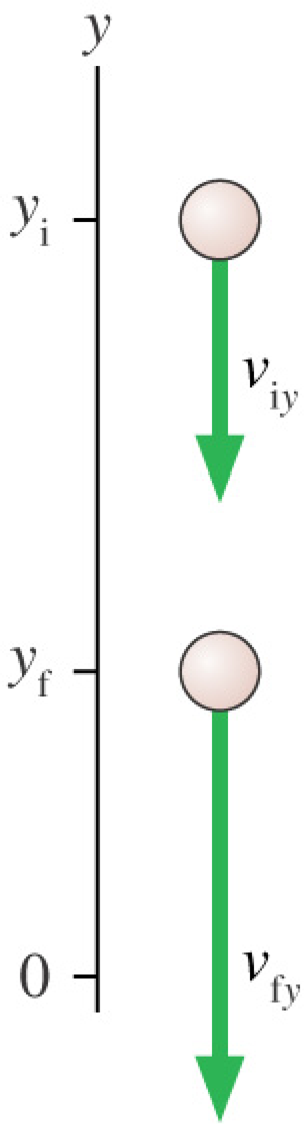
Energy

Readings: Chapter 10

Free-Fall motion



$$v_{fy}^2 - v_{iy}^2 = 2g(y_i - y_f)$$

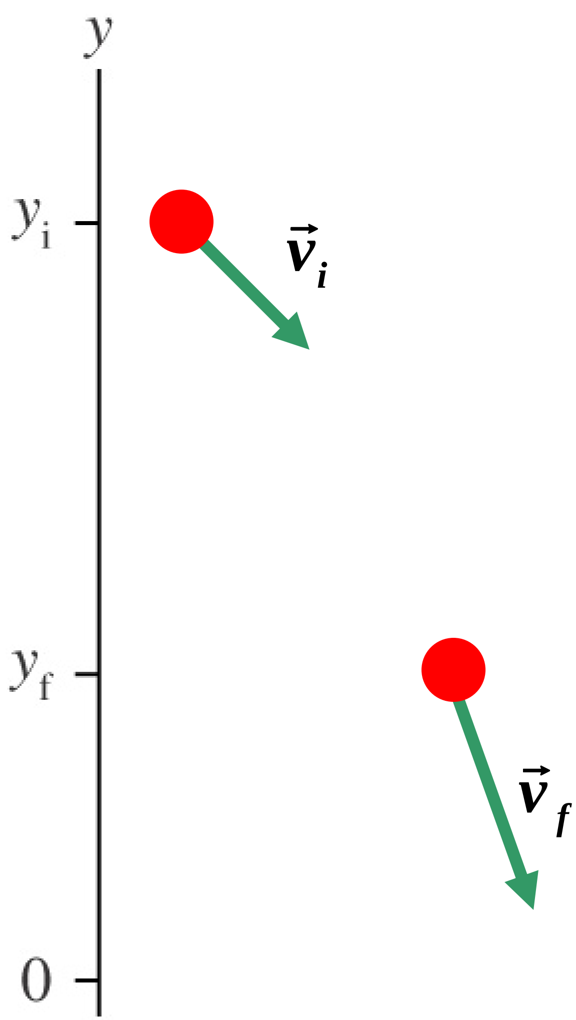
$$\frac{1}{2}v_{fy}^2 + gy_f = \frac{1}{2}v_{iy}^2 + gy_i$$

This is the conservation law for free fall motion: the quantity

$$\frac{1}{2}v_y^2 + gy$$

has the same value before and after the motion.

Free-Fall Motion



$$v_{fy}^2 - v_{iy}^2 = 2g(y_i - y_f)$$

$$v_{fx} = v_{ix}$$

Then

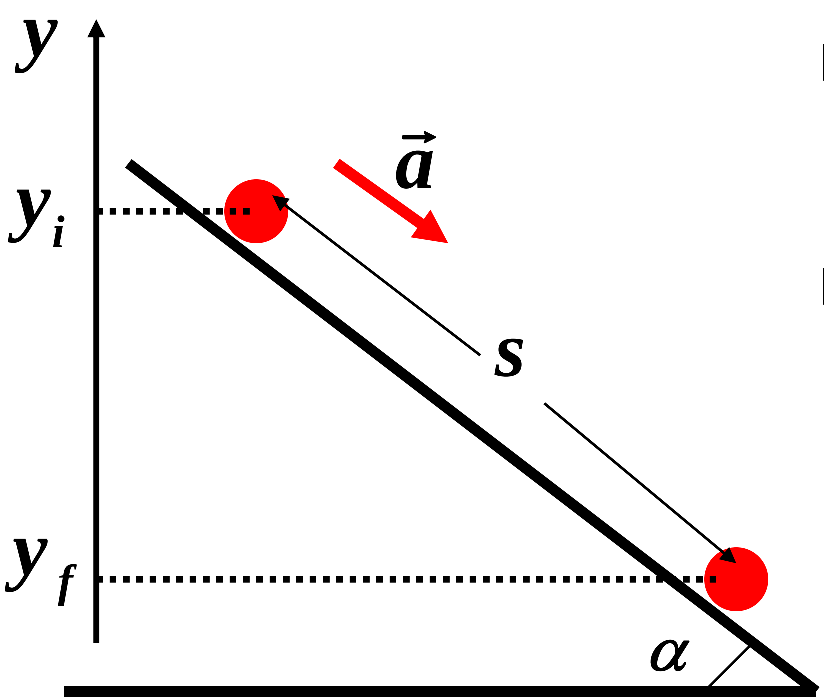
$$\frac{1}{2}v_{fx}^2 + \frac{1}{2}v_{fy}^2 + gy_f = \frac{1}{2}v_{ix}^2 + \frac{1}{2}v_{iy}^2 + gy_i$$

$$\frac{1}{2}v_f^2 + gy_f = \frac{1}{2}v_i^2 + gy_i$$

Conservation law: the quantity

$$\frac{1}{2}v^2 + gy$$

has the same value before and after the motion.



Frictionless surface: acceleration

$$a = g\sin\alpha$$

Motion with constant acceleration:

$$v_f^2 - v_i^2 = 2as$$

$$s = \frac{y_i - y_f}{\sin\alpha}$$

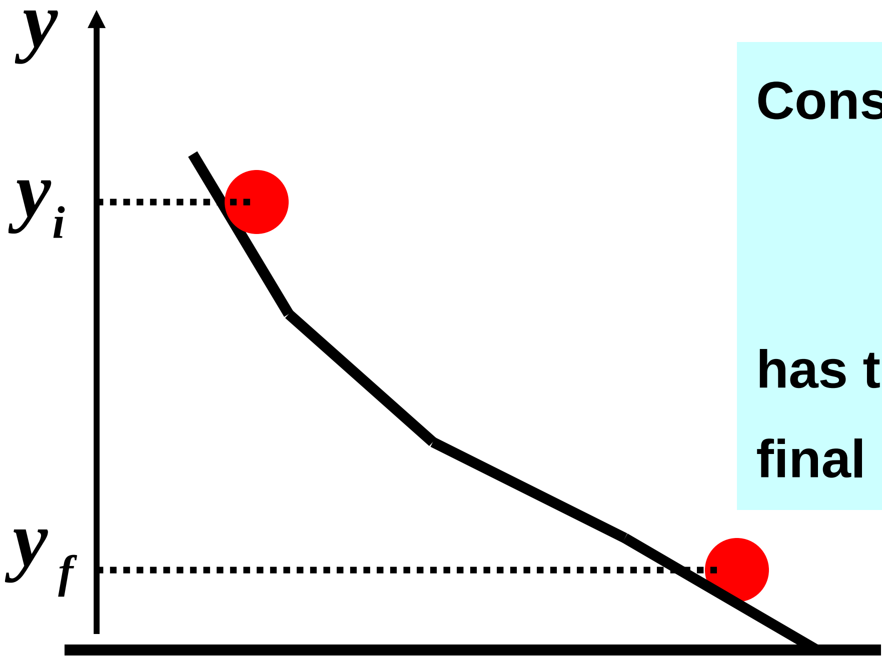
$$v_f^2 - v_i^2 = 2g\sin\alpha\,\frac{y_i - y_f}{\sin\alpha} = 2gy_i - 2gy$$

Conservation law: the quantity

$$\frac{1}{2}v^2 + gy$$

has the same value before and after the motion.

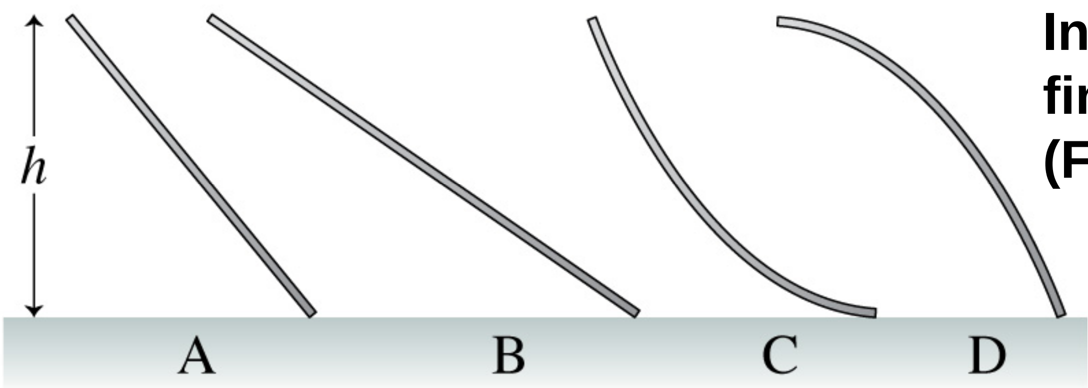
$$\frac{1}{2}v_f^2 + gy_f = \frac{1}{2}v_i^2 + gy_i$$



Conservation law: the quantity

$$\frac{1}{2}v^2 + gy$$

has the same value at the initial and the final points



In all cases the velocity at the final point is the same (FRICTIONLESS MOTION)

Conservation law: $\frac{1}{2}v^2 + gy$ or $\frac{1}{2}mv^2 + mgy$

$$K = \frac{1}{2}mv^2$$

Kinetic Energy – energy of motion

$$U_g = mgy$$

Gravitational Potential Energy – energy of position

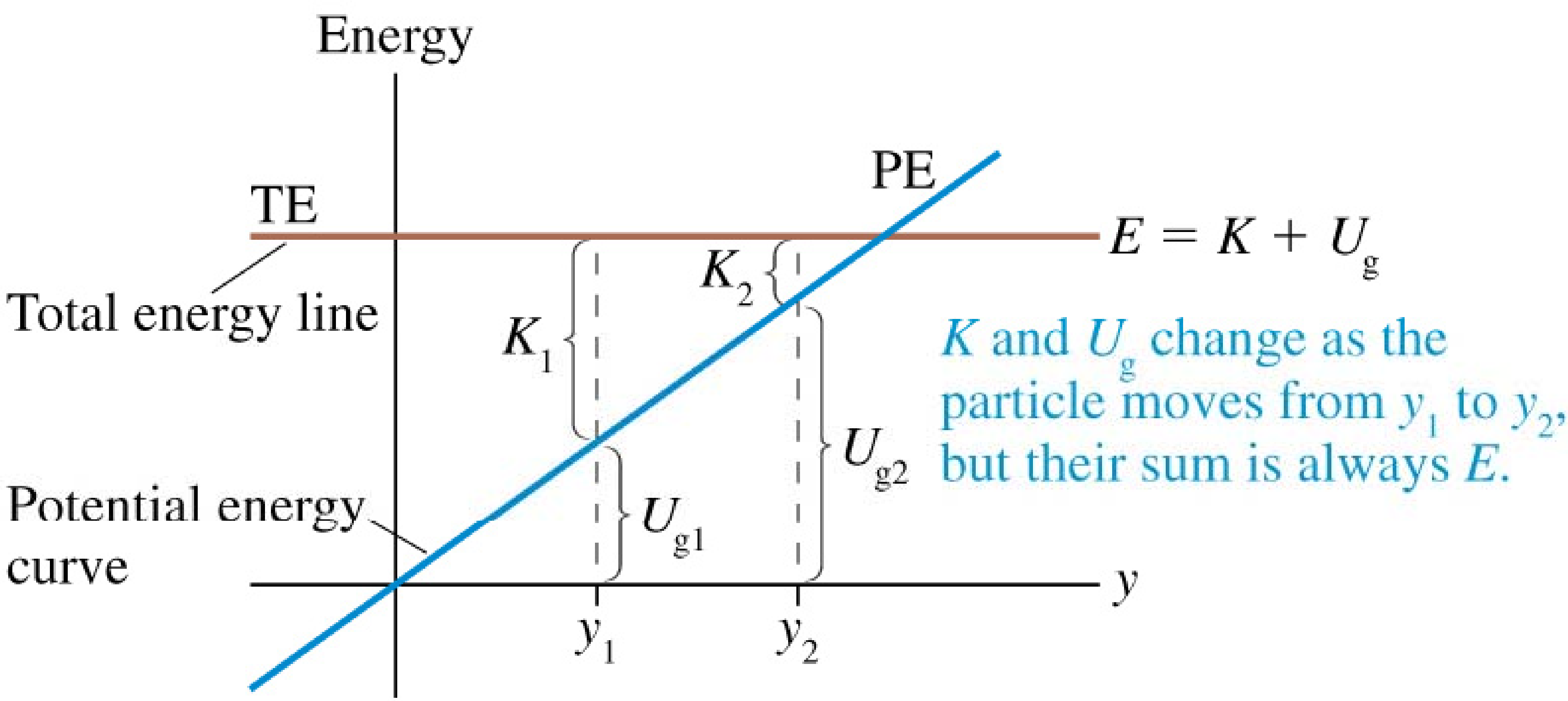
$$E_{mech} = K + U_g$$

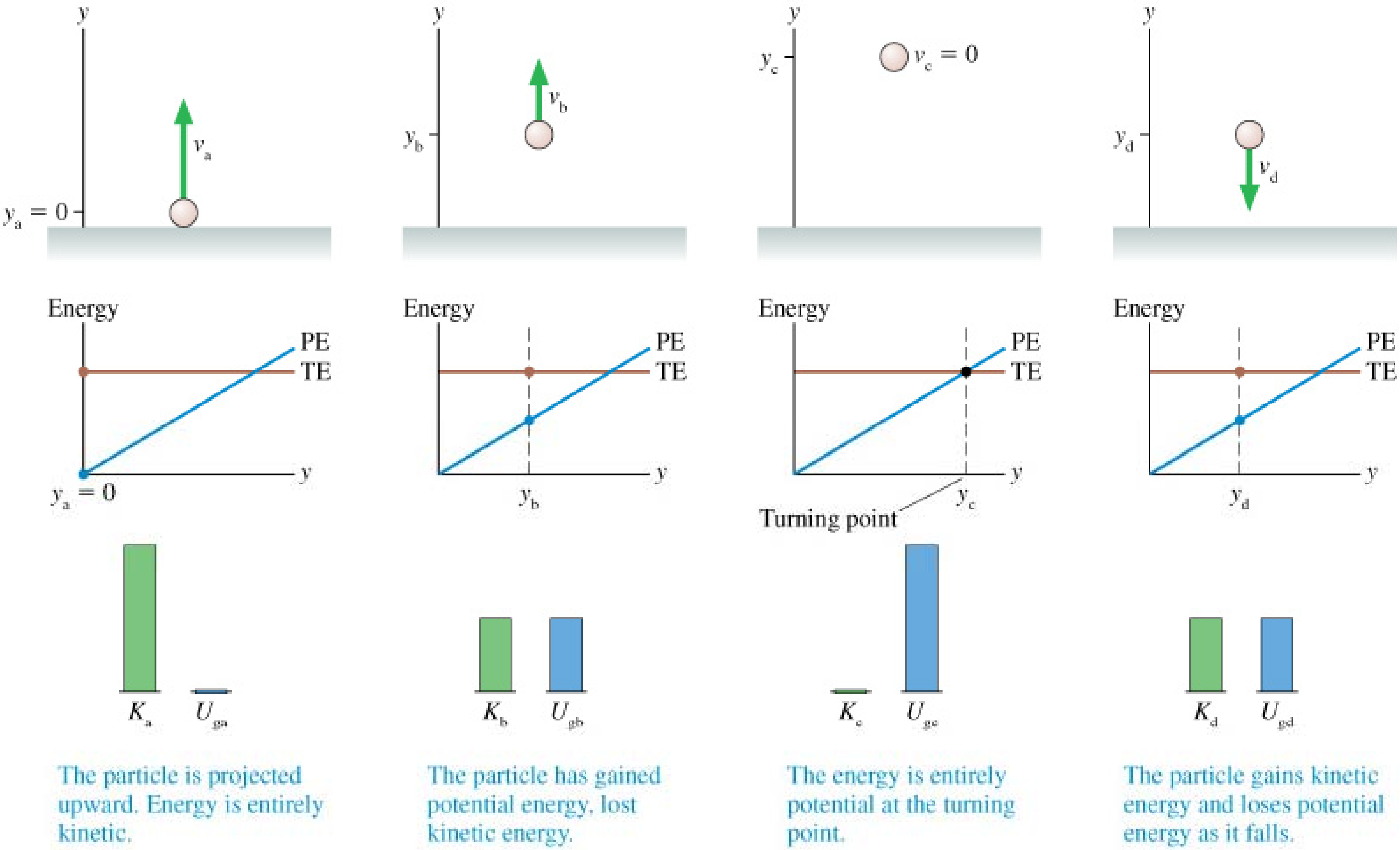
Mechanical Energy

Conservation law of mechanical energy (without friction):

$$E_{mech} = K + U = \text{constant}$$

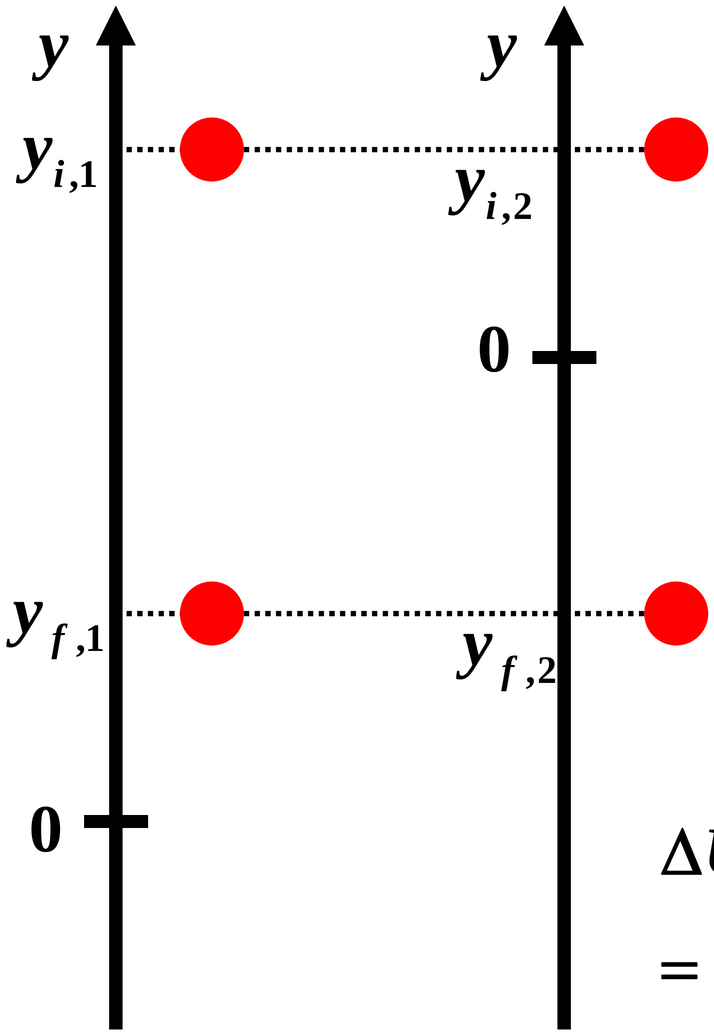
The units of energy is Joule: $J = kg\frac{m^2}{s^2}$


Energy
PE
TE
$E = K + U_g$
Total energy line
K_1
K_2
K and U_g change as the particle moves from y_1 to y_2, but their sum is always E.
U_{g2}
U_{g1}
Potential energy curve
y
y_1
y_2



y
$y_a = 0$
v_a
Energy
PE
TE
y
$y_a = 0$
K_a
U_{ga}
The particle is projected upward. Energy is entirely kinetic.
y
y_b
v_b
Energy
PE
TE
y
y_b
K_b
U_{gb}
The particle has gained potential energy, lost kinetic energy.
y
y_c
$v_c = 0$
Energy
PE
TE
y
y_c
Turning point
K_c
U_{gc}
The energy is entirely potential at the turning point.
y
y_d
v_d
Energy
PE
TE
y
y_d
K_d
U_{gd}
The particle gains kinetic energy and loses potential energy as it falls.

Zero of potential energy



$$K_i + U_{g,i,1} = K_{f,1} + U_{g,f,1}$$

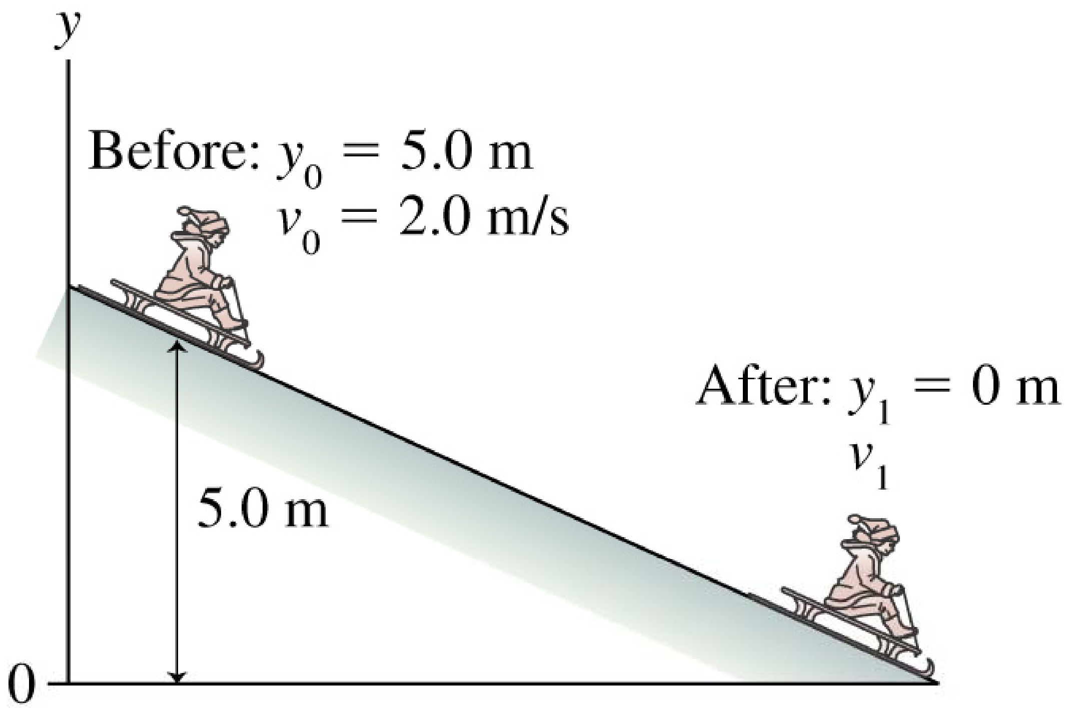
$$K_i + U_{g,i,2} = K_{f,2} + U_{g,f,2}$$

$$K_{f,1} = K_i + (U_{g,i,1} - U_{g,f,1})$$

$$K_{f,2} = K_i + (U_{g,i,2} - U_{g,f,2})$$

$$\Delta U_{g,1} = (U_{g,i,2} - U_{g,f,2}) = mg(y_{i,2} - y_{f,2}) =$$
$$= mg(y_{i,1} - y_{f,1}) = (U_{g,i,1} - U_{g,f,1}) = \Delta U_{g,2}$$

Only the change of potential energy has the physical meaning



Find: v_1

$$K_i + U_{g,i} = K_f + U_{g,f}$$

$$K_i = \frac{1}{2}mv_0^2$$

$$U_{g,i} = mgy_0$$

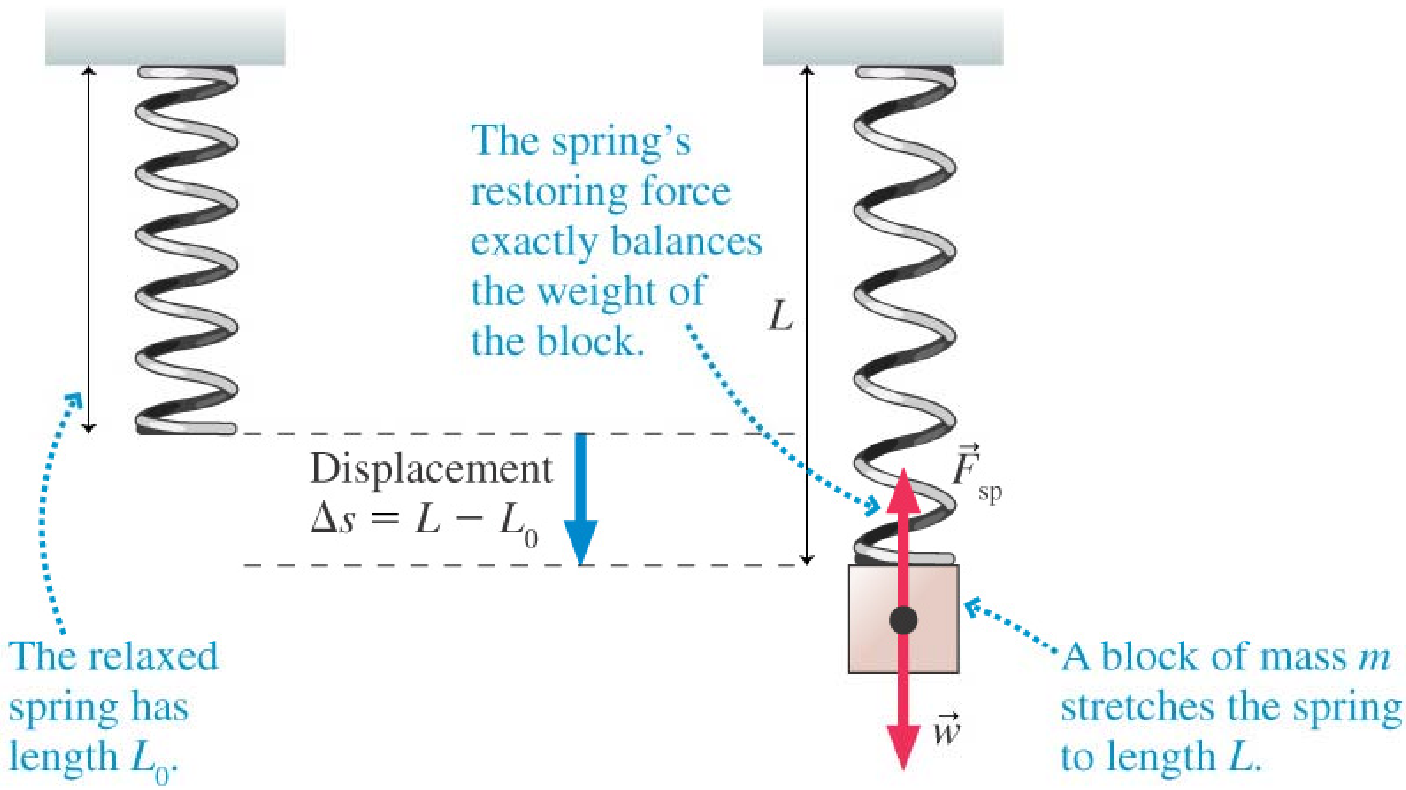
$$U_{g,f} = mgy_1 = 0$$

$$K_f = \frac{1}{2}mv_1^2$$

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_0^2 + mgy_0$$

$$v_1 = \sqrt{v_0^2 + 2gy_0} = \sqrt{4+100} \approx 10.2m/s$$

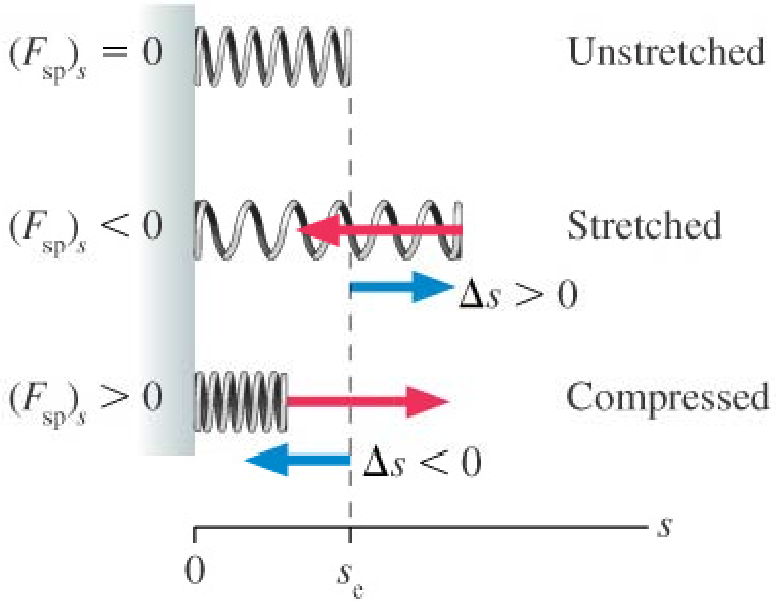
Restoring Force: Hooke's Law

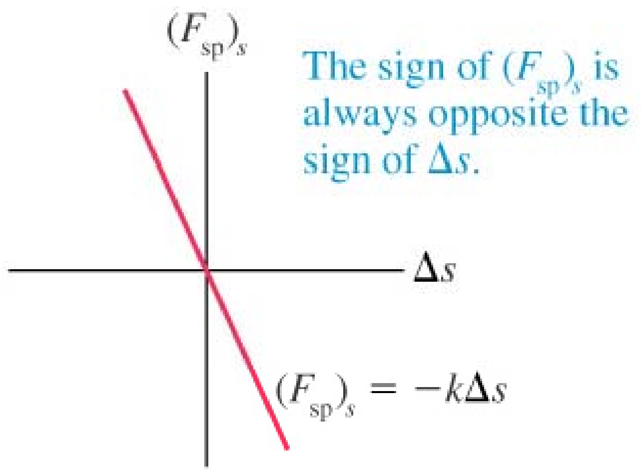


Hooke's Law: $F_{sp} = -k\Delta s$

spring constant

Restoring Force: Hooke's Law

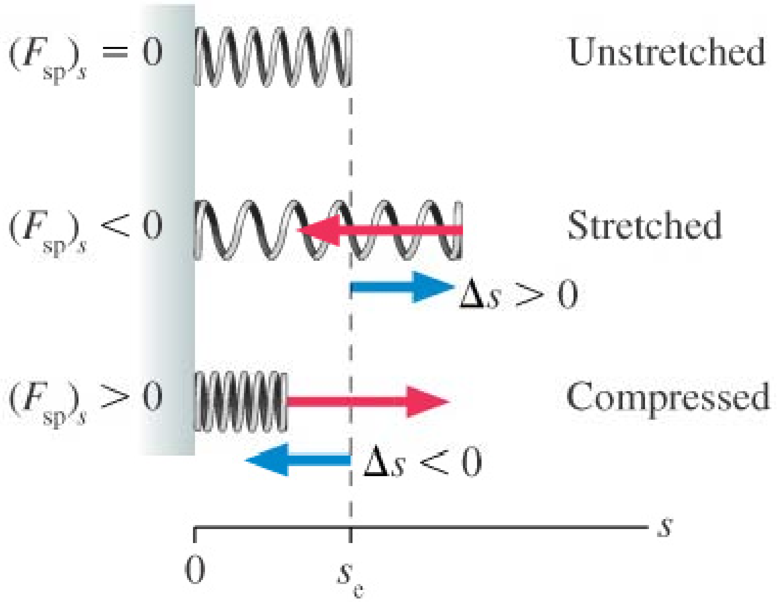




$$F_{sp} = -k\Delta s$$

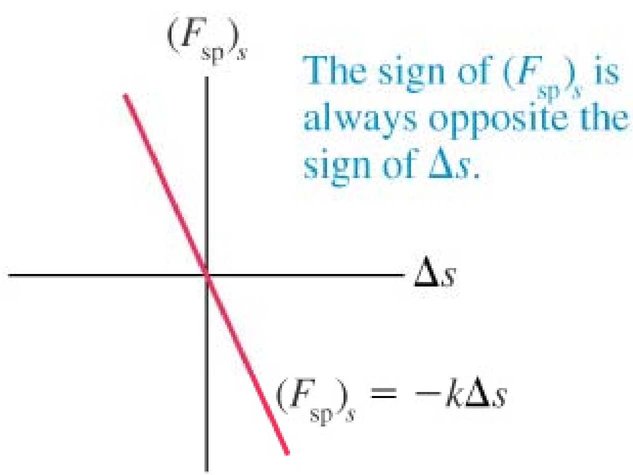
The sign of a restoring force is always opposite to the sign of displacement

Restoring Force: Elastic Potential Energy



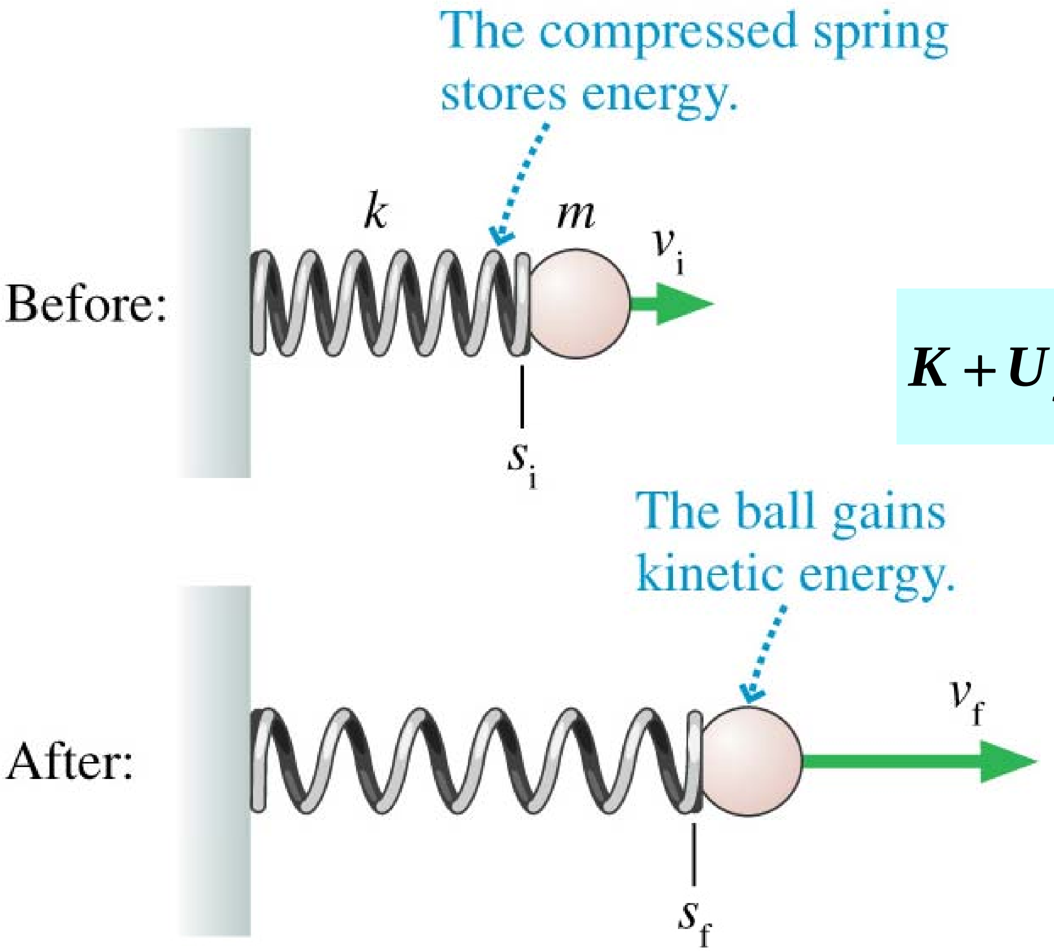
$$F_{sp} = -k\Delta s$$

$$U_s = \frac{1}{2}k(\Delta s)^2$$



The elastic potential energy is always positive

Restoring Force: Elastic Potential Energy

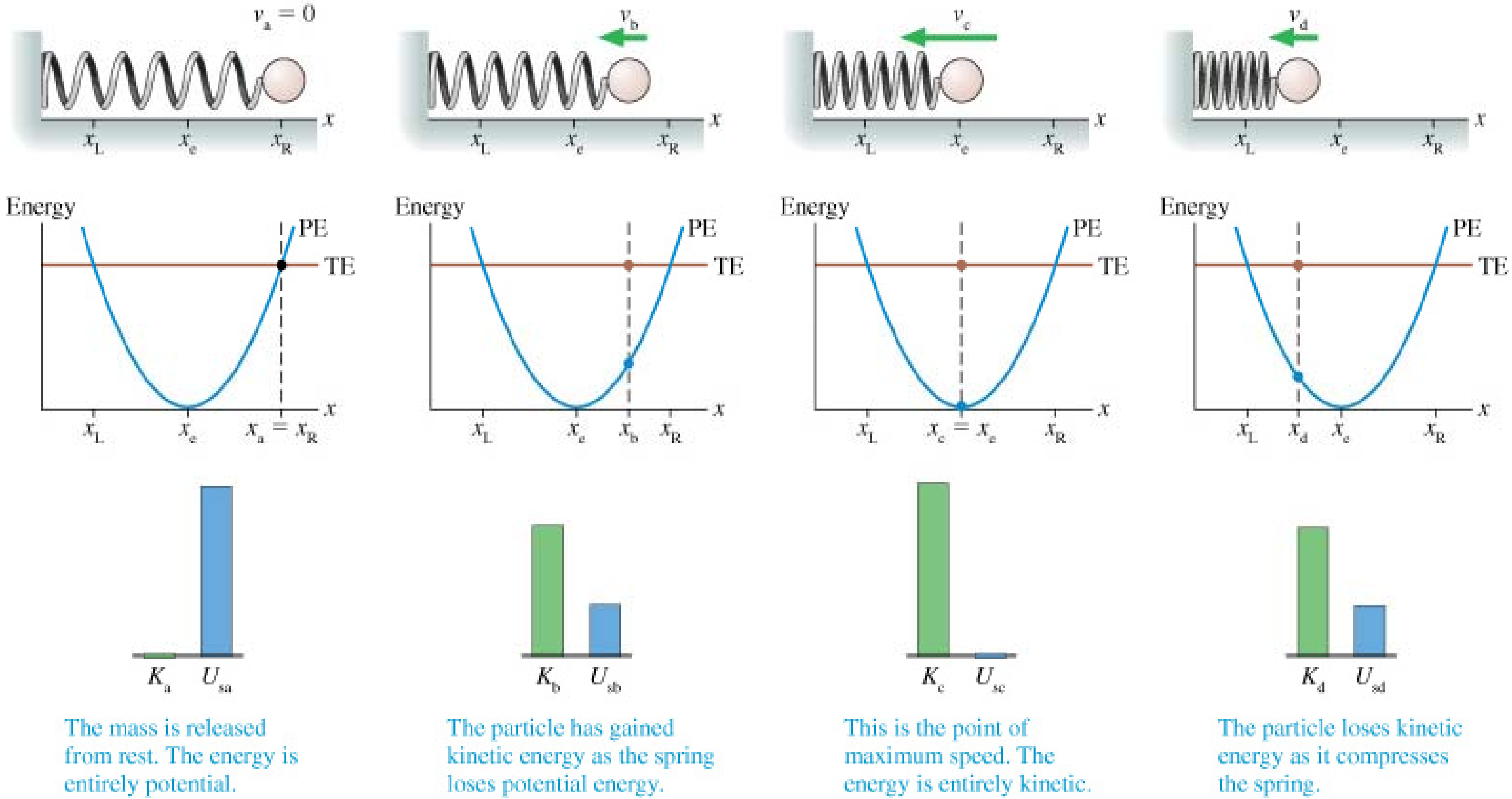


$$U_s = \frac{1}{2}k(\Delta s)^2$$

$$K + U_s = \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta s)^2 = \text{constant}$$

Restoring Force: Elastic Potential Energy

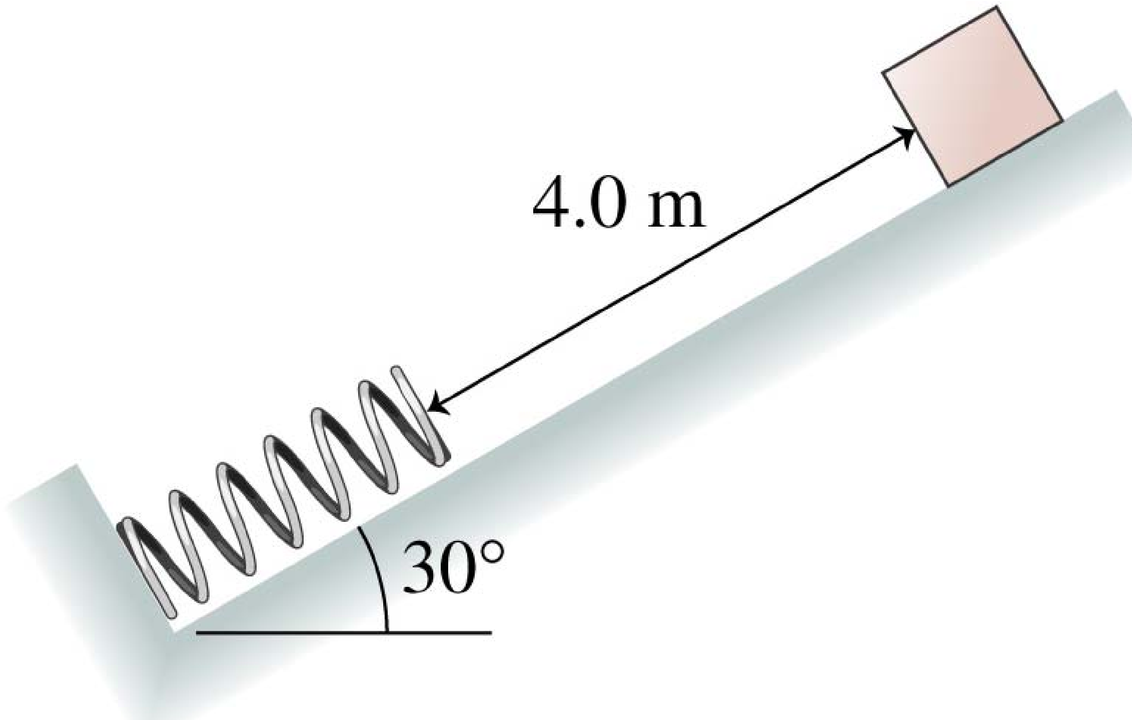
$$U_s = \frac{1}{2}k(\Delta s)^2$$

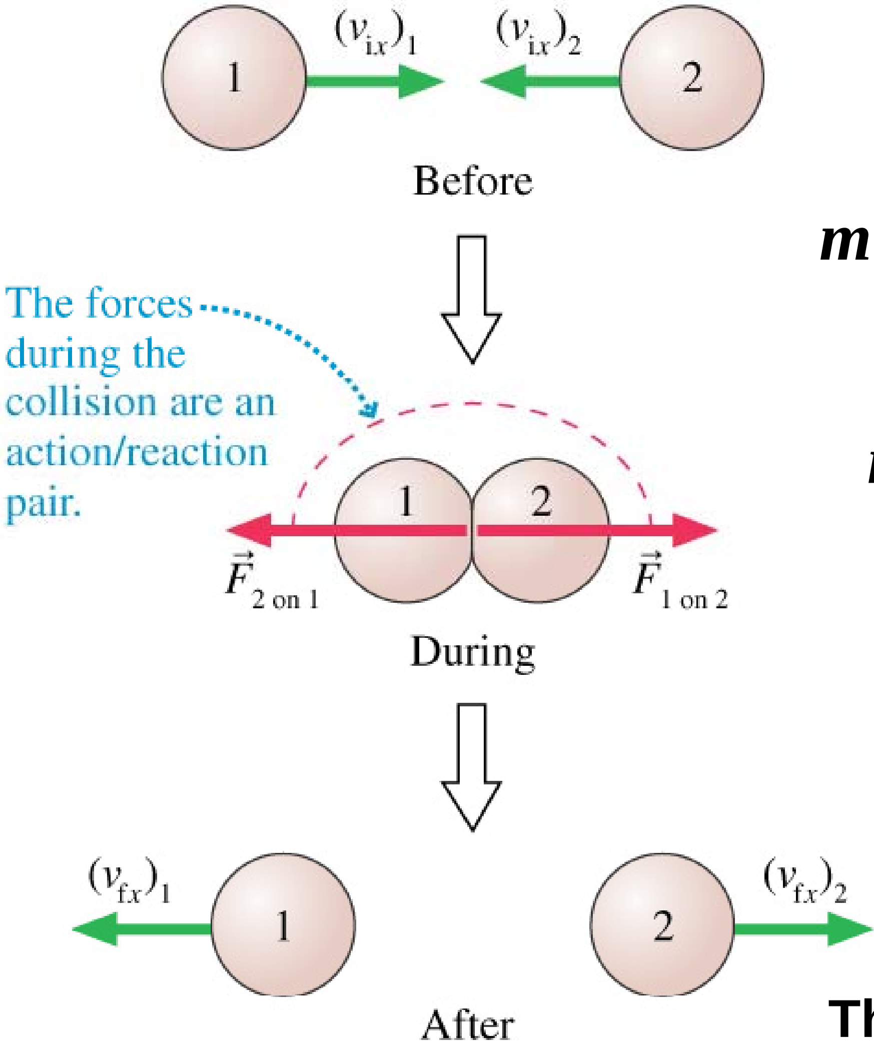


$$K + U_s = \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta s)^2 = \text{constant}$$

Law of Conservation of Mechanical Energy

$$K + U_g + U_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}k(\Delta s)^2 = \text{constant}$$





$$m_2 v_{fx,2} - m_2 v_{ix,2} = -\left(m_1 v_{fx,1} - m_1 v_{ix,1}\right)$$

$$m_1 v_{ix,1} + m_2 v_{ix,2} = m_1 v_{fx,1} + m_2 v_{fx,2}$$

$$p_{ix,1} + p_{ix,2} = p_{fx,1} + p_{fx,2}$$

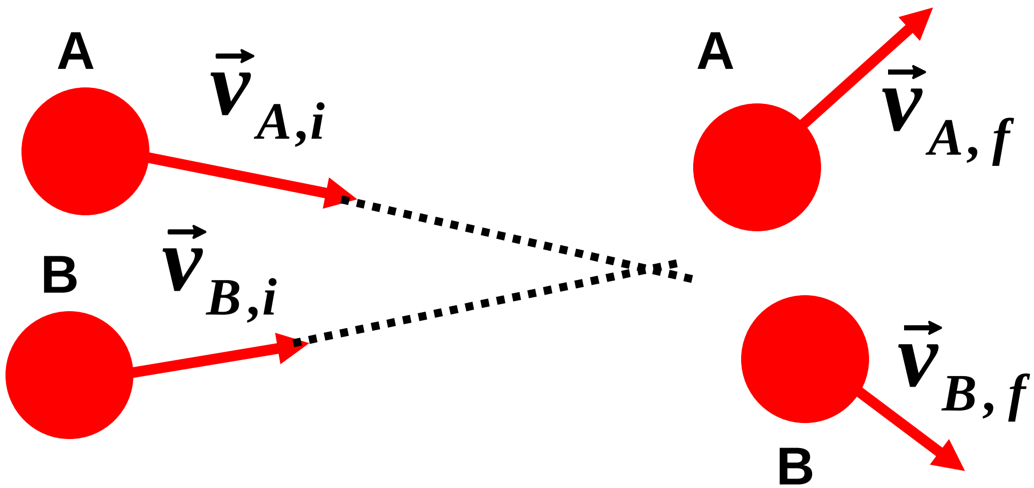
$$p_{ix,total} = p_{fx,total}$$

The law of conservation of momentum

Perfectly Elastic Collisions

During the collision the kinetic energy will be transformed into elastic energy and then elastic energy will transformed back into kinetic energy

Perfectly Elastic Collision: Mechanical Energy is Conserved (no internal friction)



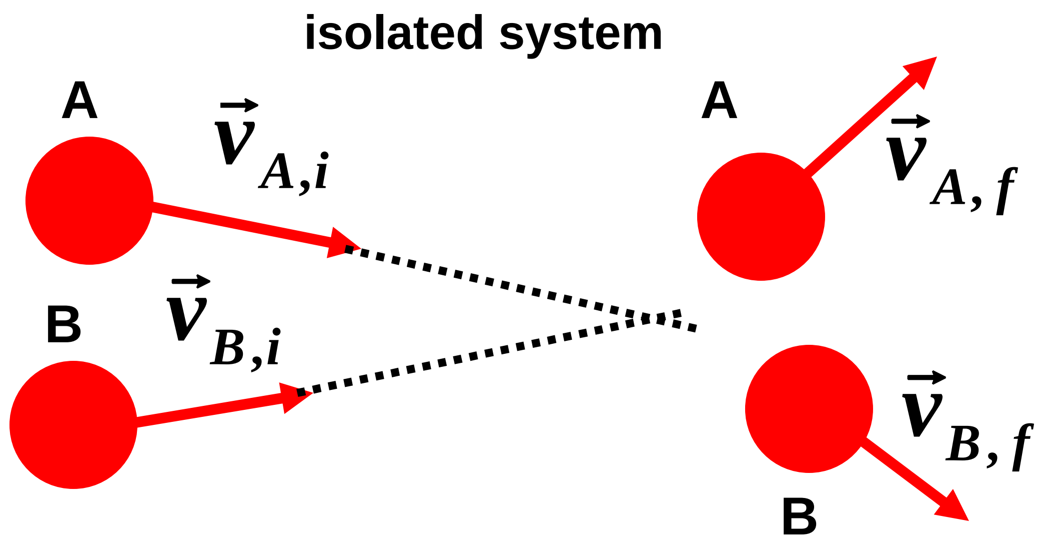
$$\frac{1}{2}m_A v_{A,i}^2 + \frac{1}{2}m_B v_{B,i}^2 = $$
$$= \frac{1}{2}m_A v_{A,f}^2 + \frac{1}{2}m_B v_{B,f}^2$$

Perfectly inelastic collision:

A collision in which the two objects stick together and move with a common final velocity – NO CONSERVATION OF MECHANICAL ENERGY

Perfectly Elastic Collisions

Conservation of Momentum and Conservation of Energy

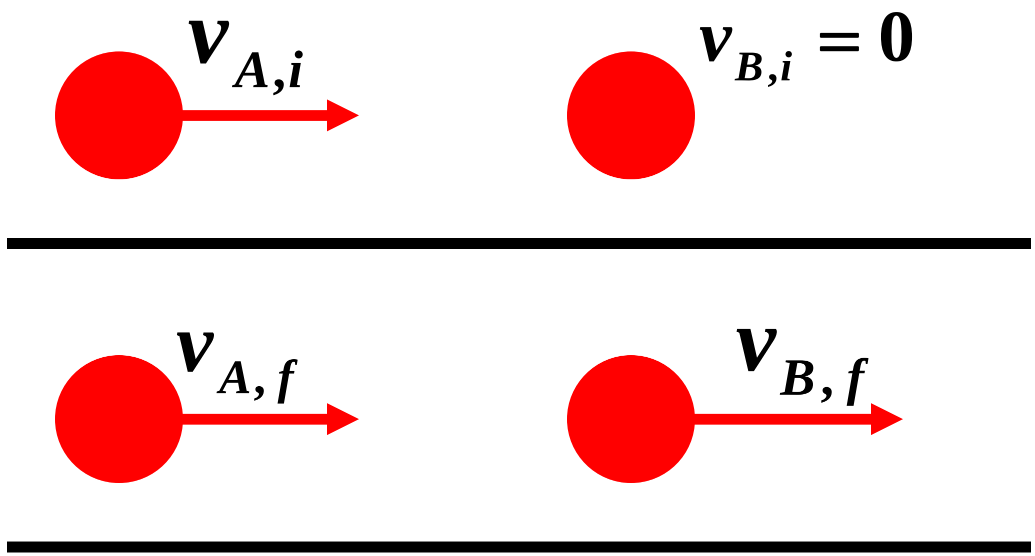


$$m_A\vec{v}_{A,i} + m_B\vec{v}_{B,i} = m_A\vec{v}_{A,f} + m_B\vec{v}_{B,f}$$

$$\frac{1}{2}m_A v_{A,i}^2 + \frac{1}{2}m_B v_{B,i}^2 = \frac{1}{2}m_A v_{A,f}^2 + \frac{1}{2}m_B v_{B,f}^2$$

Now we have enough equations to find the final velocities.

Example:



$$m_A v_{A,i} = m_A v_{A,f} + m_B v_{B,f}$$

$$\frac{1}{2} m_A v_{A,i}^2 = \frac{1}{2} m_A v_{A,f}^2 + \frac{1}{2} m_B v_{B,f}^2$$

$$v_{A,f} = \frac{m_A - m_B}{m_A + m_B} v_{A,i}$$

$$v_{B,f} = \frac{2m_A}{m_A + m_B} v_{A,i}$$

$v_{B,f}$ is always positive, $v_{A,f}$ can be positive (if $m_A > m_B$) or negative (if $m_A < m_B$)

$$v_{A,f} = 0$$
$$v_{B,f} = v_{A,i}$$
if $m_A = m_B$