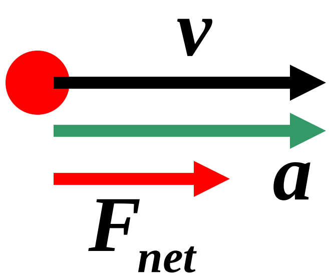
Work

Readings: Chapter 11

Newton's Second Law: Net Force is zero – acceleration is zero – velocity is constant – kinetic energy is constant

$$F_{net} = 0 \quad \Rightarrow \quad a = 0 \quad \Rightarrow \quad v = \text{const} \quad \Rightarrow \quad K = \frac{1}{2} m v^2 = \text{const}$$

Newton's Second Law: If Net Force is not zero then: What is the relation between the change of Kinetic energy and the Net Force?

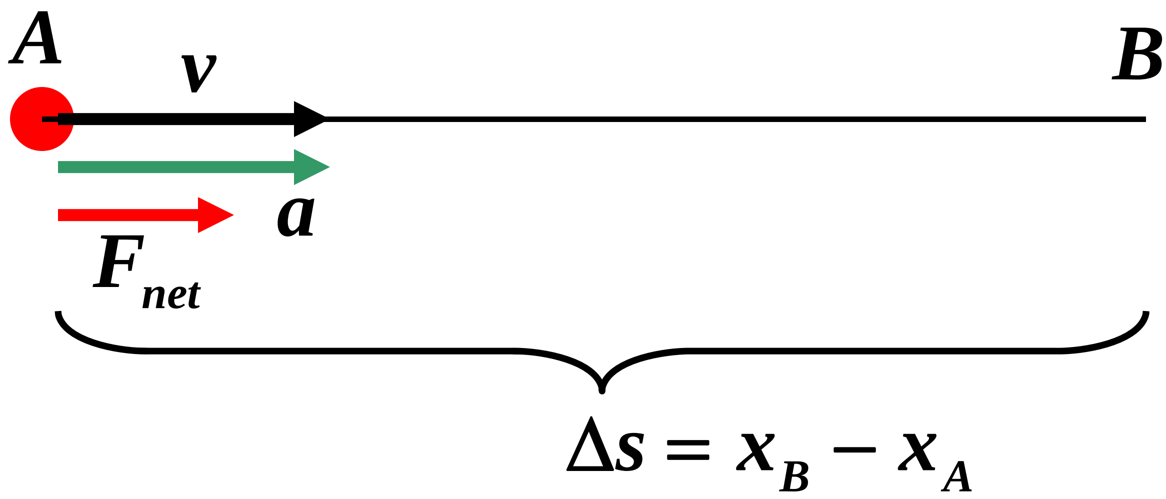


$$F_{net} = ma = m\frac{\Delta v}{\Delta t}$$

$$F_{net} v \Delta t = m\frac{\Delta v}{\Delta t} v \Delta t = m v \Delta v = \frac{1}{2} m \Delta v^2$$

$$F_{net} \Delta s = \frac{1}{2} m \Delta v^2 = \Delta K$$

Where $\Delta s = v \Delta t$ **- displacement**



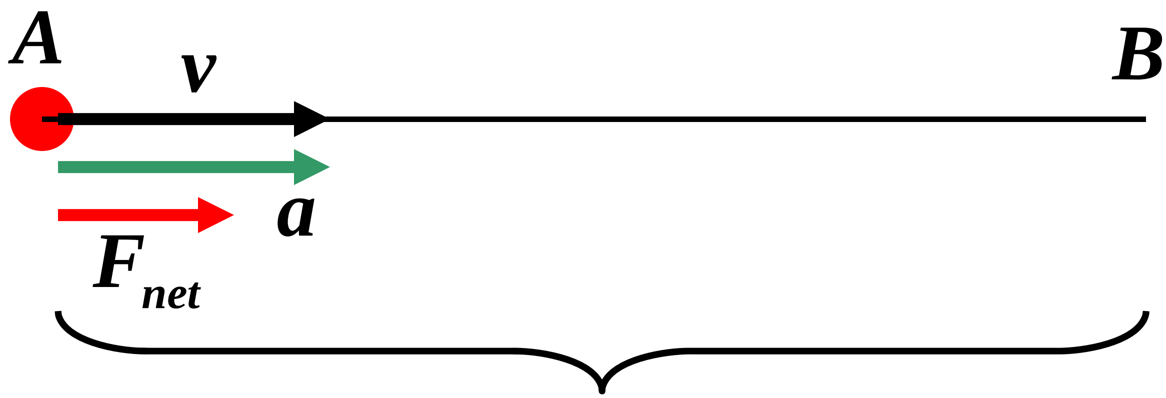
If $F_{net} = const$ then

$$F_{net}(x_B - x_A) = \Delta K = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

This equation gives the relation between the displacement and velocity. It is the same equation as

$$v_B^2 - v_A^2 = 2a(x_B - x_A)$$

for the motion with constant acceleration (constant net force)



$$\Delta s = x_B - x_A$$

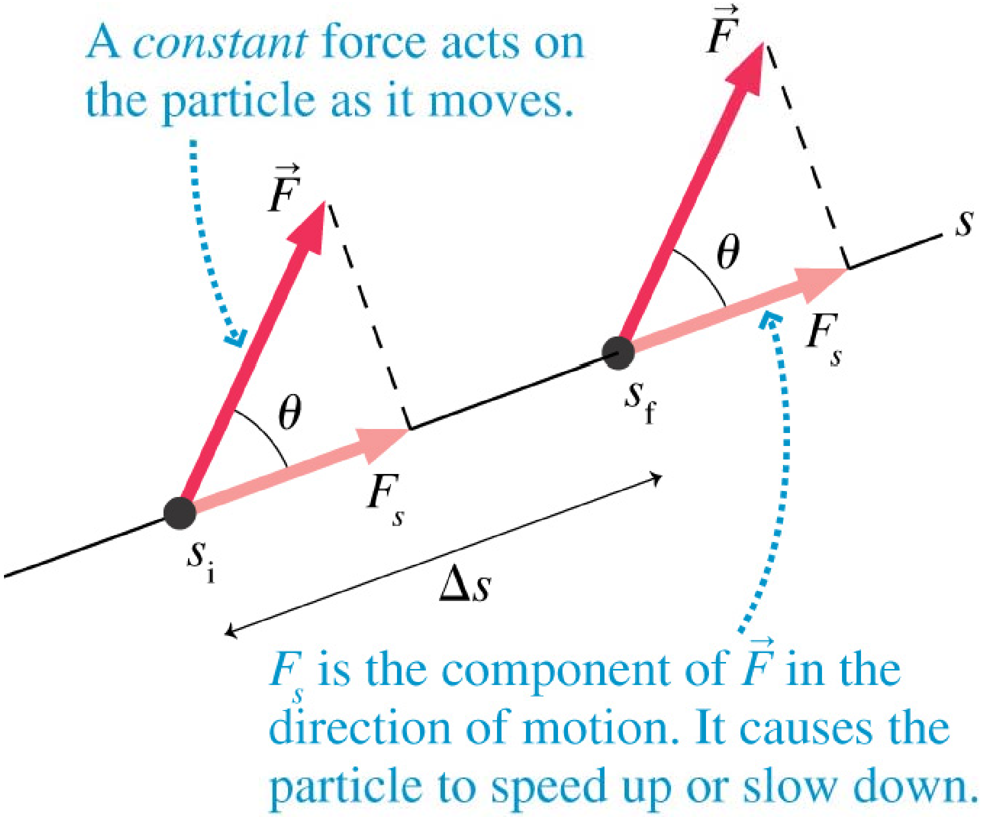
If $F_{net} \neq const$ then we have integral

$$\int_{x_A}^{x_B} F_{net}\,dx = \Delta K = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

If the net force depends also on the velocity then the relation becomes more complicated

$F_{net}\Delta s$ or $\int_{x_A}^{x_B} F_{net}\,dx$ is the Work done by net force

$$W = \int_{x_A}^{x_B} F_{net}\,dx$$

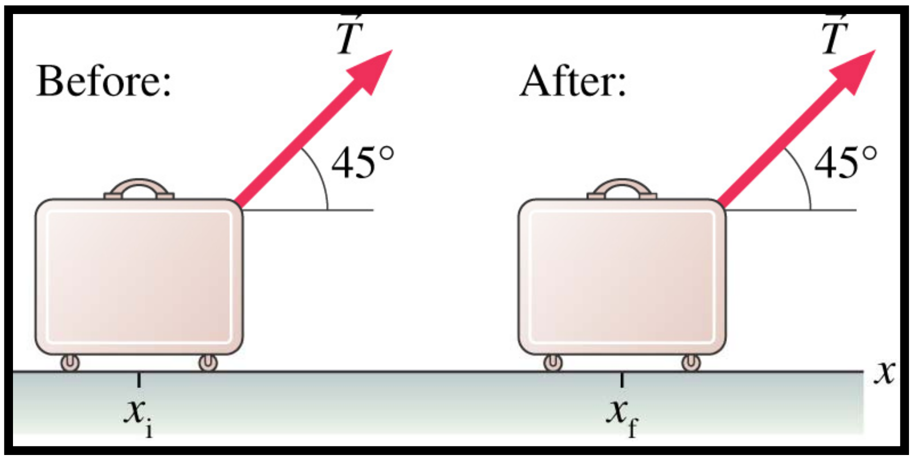


Then Work is

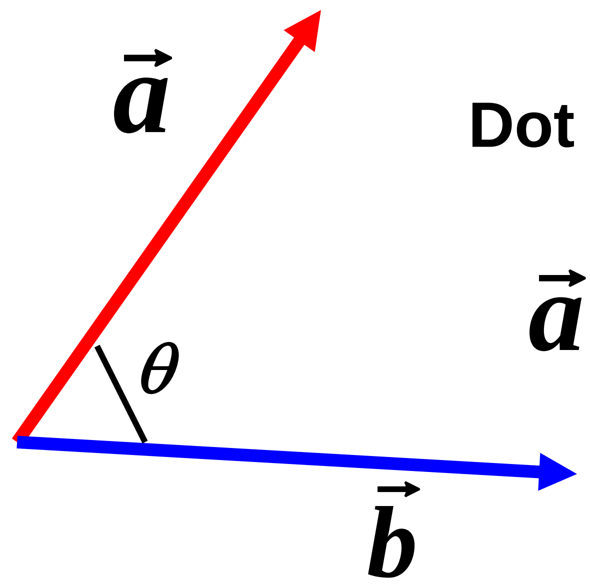
$$W = F_s \Delta s = F \cos\theta \Delta s$$

This combination is called the dot product (or scalar product) of two vectors

$$F \cos\theta \Delta s = \vec{F} \cdot \Delta\vec{s}$$



Dot Product

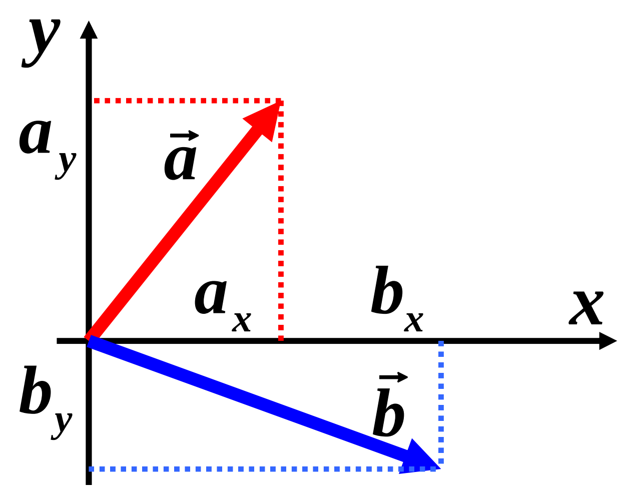


Dot product is a scalar:

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$

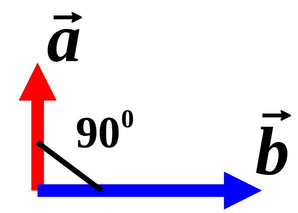
If we know the components of the vectors then the dot product can be calculated as

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$



If vectors are orthogonal then dot product is zero

$$\theta = 90^0 \Rightarrow \cos\theta = 0 \qquad \vec{a} \cdot \vec{b} = 0$$

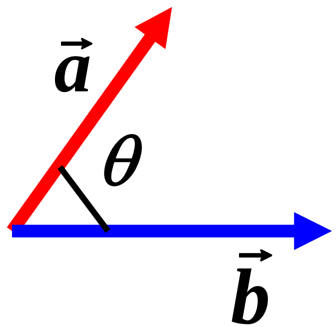


Dot Product: properties

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$

$$(c\vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b}) = cab\cos\theta$$

$$(\vec{a}_1 + \vec{a}_2) \cdot \vec{b} = \vec{a}_1 \cdot \vec{b} + \vec{a}_2 \cdot \vec{b}$$



Dot product is positive if $\theta < 90^0$ $\cos\theta > 0$

Dot product is negative if $\theta > 90^0$ $\cos\theta < 0$

The magnitude of $\vec{a}$ is 5, the magnitude of $\vec{b}$ is 2, the angle θ is 60^0

What is the dot product of $\vec{a}$ and $\vec{b}$

$$\vec{a} \cdot \vec{b} = 5 \cdot 2\cos 60^0 = 5 \cdot 2 \cdot \frac{1}{2} = 5$$

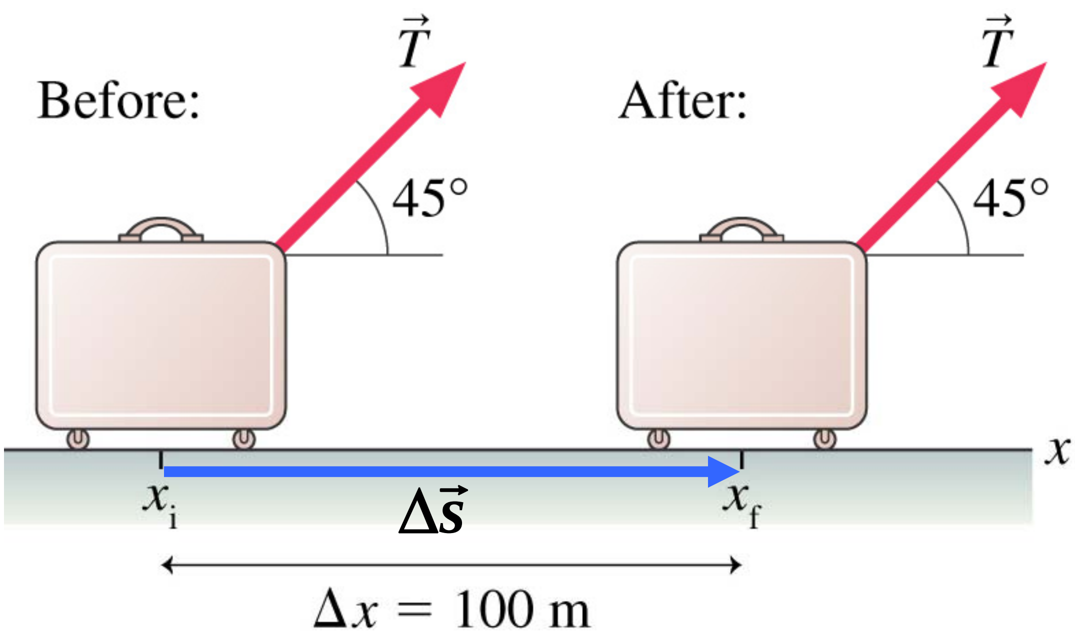
Work produced by a force, acting on an object, is the dot product of the force and displacement

$$W = \vec{F} \cdot \Delta\vec{s} \qquad W = \int_{\text{initial point}}^{\text{final point}} \vec{F} \cdot \Delta\vec{s}$$

Work has the same units as the energy:

$$1J = 1N \cdot m = 1\frac{kg \cdot m}{s^2} m = 1\frac{kg \cdot m^2}{s^2}$$

Example: What is the work produced by the tension force?



$$T = 10N$$

$$\Delta s = 100m$$

$$\theta = 45^0$$

$$W = \vec{T} \cdot \Delta\vec{s} = T\Delta s \cos\theta =$$

$$= 10 \cdot 100 \cos 45^0 =$$

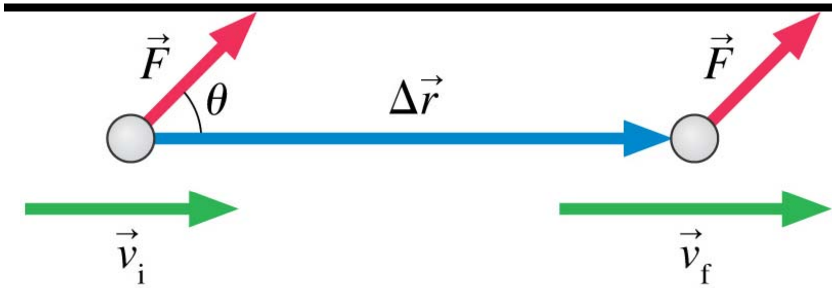
$$= 707N \cdot m = 707J$$

$$\Delta K = K_{final} - K_{initial} = W = \vec{F} \cdot \Delta\vec{s}$$

$$W = \vec{F} \cdot \Delta\vec{s} = F\,\Delta s > 0$$

$$(a > 0)$$

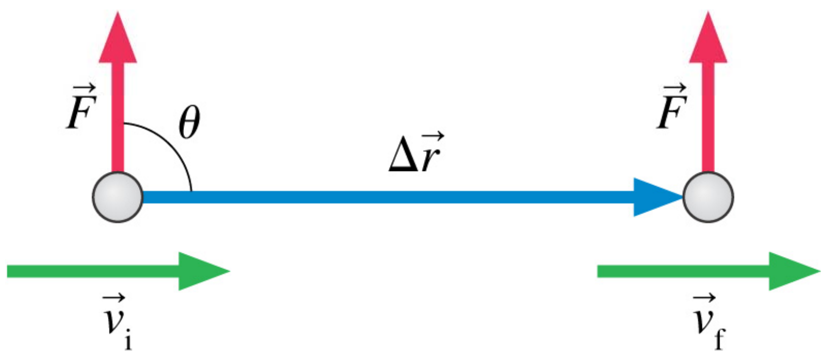
$$K_{final} > K_{initial} \qquad v_f > v_i$$



$$W = \vec{F} \cdot \Delta\vec{s} > 0$$

$$(a > 0)$$

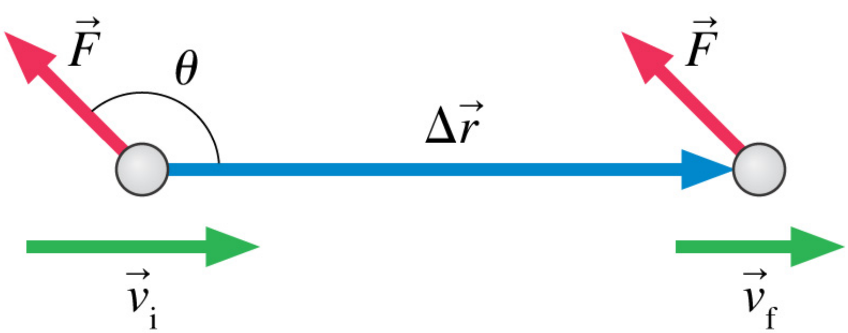
$$K_{final} > K_{initial} \qquad v_f > v_i$$



$$W = \vec{F} \cdot \Delta\vec{s} = 0$$

$$(a = 0)$$

$$K_{final} = K_{initial} \qquad v_f = v_i$$



$$W = \vec{F} \cdot \Delta\vec{s} < 0$$

$$(a < 0)$$

$$K_{final} < K_{initial} \qquad v_f < v_i$$

Work produced by a NET FORCE is equal to a change of KINETIC ENERGY (it follows from the second Newton's law)

$$W_{net} = \vec{F}_{net} \cdot \Delta\vec{s} = \Delta K = K_{final} - K_{initial}$$

The NET FORCE is equal to (vector) sum of all forces acting on the object

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + ...$$

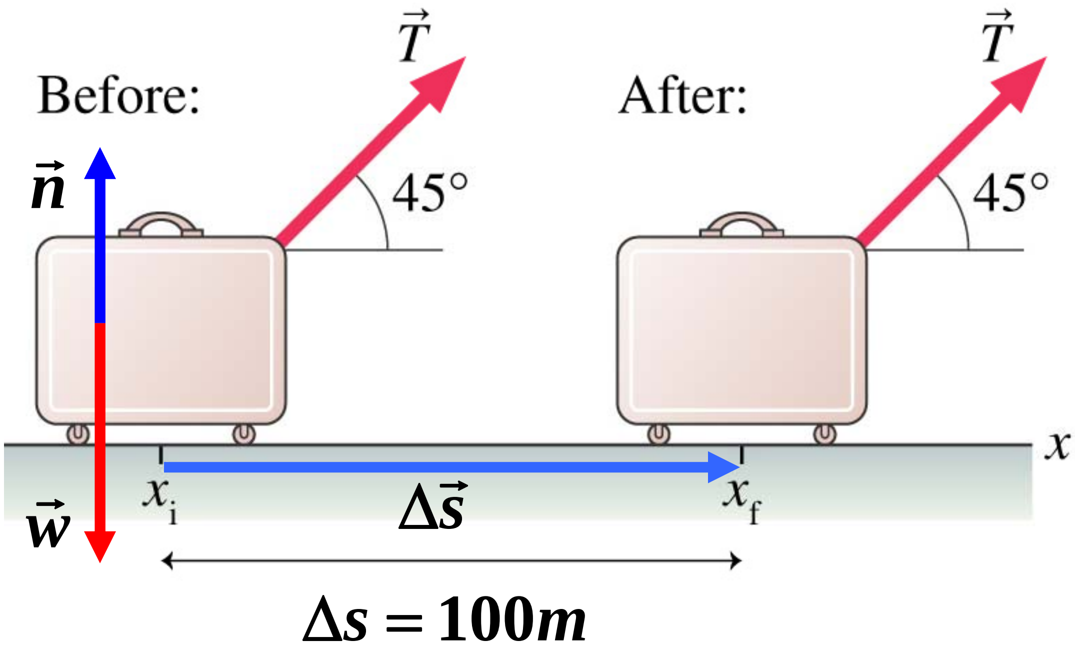
then

$$W_{net} = (\vec{F}_1 + \vec{F}_2 + ...) \cdot \Delta\vec{s} = \vec{F}_1 \cdot \Delta\vec{s} + \vec{F}_2 \cdot \Delta\vec{s} + ... = W_1 + W_2 + ... = \Delta K$$

so the sum of the works produced by all forces acting on the object is equal to the change of kinetic energy,

$$W_1 + W_2 + ... = \Delta K$$

Example: What is the change of kinetic energy? Or what is the final velocity of the block if the initial velocity is 5 m/s? The mass of the block is 5 kg.



Forces:

normal force $\vec{n}$

gravitational force $\vec{w}$

tension $\vec{T}$

$T = 10N \qquad \theta = 45^0$

Work produced by the net force is equal to the change of kinetic energy.

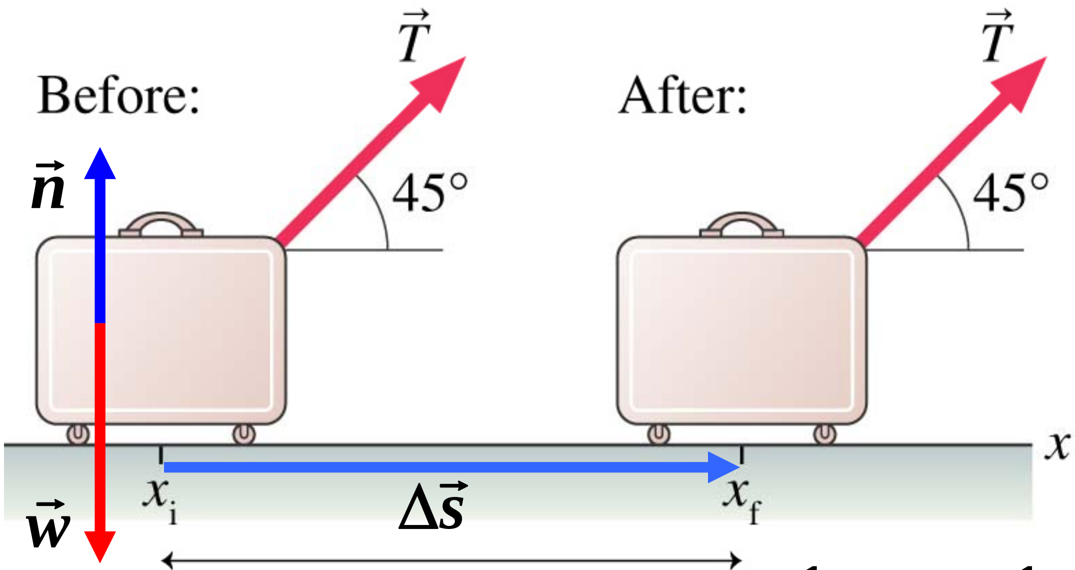
$$W_{net} = \vec{F}_{net} \cdot \Delta\vec{s} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Since $\vec{F}_{net} = \vec{n} + \vec{w} + \vec{T}$ **then**

$$W_{net} = W_n + W_w + W_T = \vec{n}\cdot\Delta\vec{s} + \vec{w}\cdot\Delta\vec{s} + \vec{T}\cdot\Delta\vec{s} = T\Delta s\cos\theta$$

$$W_n = \vec{n}\cdot\Delta\vec{s} = 0 \qquad W_w = \vec{w}\cdot\Delta\vec{s} = 0 \qquad W_T = \vec{T}\cdot\Delta\vec{s} = T\Delta s\cos\theta$$

Example: What is the change of kinetic energy? Or what is the final velocity of the block if the initial velocity is 5 m/s? The mass of the block is 5 kg.



$$\Delta s = 100m$$

Forces:

normal force $\vec{n}$

gravitational force $\vec{w}$

tension $\vec{T}$

$$T = 10N \qquad \theta = 45^0$$

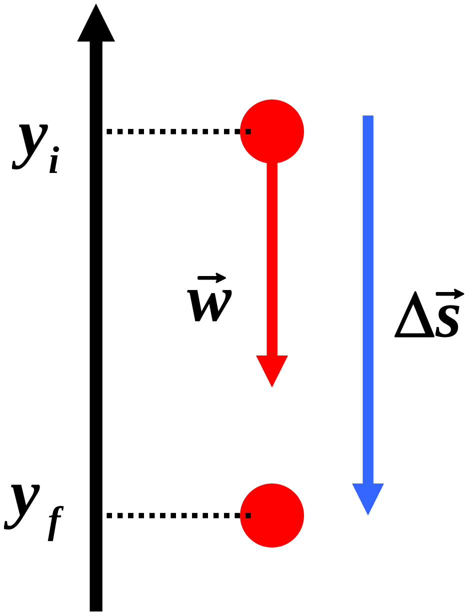
$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + W_{net} = \frac{1}{2}mv_i^2 + T\Delta s\cos\theta$$

$$v_f^2 = v_i^2 + 2\frac{T\cos\theta}{m}\Delta s$$

This problem can be also solved by finding acceleration (this is the motion with constant acceleration). Then

$$v_f^2 = v_i^2 + 2a\Delta s \qquad a = \frac{T\cos\theta}{m}$$

Example: Free fall motion.



Forces:

gravitational force $\vec{w}$

Work produced by the net force (gravitational force) is equal to the change of kinetic energy.

$$W_{net} = \vec{F}_{net} \cdot \Delta\vec{s} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = W_w = \vec{w} \cdot \Delta\vec{s} = mg\Delta s = mg(y_i - y_f) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

then

$$mgy_i + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}mv_f^2$$

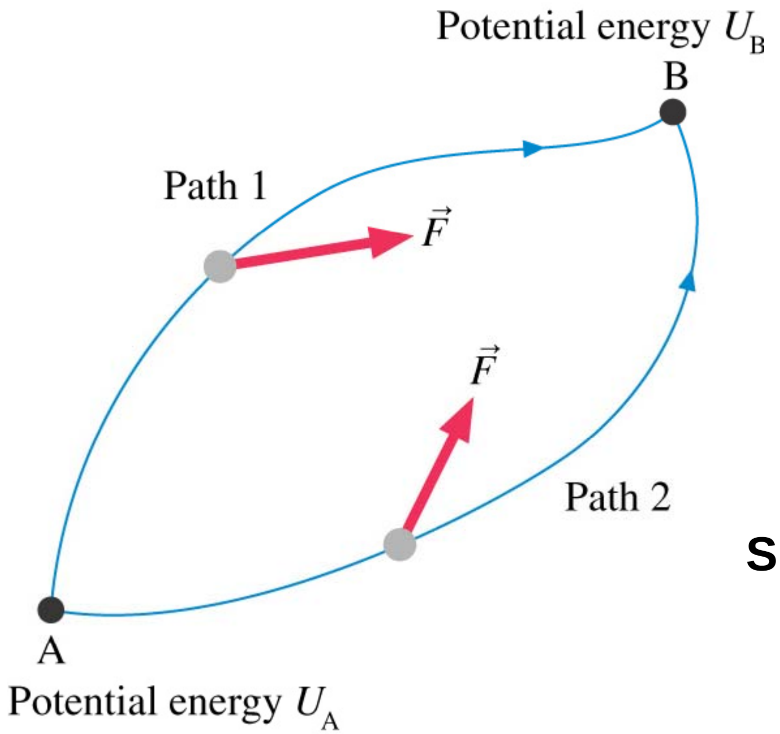
Conservation of mechanical energy

Work produced by gravitational force can be written as the change of gravitational potential energy

Not for all forces the work can be written as the difference between the potential energy between two points. Only if the force does not depend on velocity then we can introduce potential energy as

$$W = \vec{F} \cdot \Delta \vec{s} = U_{initial} - U_{final}$$

Potential energy depends only on the position of the object

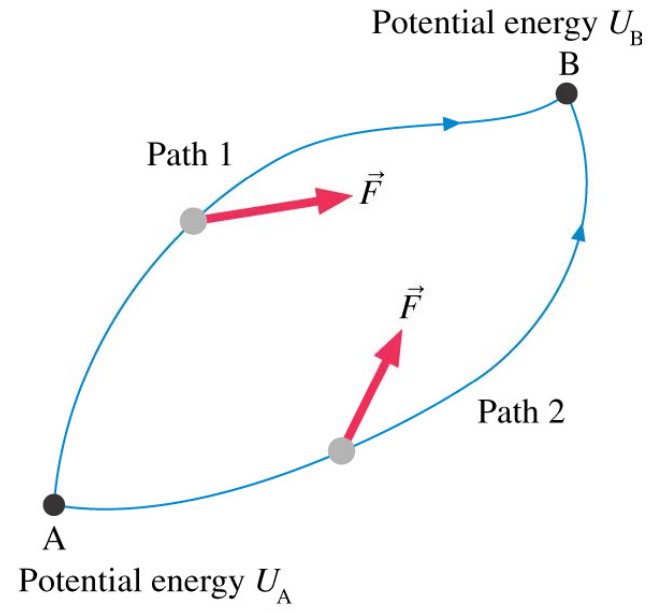


It means also that the work done by the force does not depend on the trajectory (path) of the object:

$$W_{path1} = W_{path2} = U_A - U_B$$

Such forces are called conservative forces

Conservative Forces



Example 1: Gravitational Force

$$U = mgh$$

Example 2: Elastic Force

$$W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} (-kx) dx = -k \frac{x^2}{2} \Bigg|_{x_i}^{x_f} =$$

$$= k \frac{x_i^2}{2} - k \frac{x_f^2}{2} = U_{initial} - U_{final}$$

$$U = k \frac{x^2}{2}$$

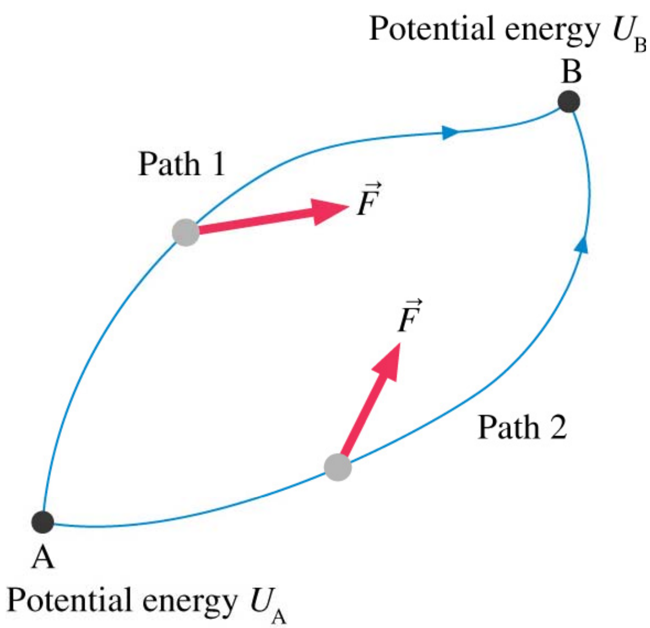
Nonconservative (dissipative) forces

Example: Friction

Direction of friction force is always opposite to the direction of velocity, so the friction force depends on velocity.

The work done by friction force depends on path.

If we know force we can find the corresponding potential energy

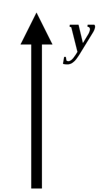


$$W = \vec{F} \cdot \Delta\vec{s} = -\Delta U = -(U_{final} - U_{initial})$$

We assume that the potential (for example) at point A is 0, then the potential energy at point B is

$$U_B = U_B - \int_A^B \vec{F} \cdot d\vec{s} = -\int_A^B \vec{F} \cdot d\vec{s}$$

If we know potential we can find the force as $F_s = -\dfrac{\Delta U}{\Delta s}$



Example: Gravitational force: $U = mgy$ $\quad F_y = -\dfrac{mg\Delta y}{\Delta y} = -mg$

Example: Elastic force: $U = k\dfrac{x^2}{2}$ $\quad F_x = -\dfrac{k\Delta\left(x^2/2\right)}{\Delta x} = -kx$

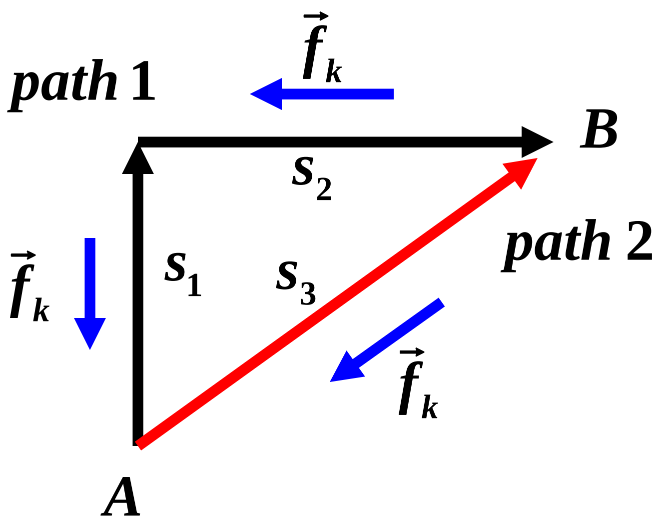
The exact relation between the force and potential: $F_x = -\dfrac{\partial U(x,y,z)}{\partial x}$ $\quad F_y = -\dfrac{\partial U(x,y,z)}{\partial y}$ $\quad F_z = -\dfrac{\partial U(x,y,z)}{\partial z}$

Nonconservative (dissipative) forces

Example 1: Friction

Direction of friction force is always opposite to the direction of velocity, so the friction force depends on velocity.

The work done by friction force depends on path.



$$W_{path1} = -f_k s_1 - f_k s_2 = -f_k(s_1 + s_2)$$

$$W_{path2} = -f_k s_3$$

$$s_3 \neq (s_1 + s_2)$$

$$W_{path1} \neq W_{path2}$$

Example 2: External Force

The work done by the net force is equal to the change of kinetic energy.

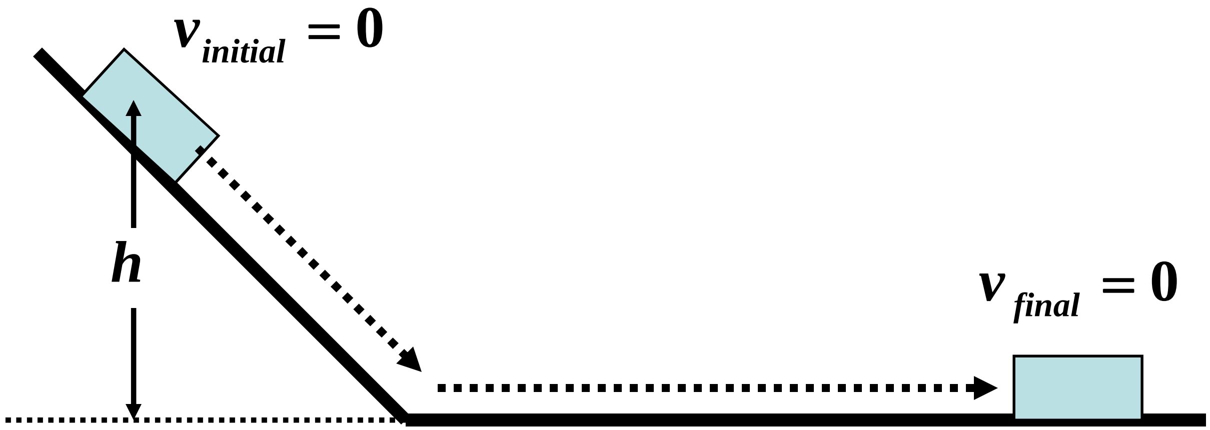
The net force is the sum of conservative forces (gravitational force, elastic force, …) and nonconservative force (friction force ..). The work done by conservative forces can be written as the change of potential energy. Then

$$\begin{aligned} W_{net} &= \vec{F}_{net} \cdot \Delta\vec{s} = \\ &= \vec{F}_{conservative} \cdot \Delta\vec{s} + \vec{F}_{nonconservative} \cdot \Delta\vec{s} = \\ &= U_{initial} - U_{final} + \vec{F}_{nonconservative} \cdot \Delta\vec{s} = K_{final} - K_{initial} \end{aligned}$$

$$\begin{aligned} \vec{F}_{nonconservative} \cdot \Delta\vec{s} &= (U_{final} + K_{final}) - (U_{initial} + K_{initial}) = \\ &= E_{mech,\, final} - E_{mech, initial} \end{aligned}$$

Work done by nonconservative forces is equal to the change of mechanical energy of the system (sum of kinetic energy, gravitational potential energy, elastic potential energy, and so on).

Example



Work done by friction force is equal to the change of mechanical energy:

$$W_{friction} = \int \vec{f}_k \cdot d\vec{s} = E_{mech,\,final} - E_{mech,initial} = -mgh$$

Nonconservative forces: friction (dissipative) forces and external forces.

Friction force decreases the mechanical energy of the system but increases the TEMPERATURE of the system – increases thermal energy of the system. Then

$$\vec{F}_{friction} \cdot \Delta\vec{s} = E_{thermal,initial} - E_{thermal,final}$$

$$\vec{F}_{nonconservative} \cdot \Delta\vec{s} = \vec{F}_{external} \cdot \Delta\vec{s} + E_{thermal,initial} - E_{thermal,final} =$$
$$= E_{mech,final} - E_{mech,initial}$$

$$\vec{F}_{external} \cdot \Delta\vec{s} = (E_{mech,final} + E_{thermal,final}) - (E_{mech,initial} + E_{thermal,initial})$$

Work done by external forces is equal to the change of the total energy of the system (mechanical + thermal)

Work done by external forces is equal to the change of the total energy of the system (mechanical + thermal)

Thermal energy is also a mechanical energy – this is the energy of motion of atoms or molecules inside the objects.

$$\vec{F}_{external} \cdot \Delta\vec{s} = (E_{mech,\,final} + E_{thermal,\,final}) - (E_{mech,initial} + E_{thermal,initial})$$