\right) = A\sin(2\pi ft)$$

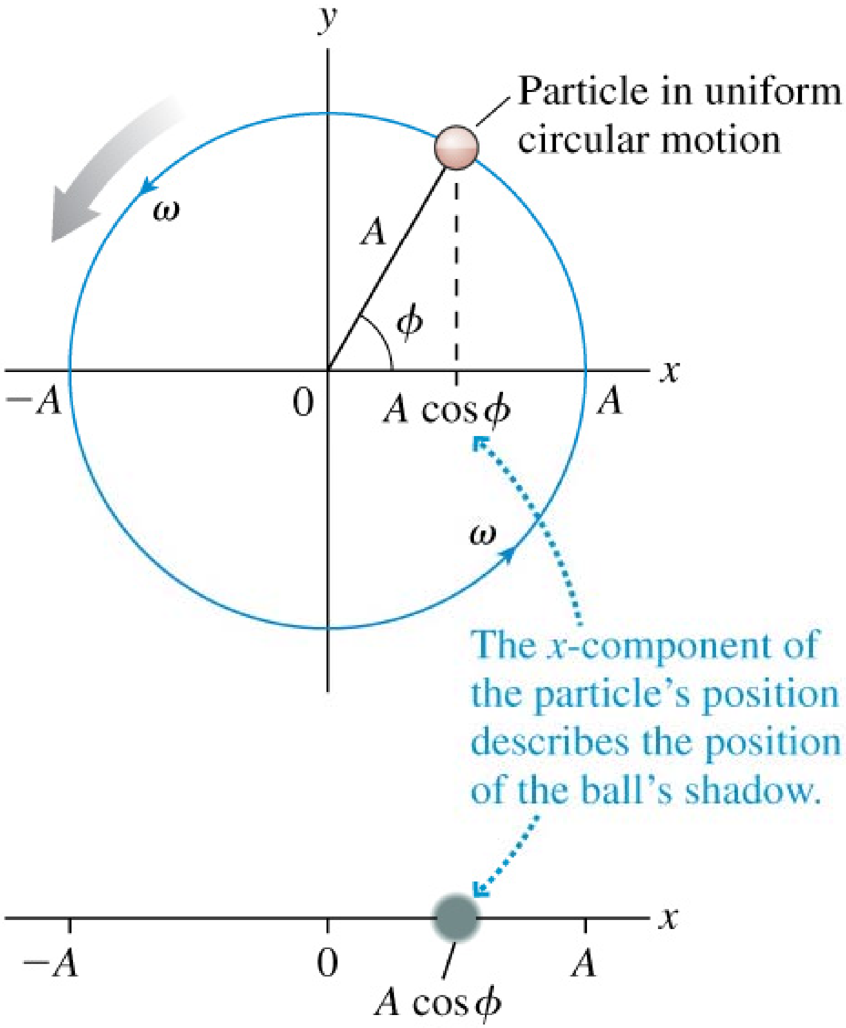
The most general expression for sinusoidal motion

$$x(t) = A\cos\left(\frac{2\pi}{T}t + \varphi_0\right) = A\cos(2\pi ft + \varphi_0)$$

φ_0 **- phase constant** A **- amplitude**

If $\varphi_0 = 90^0$ **then** $x(t) = A\cos(2\pi ft + 90^0) = A\sin(2\pi ft)$

Oscillation: Periodic Motion: Simple Harmonic Motion



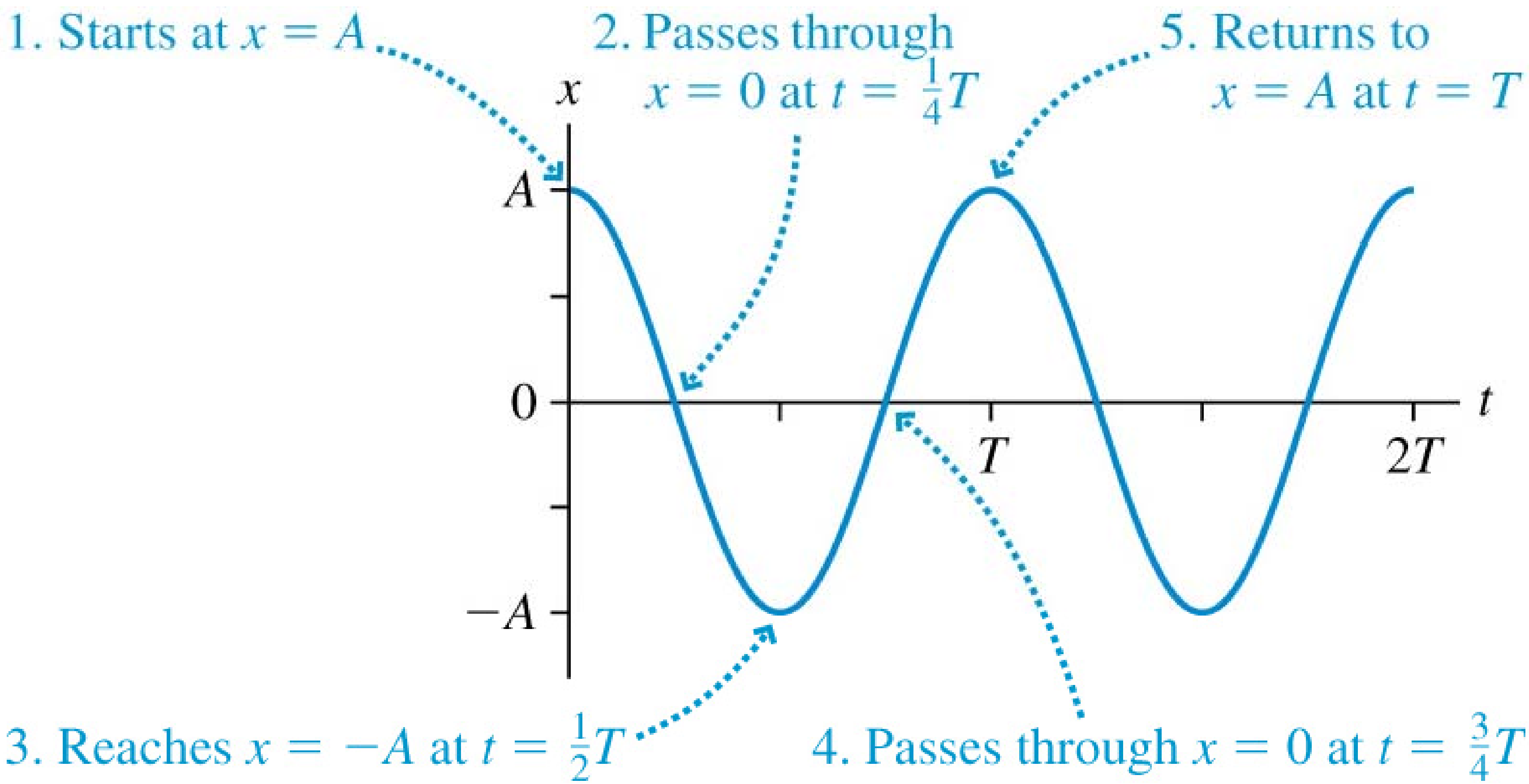
Example: **Uniform circular motion**

x- component:

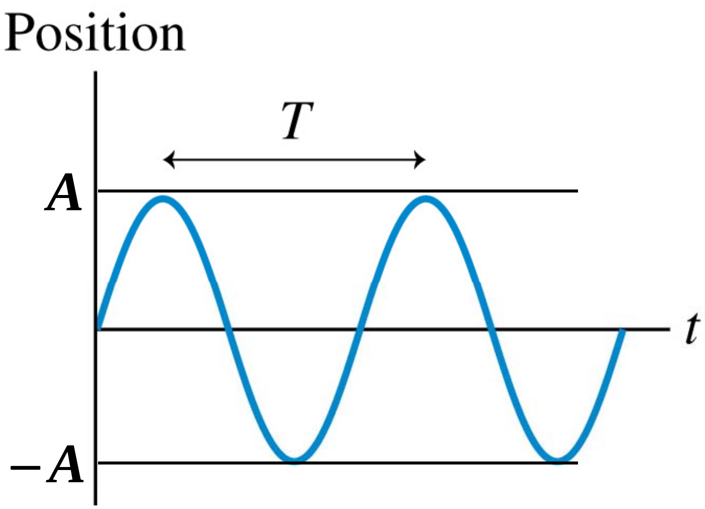
$$x(t) = A\cos(\varphi) = A\cos(\omega t)$$

ω **- angular frequency**

$$x(t) = A\cos\left(\frac{2\pi}{T}t\right) = A\cos(2\pi ft)$$



Oscillation: Periodic Motion: Simple Harmonic Motion



$$x(t) = A\cos\left(\frac{2\pi}{T}t + \varphi_0\right) = A\cos\left(2\pi ft + \varphi_0\right) = A\cos\left(\omega t + \varphi_0\right)$$

A - amplitude

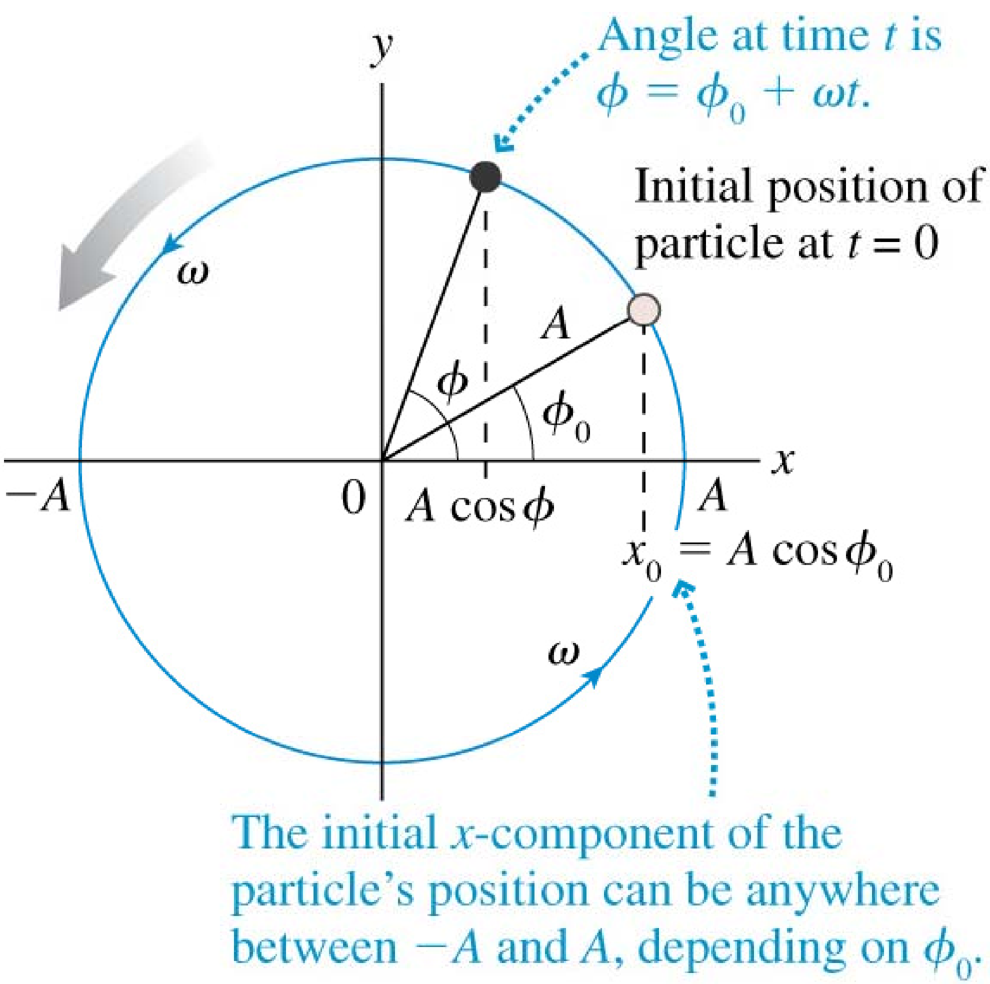
φ_0 - phase constant

T **– period of motion (units – s)**

$f = \frac{1}{T}$ **- frequency (units – hertz – Hz =1/s)**

$\omega = 2\pi f = 2\pi \frac{1}{T}$ **- angular frequency (units – rad/s)**

$$x(t) = A\cos\left(\frac{2\pi}{T}t + \varphi_0\right) = A\cos\left(2\pi ft + \varphi_0\right) = A\cos\left(\omega t + \varphi_0\right)$$



φ_0 - phase constant

Phase constant specifies the initial position of the oscillator

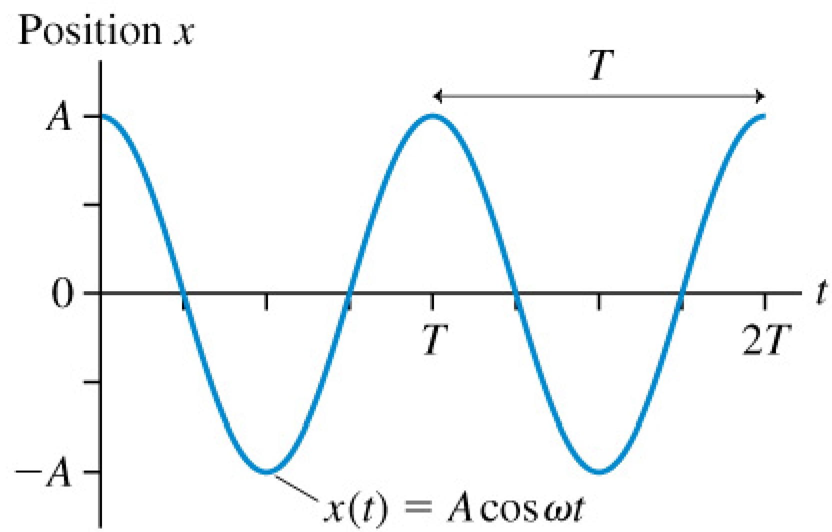
Simple Harmonic Motion: Position, Velocity, and Acceleration

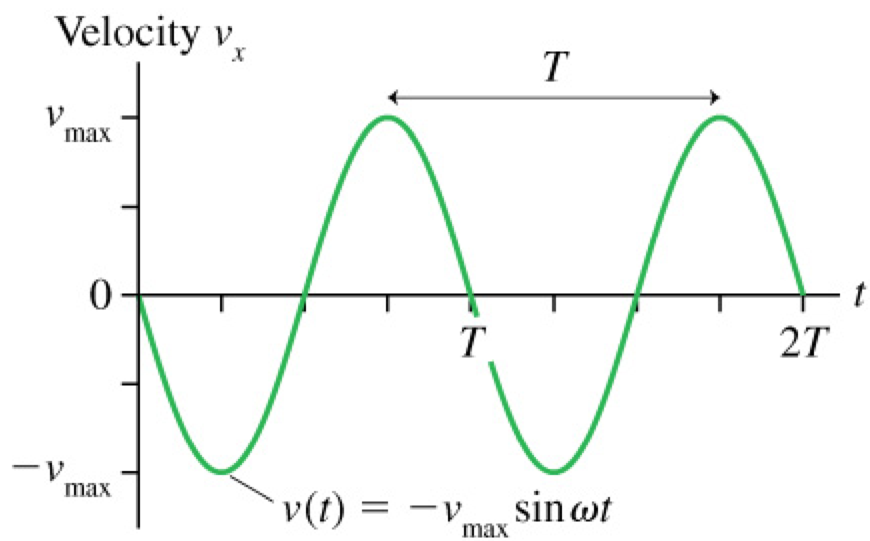
$$x(t) = A\cos(\omega t + \varphi_0)$$

$$v(t) = \frac{dx(t)}{dt} = A\frac{d\cos(\omega t + \varphi_0)}{dt} =$$
$$= -\omega A\sin(\omega t + \varphi_0) = -v_{max}\sin(\omega t + \varphi_0)$$

$$v_{max} = \omega A$$

$$a(t) = \frac{dv(t)}{dt} = -\omega A\frac{d\sin(\omega t + \varphi_0)}{dt} =$$
$$= -\omega^2 A\cos(\omega t + \varphi_0) = -\omega^2 x(t)$$





Simple Harmonic Motion: object oscillating on a spring

$$x(t) = A\cos\left(\omega t + \varphi_0\right)$$

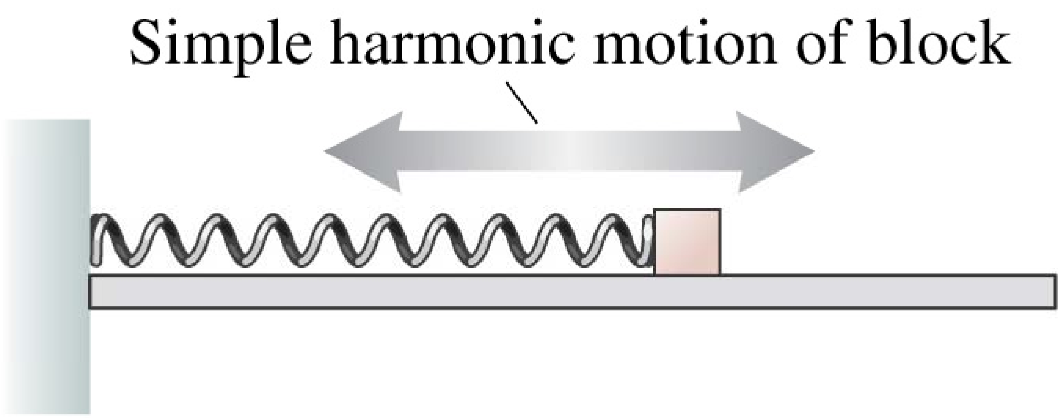
$$a(t) = -\omega^2 A\cos\left(\omega t + \varphi_0\right) = -\omega^2 x(t)$$

Newton's second law:

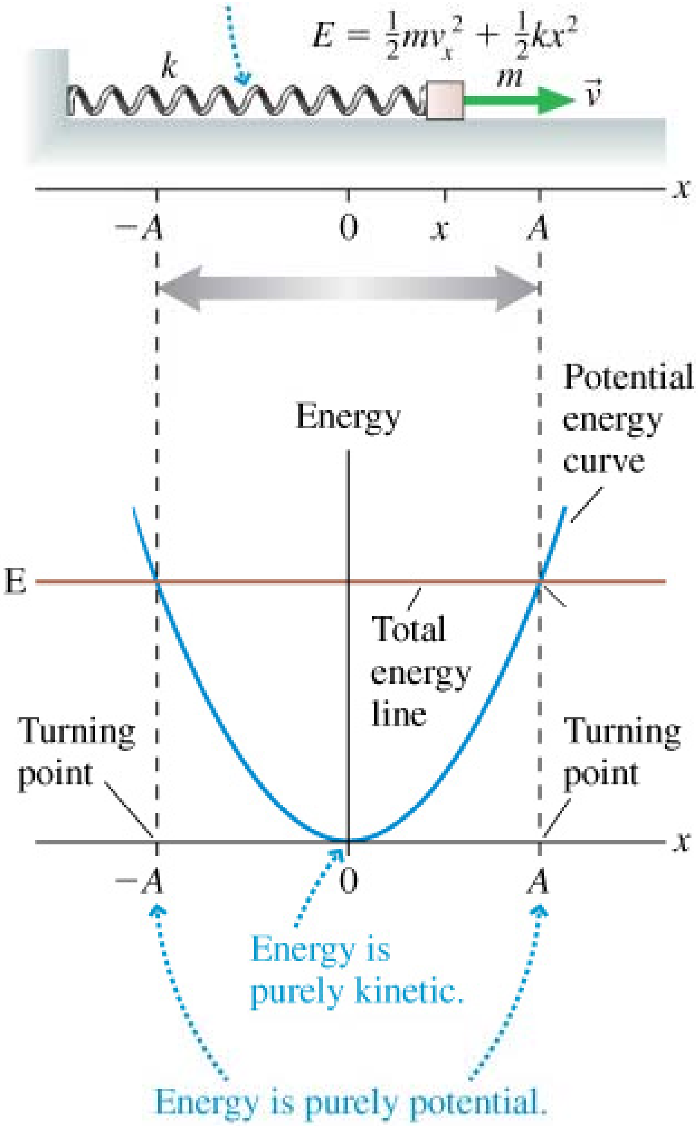
$$ma = F = -m\omega^2 x = -kx$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$F = -kx$$ **Hooke's law:**



Simple Harmonic Motion: Conservation of energy



$$x(t) = A\cos(\omega t + \varphi_0)$$

$$v(t) = -\omega A\sin(\omega t + \varphi_0)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Kinetic energy:

$$K = \frac{mv^2}{2} = \frac{m}{2}\omega^2 A^2 \sin^2(\omega t + \varphi_0)$$

Potential energy:

$$U = \frac{kx^2}{2} = \frac{k}{2}A^2\cos^2(\omega t + \varphi_0)$$

Total energy:

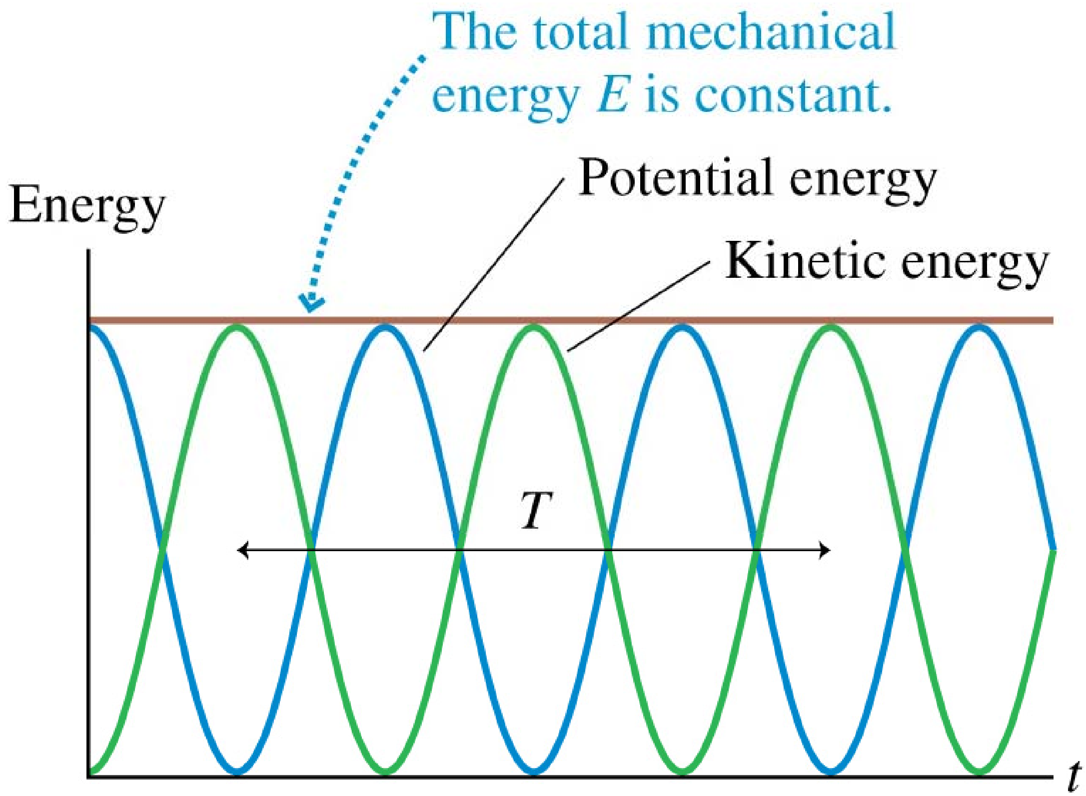
$$E = K + U = \frac{m}{2}\omega^2 A^2 \sin^2(\omega t + \varphi_0) + \frac{k}{2}A^2\cos^2(\omega t + \varphi_0) =$$

$$= \frac{k}{2}A^2\left(\sin^2(\omega t + \varphi_0) + \cos^2(\omega t + \varphi_0)\right) = \frac{k}{2}A^2$$

$$K = \frac{mv^2}{2} = \frac{m}{2}\omega^2 A^2 \sin^2\left(\omega t + \varphi_0\right) = \frac{k}{2} A^2 \sin^2\left(\omega t + \varphi_0\right)$$

$$U = \frac{kx^2}{2} = \frac{k}{2} A^2 \cos^2\left(\omega t + \varphi_0\right)$$

$$\omega = \sqrt{\frac{k}{m}}$$



Example: Find the relation between kinetic and potential energy at x=A/3.

$$x(t) = A\cos\left(\omega t + \varphi_0\right)$$

The total energy is the sum of kinetic and potential energy. At x=A the kinetic energy is 0.

$$E = K + U = \frac{k}{2}A^2$$

At x=A/3 the potential energy is

$$U = \frac{kx^2}{2} = \frac{k}{2}\frac{A^2}{9} = \frac{kA^2}{18} = \frac{1}{9}E$$

Then the kinetic energy at this point is

$$K = E - U = \frac{k}{2}A^2 - \frac{k}{18}A^2 = \frac{4k}{9}A^2 = \frac{8}{9}E$$

Then

$$\frac{U}{K} = \frac{\frac{kx^2}{18}}{\frac{4kA^2}{9}} = \frac{1}{8}$$

Simple Harmonic Motion:

If force is proportional to displacement (it is not necessary the spring system)

$$F = -kx$$

then

$$ma = F = -kx$$

$$a = \frac{d^2x}{dt^2}$$

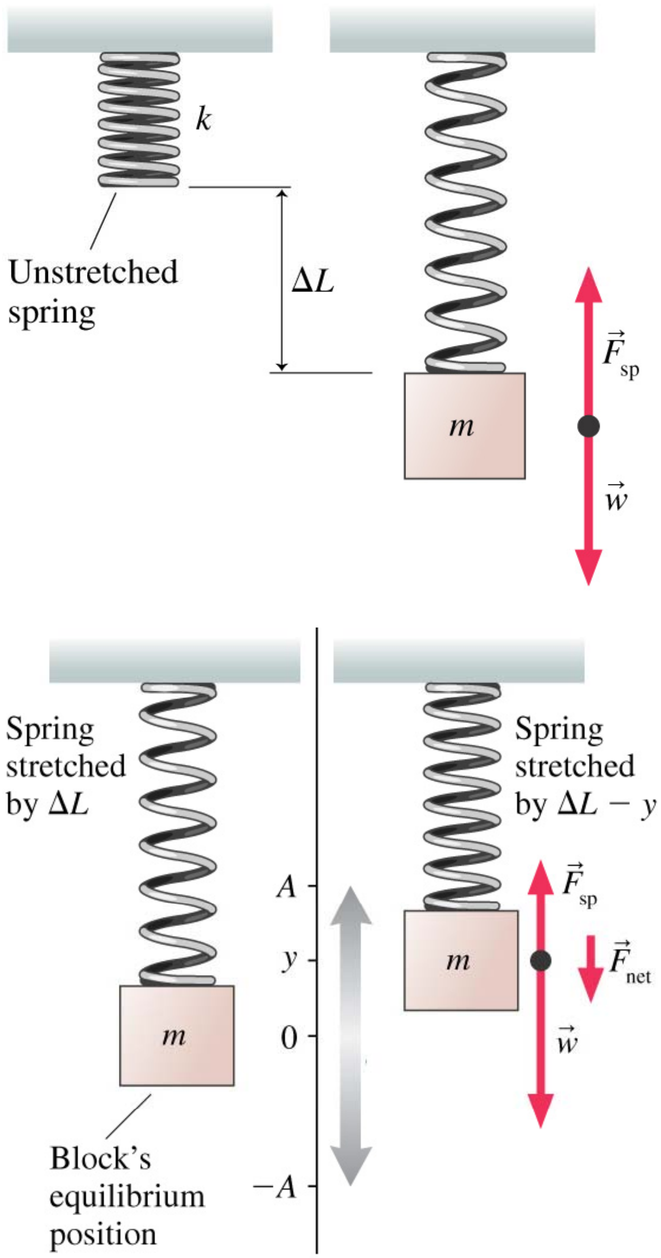
$$m\frac{d^2x}{dt^2} = -kx$$

Solution of this equation:

$$x(t) = A\cos\left(\omega t + \varphi_0\right)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Oscillations about equilibrium position



Equilibrium:

$$k\Delta L = w$$

$$\Delta L = \frac{w}{k}$$

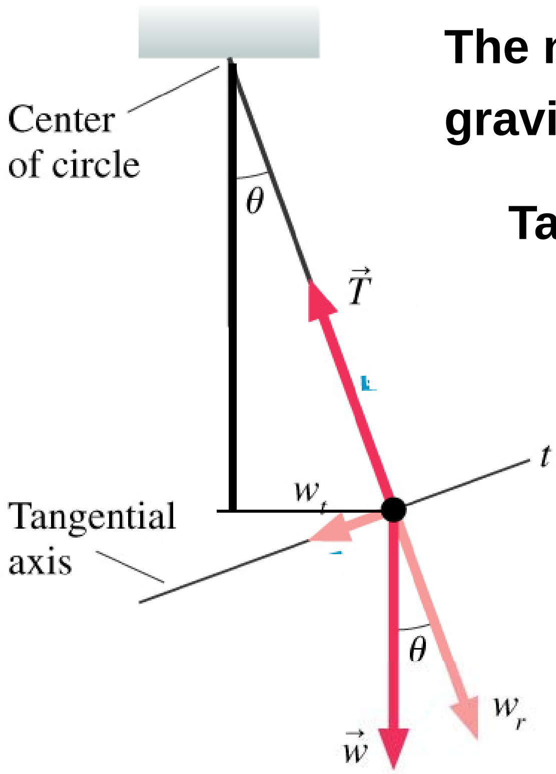
Net force:

$$F_{net} = k(\Delta L - y) - w = -ky + (k\Delta L - w) = -ky$$

$$y(t) = A\cos\left(\omega t + \varphi_0\right)$$

Oscillations about equilibrium position

The pendulum: small-angle approximation



The net force is the sum of two forces: tension and gravitational force.

Tangential component of the net force is

$$F_{net,t} = w_t = -w\sin\theta$$

If y is the arc length then

$$\theta = \frac{y}{l}$$

$$F_{net,t} = w_t = -w\sin\frac{y}{l}$$

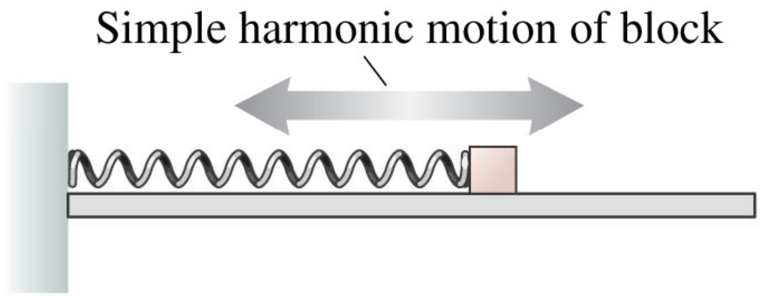
If y<<l then

$$F_{net,t} = w_t = -w\sin\frac{y}{l} \approx -w\frac{y}{l} = -k_0 y$$

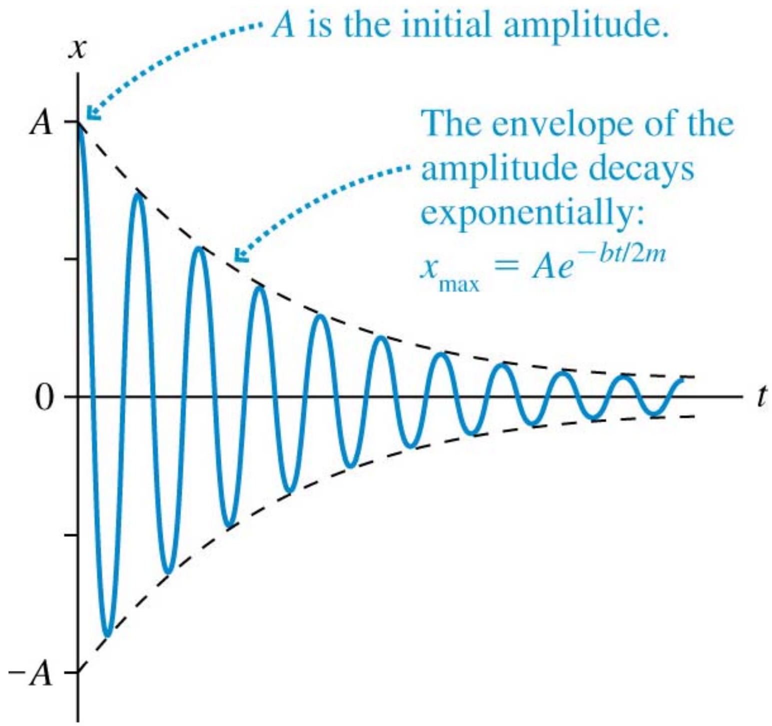
$$k_0 = \frac{w}{l} = m\frac{g}{l}$$

then $$y(t) = A\cos\left(\omega t + \varphi_0\right)$$

$$\omega = \sqrt{\frac{k_0}{m}} = \sqrt{\frac{mg}{ml}} = \sqrt{\frac{g}{l}}$$



If the system has a friction then there is an energy loss



$$E = \frac{k}{2} A^2$$

The energy is determined by the amplitude of oscillations, so the energy loss means that the amplitude is decreasing