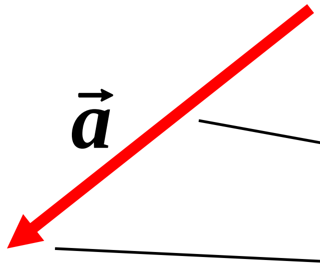
Vectors and Integrals

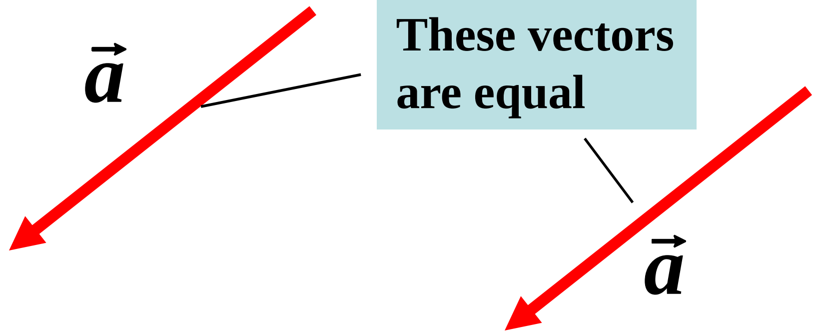
Readings: Chapter 3

Vectors



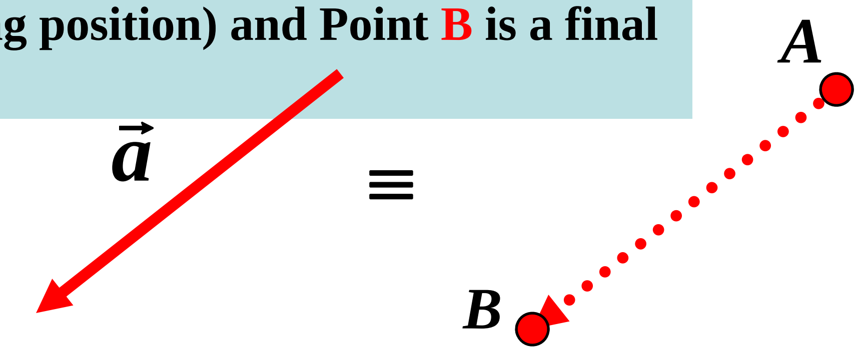
Vectors are objects which are characterized by two parameters:

- **magnitude (length)**
- **direction**

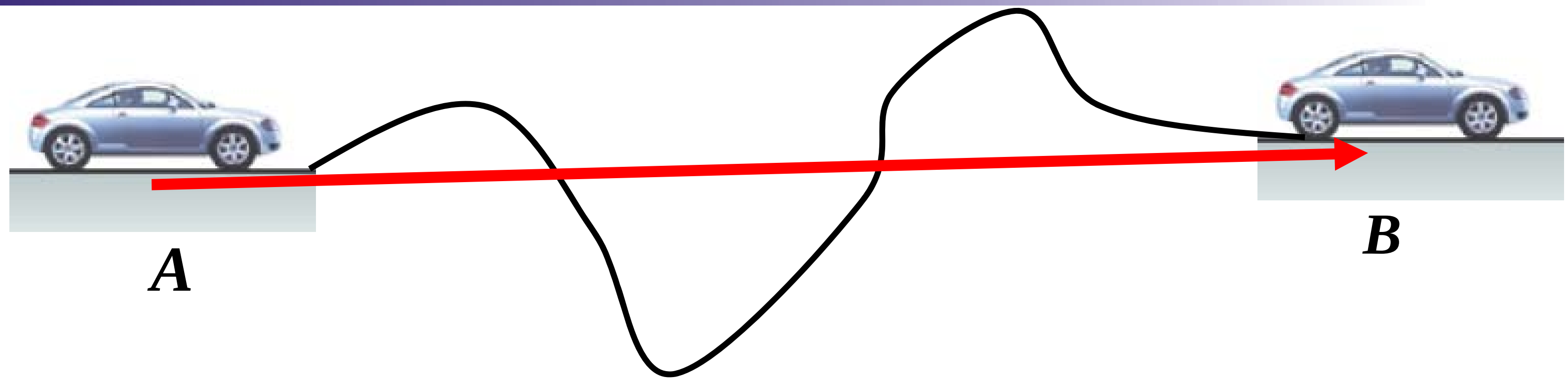


Vector can be considered as a set of two points in space:

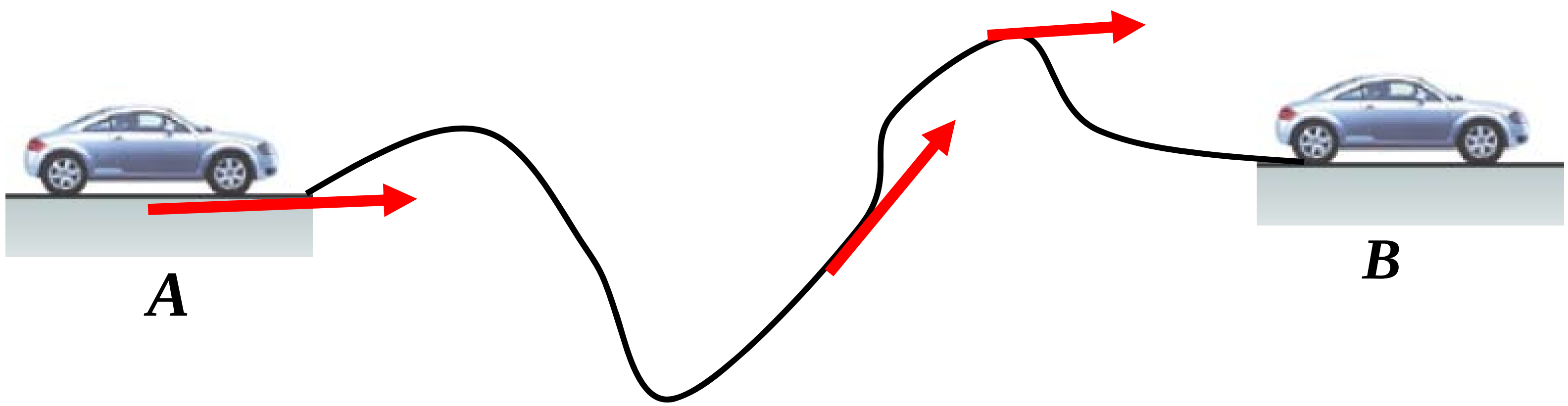
Point A is an initial point (starting position) and Point B is a final point (ending position)



Vectors: example



Vector: displacement of a car: initial point A, final point B

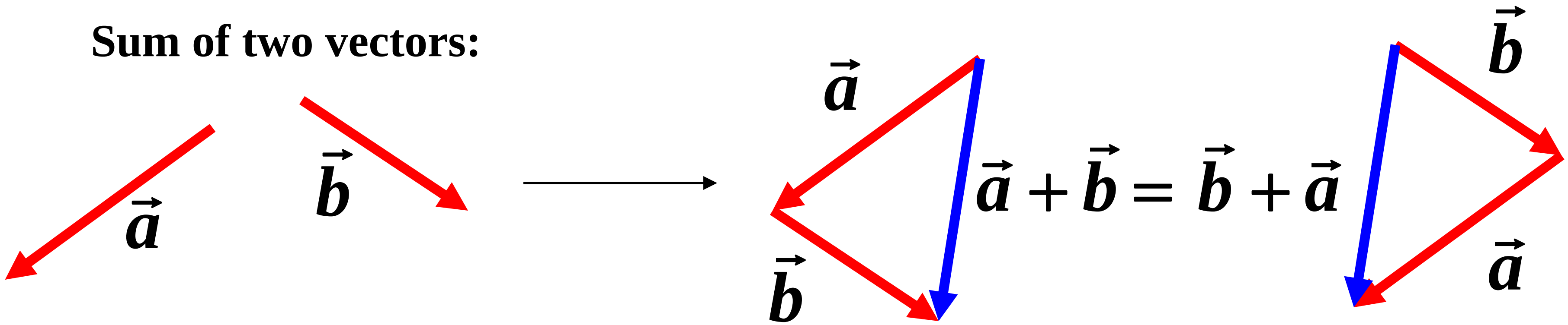


Vector: velocity of a car, direction is the direction of the motion

The speed of the car could be the same (the magnitude of velocity could be the same) but the directions of the motion could be different - different vectors – different velocities.

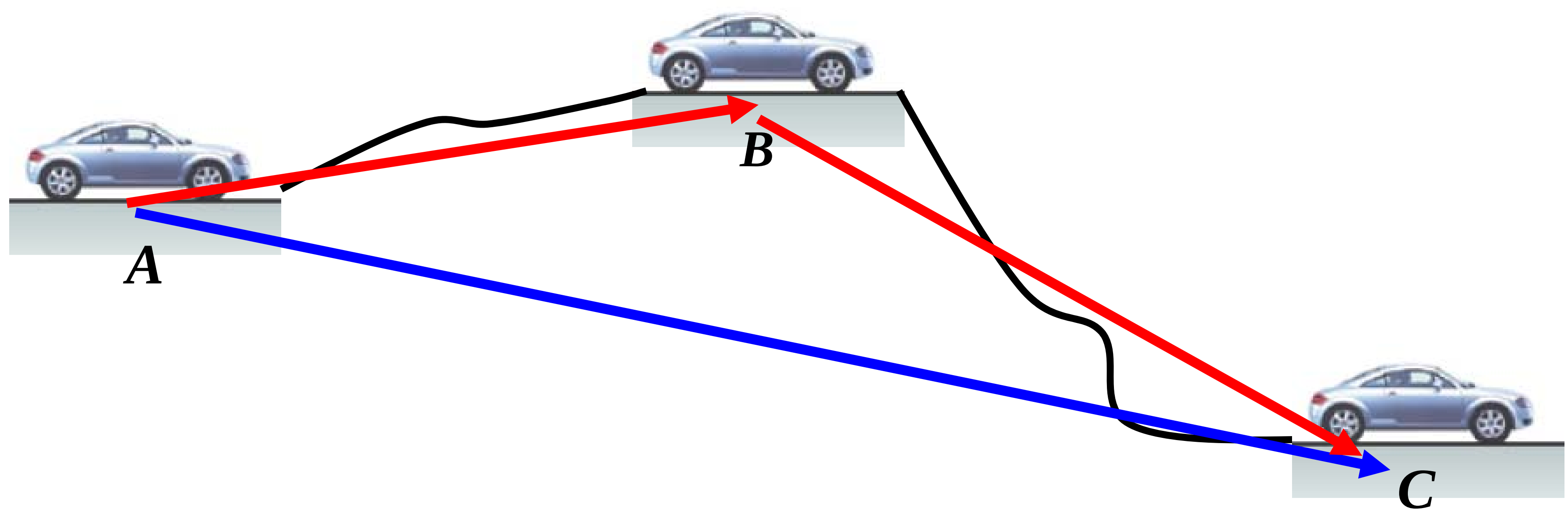
Sum of two vectors

Sum of two vectors:



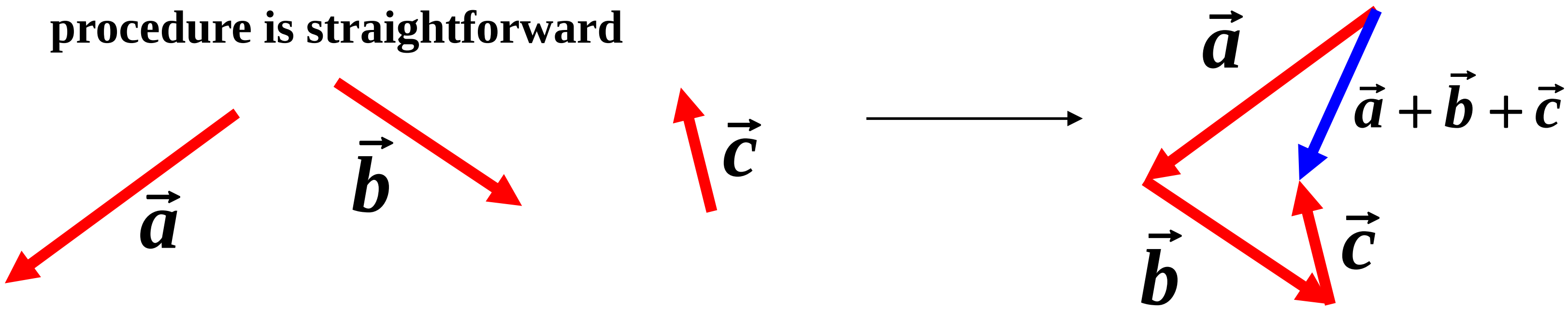
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

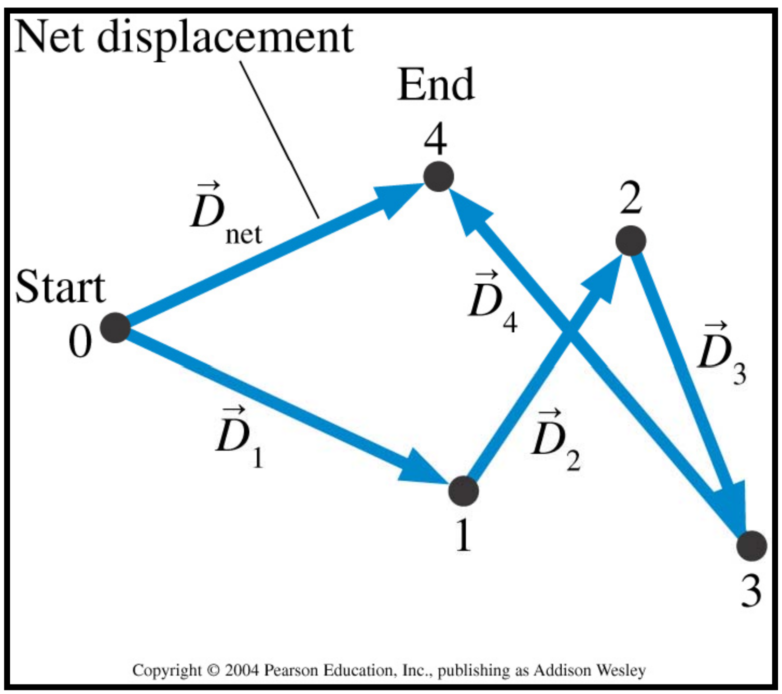
Sum of two vectors: net displacement



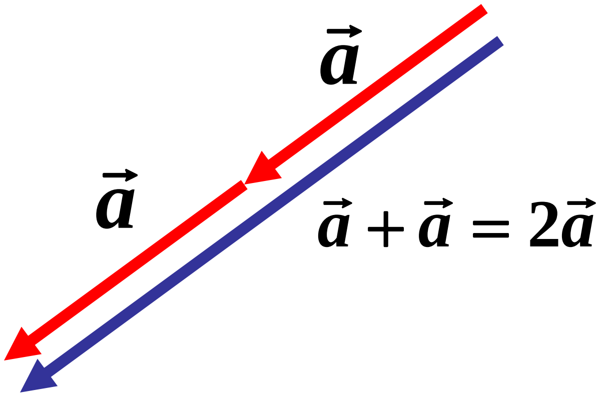
Sum of vectors

Sum of vectors: for many vectors the procedure is straightforward

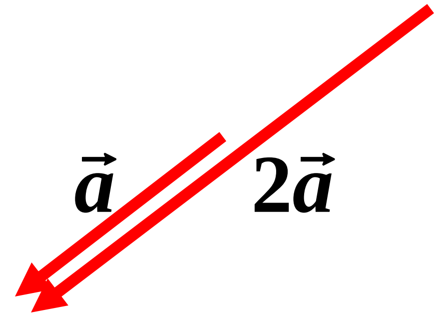




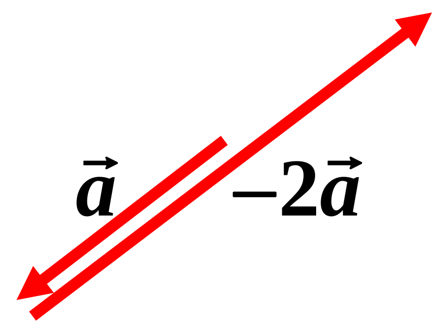
Vectors: multiplication by a number

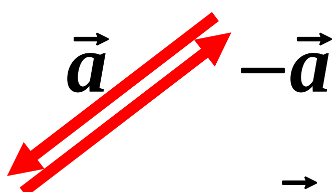


Vector $c\vec{a}$ (where c is a positive number) has the same direction as $\vec{a}$, but its magnitude is c times larger



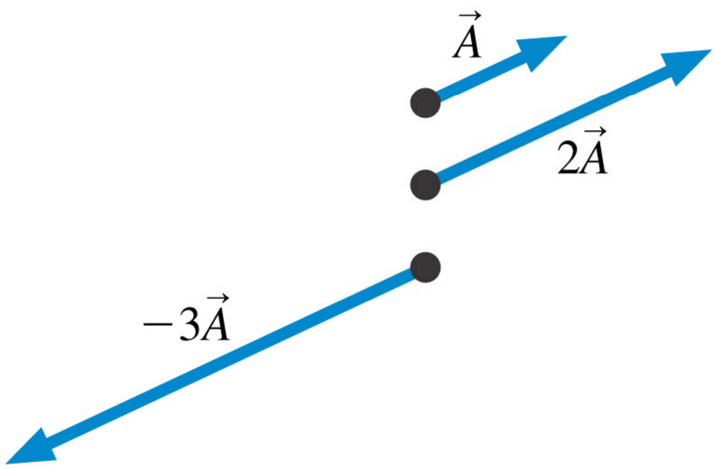
Vector $c\vec{a}$ (where c is a negative number) has the direction opposite to $\vec{a}$, and c times larger magnitude

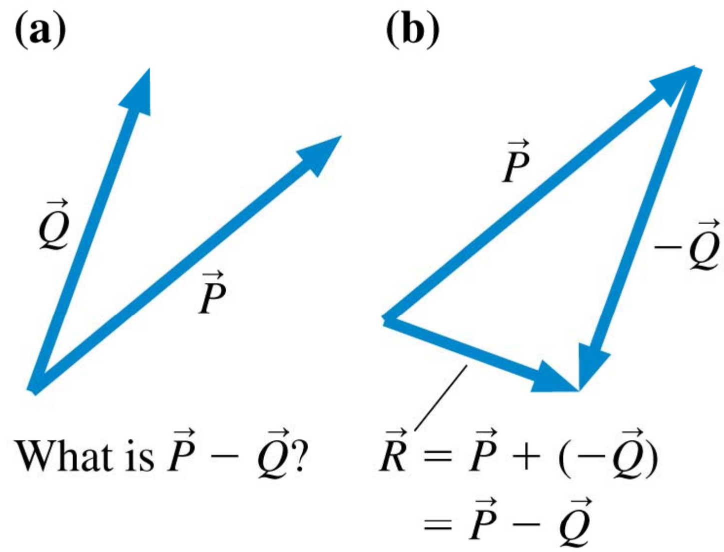




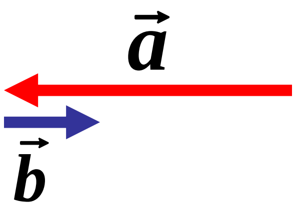
$$\vec{a} + (-\vec{a}) = \vec{a} - \vec{a} = 0$$

Vectors: multiplication by number





Vectors: Examples



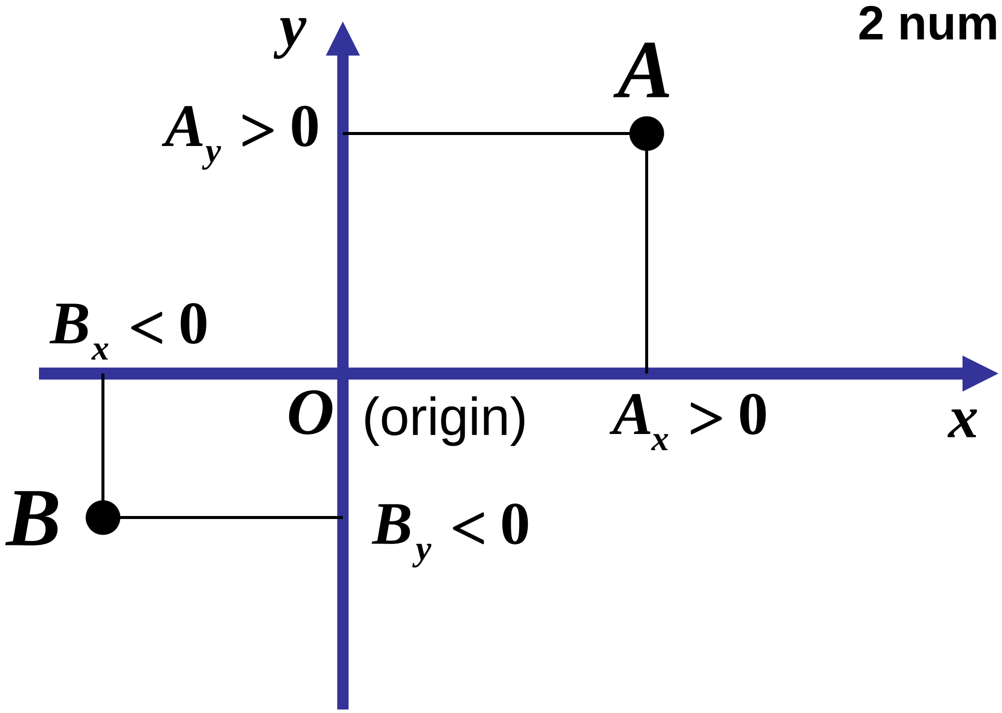
The magnitude of $\vec{a}$ is 5

What is the direction and the magnitude of $\vec{b} = -0.2\vec{a}$

The magnitude of $\vec{b}$ is $b = 0.2 \cdot 5 = 1$, the direction is opposite to $\vec{a}$

Vectors: coordinate system and vector component

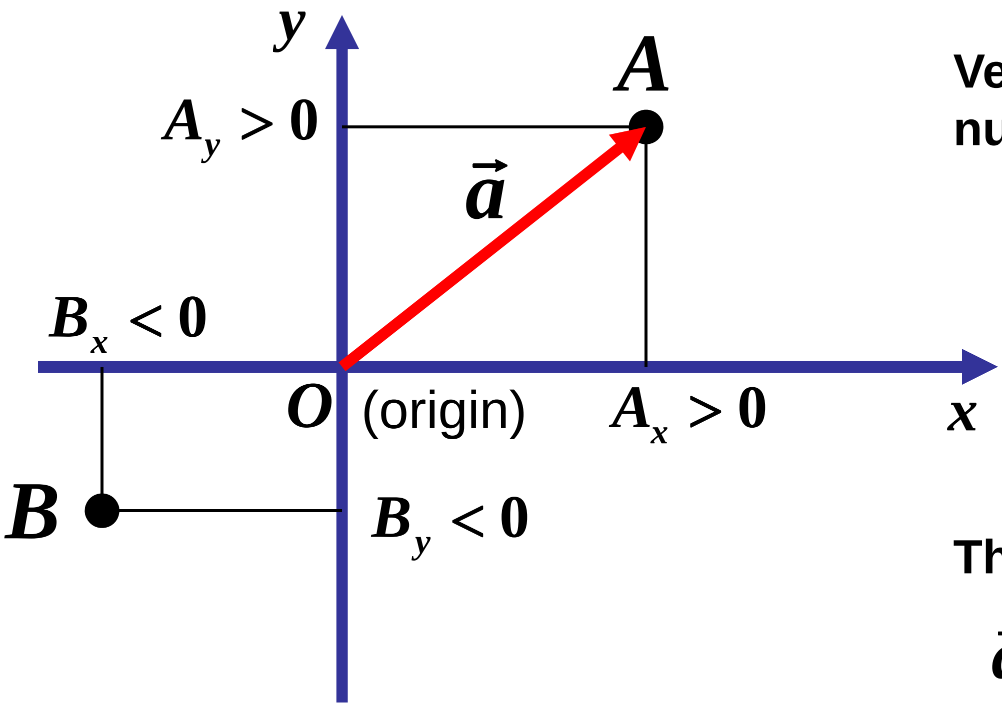
coordinate system:



Position of point *A* is characterized by 2 numbers A_x, A_y

Vectors: coordinate system and vector components

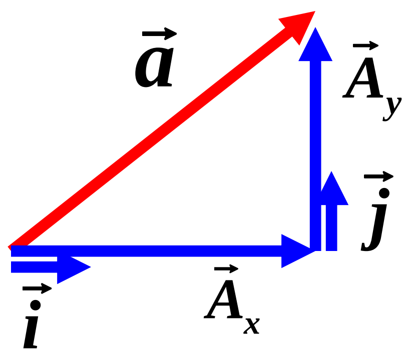
coordinate system:



Vector $\vec{a}$ is characterized by two numbers A_x, A_y

The meaning of it is the following:

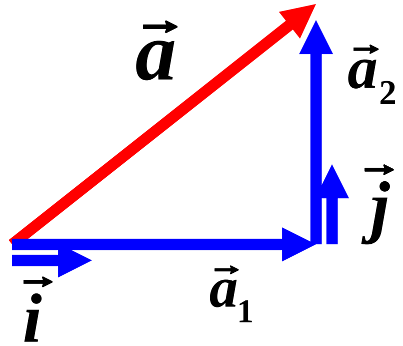
$$\vec{a} = \vec{A}_x + \vec{A}_y = A_x\vec{i} + A_y\vec{j}$$



$\vec{i}, \vec{j}$ - vectors with unit magnitude

Vectors: coordinate system and vector components

Then the sum of the vectors is the sum of their components:



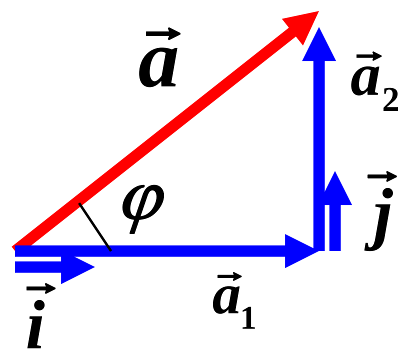
$$\vec{a} = (a_1, a_2) \quad \vec{b} = (b_1, b_2) \longrightarrow \vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2)$$

$$\vec{a} = (a_1, a_2) \longrightarrow c\vec{a} = (ca_1, ca_2)$$

$$\vec{a} + \vec{b} = a_1\vec{i} + a_2\vec{j} + b_1\vec{i} + b_2\vec{j} = (a_1 + b_1)\vec{i} + (a_2 + b_2)\vec{j}$$

Magnitude of the vector: $a = \sqrt{a_1^2 + a_2^2}$

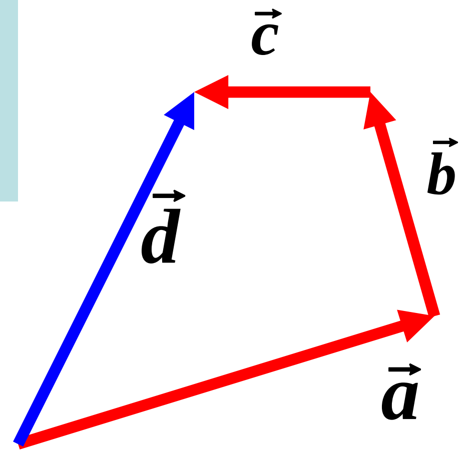
Direction of the vector: $\tan\varphi = \dfrac{a_2}{a_1}$



Vectors: Example

Find the magnitude and direction of the sum of three vectors with components

(5,2) (-1,4) and (0,-2)



$$\vec{d} = \vec{a} + \vec{b} + \vec{c}$$

Components of vector $\vec{d}$

$$d_1 = 5 - 1 + 0 = 4$$

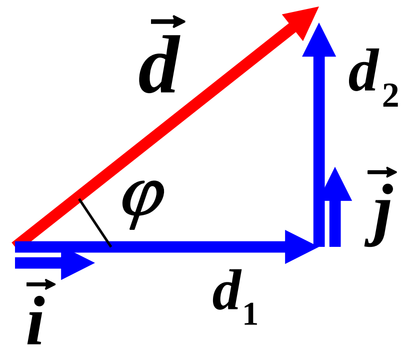
$$d_2 = 2 + 4 - 2 = 4$$

Magnitude

$$d = \sqrt{d_1^2 + d_2^2} = \sqrt{4^2 + 4^2} = 5.6$$

Direction

$$\tan\varphi = \frac{d_2}{d_1} = 1 \qquad \varphi = 45^0$$



USING VECTORS

Components

The component vectors are parallel to the x- and y-axes.

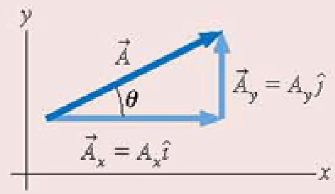
$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$$

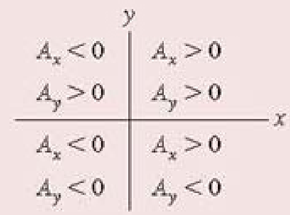
In the figure at the right, for example:

$$A_x = A\cos\theta \quad A = \sqrt{A_x^2 + A_y^2}$$

$$A_y = A\sin\theta \quad \theta = \tan^{-1}(A_y/A_x)$$

► Minus signs need to be included if the vector points down or left.

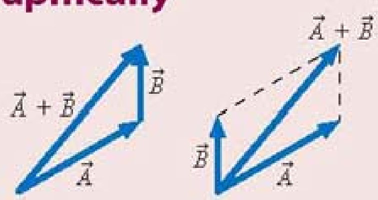




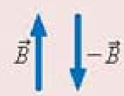
The components A_x and A_y are the magnitudes of the component vectors $\vec{A}_x$ and $\vec{A}_y$ *and* a plus or minus sign to show whether the component vector points toward the positive end or the negative end of the axis.

Working Graphically

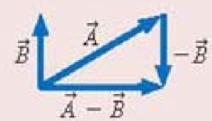
Addition



Negative



Subtraction



Multiplication

Working Algebraically

Vector calculations are done component by component.

$$\vec{C} = 2\vec{A} + \vec{B} \quad \text{means} \quad \begin{cases} C_x = 2A_x + B_x \\ C_y = 2A_y + B_y \end{cases}$$

The magnitude of $\vec{C}$ is then $C = \sqrt{C_x^2 + C_y^2}$ and its direction is found using $\tan^{-1}$.